Phys 471 – MODERN EXPERIMENTAL PHYSICS II

Lab 1 - Transmission Lines and Electronic Signal Handling

> The difference between electricity and electronics is the difference between a toaster and a television set.
>
> Isaac Asimov (1920 - 1992)

I. BACKGROUND

A great many modern physics experiments involve sending and receiving electronic signals, sometimes over long distances. An electronic signal can be analog, which is simply a voltage as a function of time (e.g. the basis of cable television), or digital, which consists of a series of voltage pulses. For low-speed applications over short distances, for example audio signals between a CD player and a set of speakers, one doesn't need to worry much about signal transmission - a cable can be thought of transmitting a signal instantaneously, so the voltage at one end is equal to the voltage at the other. However, electronic signals cannot propagate faster than the speed of light, roughly a meter in three nanoseconds, and at GHz frequencies we have to worry about signal transmission even over laboratory distances. For these high-speed and/or long-distance applications one must take into account the fact that the voltage on one end of a cable is not equal to the voltage at the other end, simply because it takes some time for any voltage changes (such as a pulse) to propagate along the cable. Thus we need to think of a cable as a transmission line.

It is worth noting at the beginning that essentially everything in this lab relating to electronic cables carries over pretty nicely to optical cables, *a.k.a.* fiber optics. Both transmit electromagnetic signals, and in both cases we will have to worry about how the signals are transmitted and whether pulses are reflected at the cable ends. Usually, signal reflections from the ends of cables are undesirable, and usually they can be avoided with a small amount of care.

There are many examples of transmission lines in common use. Twin lead, often used as television antenna cable, consists of two parallel conductors separated by a plastic spacer. A better system for transmitting radio frequency signals (up to about 10 GHz) is the coaxial cable. The coaxial cable has cylindrical symmetry: a center conductor is surrounded by a "shield" conductor which is usually at ground potential. The shield helps to reduce noise pickup in the signal line. These are like the cables used for cable television, and are often called BNC cables by physicists, after the Berkeley Nucleonics Corporation, which was a big manufacturer of coaxial cables and connectors in the 1960's. At microwave frequencies, 3 GHz ~ 300 GHz, radiation can be transmitted in hollow metal pipes called wave guides. Wave guides usually have a rectangular cross section whose sides are approximately one half wavelength (~1 mm ~ 10 cm) long. Finally, in the optical domain waves can be transmitted by the optical fiber mentioned above. Optical fiber is a transparent material which has a higher index of refraction than the surroundings so that propagating waves are totally internally reflected.

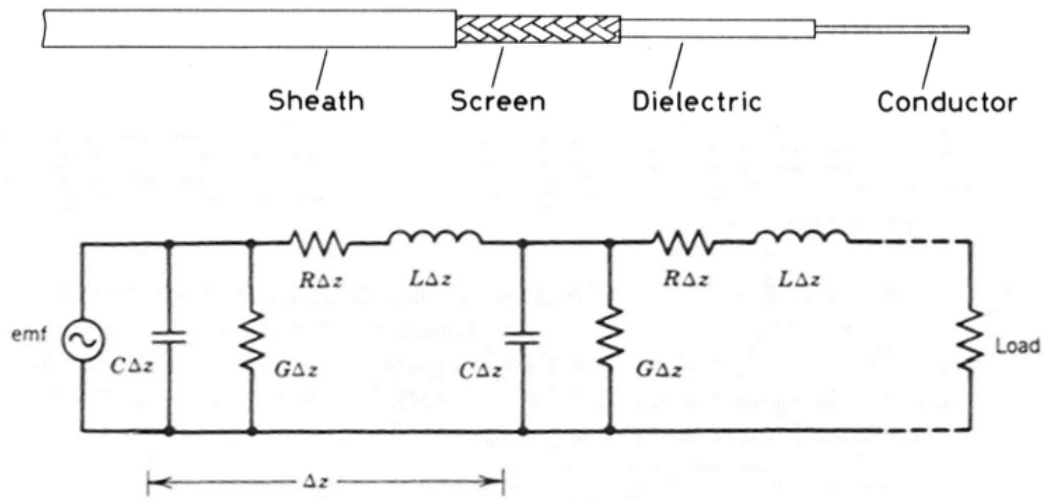


Figure 1. Schematic diagram of a coaxial cable (above), along with its equivalent circuit. (The convention here is that per unit length, R is the series resistance and G is the parallel conductance, so the parallel resistance is $1/G$. For a near-ideal cable both R and G are small).

Consider the coaxial cable shown in Fig 1. At low frequencies, a current flows down the center conductor and returns via the outer conductor. All we have to worry about is the resistance of the wire conductors (R in Fig. 1) and the cable capacitance, which is typically 50-150 picoFarads/metre. The cable also has some inductance L, but this is usually negligible at low frequencies (the inductive impedance, $Z = i\omega L$, being proportional to ω, gets smaller at low frequencies). At high frequencies, we cannot ignore the inductance or capacitance of the cable, and we have to solve the full system for the voltage and current as a function of both time and position along the cable.

THE IDEAL (LOSSLESS) COAXIAL CABLE.

If we ignore the series and parallel resistances in the cable ($R \rightarrow 0$ and $G \rightarrow 0$ in Fig. 1) and look at a small section in the middle of a long cable, then the voltage between the conductors is

$$V = \frac{q\Delta z}{C\Delta z} = \frac{q}{C}$$

where q is the charge per unit length of the conductors and C is the cable capacitance per unit length (see Fig. 1). The rate of change of charge is the current, so

$$\Delta I = \frac{\partial (q\Delta z)}{\partial t}$$

and thus

$$\frac{\partial I}{\partial z} = -\frac{\partial q}{\partial t}$$
$$= -C\frac{\partial V}{\partial t}.$$

The voltage drop along the cable comes from the cable inductance, giving

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t},$$

where L is now the cable inductance per unit length. Differentiating both of these expressions to eliminate I then yields

$$\frac{\partial^2 V}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 V}{\partial z^2}$$

which should be recognizable as a standard wave equation. The reader can verify that the same expression is obtained for I. Thus, the cable supports electromagnetic waves that propagate along the cable, at a propagation speed of

$$v_{prop} = \frac{1}{\sqrt{LC}}.$$

The complete solution of the wave equation yields forward and backward propagating waves

$$V(z,t) = F_+\left(t - \frac{z}{v}\right) + F_-\left(t + \frac{z}{v}\right)$$

where F_+ and F_- are arbitrary functions. The reader can show that with this $V(z,t)$ the corresponding current is

$$I(z,t) = \frac{V(z,t)}{Z_c}$$

where $Z_c = \sqrt{L/C}$ called the *characteristic impedance* of the cable, and for a lossless cable Z_C is purely resistive (i.e. it's a real number). Fig. 2 shows a cross section of a coaxial cable along with the electric and magnetic fields present when a sine wave signal is propagating down the cable.

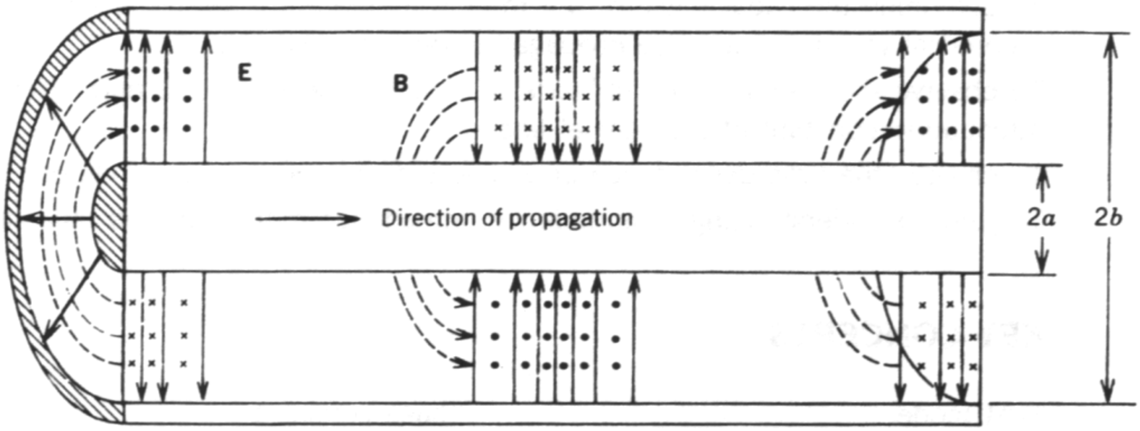


Figure 2. A section of a coaxial cable with the instantaneous electric and magnetic fields that are present when a sinusoidal signal is propagating down the cable. These fields move down the cable at the propagation speed.

The reader can also show (remember Phys 201) that the capacitance and inductance of the cable are

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} \text{ (F/m)}$$

$$L = \frac{\mu}{2\pi}\ln(b/a) \text{ (H/m)}$$

where *a* and *b* are shown in Fig. 2. Thus the velocity of transmission, at least for this ideal (lossless) cable, is equal to

$$v_{prop} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

Note, if there is no material between the (lossless) conductors, then $\mu=\mu_0$, $\varepsilon=\varepsilon_0$, and $v_{prop} = 1/\sqrt{\mu_0\varepsilon_0} = c$, the speed of light.

THE NOT-SO-IDEAL COAXIAL CABLE.

For the case that *R* and *G* are not negligible the equations are a bit more complex, and the wave equation for the signal voltage becomes

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} + (LG+RC)\frac{\partial V}{\partial t} + RGV$$

which is the most general wave equation for any transmission line having the equivalent circuit shown in Fig. 1. If we look for wave solutions to this equation, so that $V(z,t)=V(z)e^{i\omega t}$, then the general solution is

$$V(z,t) = V_1 e^{\alpha z} e^{i(\omega t + kz)} + V_2 e^{-\alpha z} e^{i(\omega t - kz)}$$

where

$$\alpha = \mathrm{Re}(\gamma)$$
$$k = \mathrm{Im}(\gamma)$$

with

$$\gamma = \sqrt{(R + i\omega L)(G + i\omega C)}.$$

We see the solution is a traveling wave with velocity $v = \omega/k$ that is substantially attenuated over distances larger than $\sim 1/\alpha$. The solution for the current is still

$$I(z,t) = \frac{V(z,t)}{Z_c}$$

where now the characteristic impedance is given by

$$\begin{aligned} Z_c &= \sqrt{\frac{R + i\omega L}{G + i\omega C}} \\ &\approx \sqrt{\frac{L}{C}}\left[1 + i\left(\frac{G}{2\omega C} - \frac{R}{2\omega L}\right)\right]. \end{aligned}$$

A major contributor to cable losses is the *skin effect*, which causes the current in the cable conductors to confine itself to a thinner and thinner layer near the conductor surface as the frequency is increased. The effective cross-sectional area of the conductor is then reduced, and the resistance subsequently increased, as the frequency increases. For a coaxial cable, this results in R being proportional to $\omega^{1/2}$, and so $\alpha \sim \omega^{1/2}$ as well. At higher frequencies, typically above 1 GHz, leakage across the dielectric can become the dominant loss mechanism, giving a nonzero G which is proportional to ω.

Whenever the cable parameters depend on frequency, we can find that the propagation velocity and/or attenuation also depend on frequency, which in turn results in *pulse dispersion or pulse distortion*. For most coaxial cables, C and L are essentially independent of frequency up to very high frequencies, so dispersion is not usually a huge problem. The attenuation does increase rapidly with frequency, however, which tends to round out the corners of square wave pulses, since the higher frequency components of the square wave damp away most quickly.

REFLECTIONS IN CABLE TRANSMISSION.

The above formalism for transmission down a coaxial cable included the implicit assumption that the properties of the cable, specifically C, L, etc., were independent of z, and in this case we found traveling wave solutions. When we consider the ends of the cable, we then have to add some boundary conditions to the differential equation, and doing this gives us reflections. That is, when a traveling wave pulse encounters the end of the cable (where it usually is connected to some other electronic device), some part of the pulse may get reflected and thus go traveling down the cable in the opposite direction.

Reflections occur whenever a traveling wave encounters a new medium in which the speed of propagation is different. For optical media, reflections occur whenever there is a change in the index of refraction. In coaxial cables, one gets reflections whenever there is a change in the characteristic impedance. To see how this works in detail, consider a long length of cable connected to a load with impedance Z_L, as shown in Figure 3. We can forget about the signal generator for now, and just look at the equations and boundary conditions at the load end of the cable. We will also neglect cable losses.

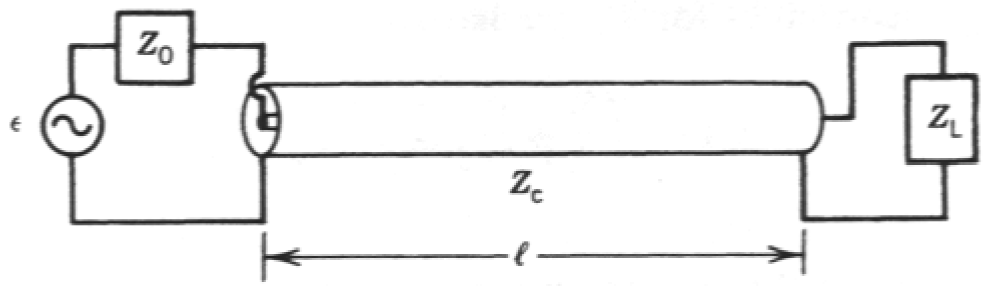


Figure 3. A signal generator (which is modeled here by a perfect voltage source ε in series with a "source impedance" Z_O) connected to a cable, which is in turn connected to a load with impedance Z_L .

With this, the general solution for a traveling wave in the cable is

$$V(z,t) = V_1 e^{i(\omega t - kz)} + V_2 e^{i(\omega t + kz)}$$

where V_1 is the amplitude of the incident wave (wave fronts move in the $+z$ direction) and V_2 is the amplitude of the reflected wave. At the end of the cable the load impedance demands that

$$Z_L = \frac{V(\ell,t)}{I(\ell,t)}$$

$$= Z_c\left(\frac{V_1 e^{i(\omega t - k\ell)} + V_2 e^{i(\omega t + k\ell)}}{V_1 e^{i(\omega t - k\ell)} - V_2 e^{i(\omega t + k\ell)}}\right).$$

(To see the sign flip in the denominator, recall from above that $\partial I/\partial z = -C\partial V/\partial t$ and integrate $\partial V/\partial t$ with respect to z. This is one of those subtle sign issues that had to be done right just once in order to get the right physics out.) Rearranging this gives

$$V_1 = e^{i2k\ell} V_2 \frac{Z_L + Z_C}{Z_L - Z_C}$$

so the ratio of the reflected wave to the incident wave is

$$\rho \equiv \frac{V_{reflected}(\ell,t)}{V_{incident}(\ell,t)} = \frac{V_2 e^{i(\omega t + k\ell)}}{V_1 e^{i(\omega t - k\ell)}} = \frac{Z_L - Z_C}{Z_L + Z_C}$$

where ρ is known as the reflection coefficient.

The above treats reflections at the load end of the cable. Looking at Fig. 3, we can also have leftward propagating pulses which reflect off the source. It can be shown that the reflection coefficient in this case is simply

$$\rho = \frac{Z_0 - Z_C}{Z_0 + Z_C}$$

IMPEDANCE MATCHING.

When the load impedance equals the cable impedance, $Z_L = Z_C$, then we have the desirable result that $\rho = 0$ and there is no reflected wave. In this case, we say the load is *impedance matched* to the cable, and all the electromagnetic energy in the incident wave (or pulse) is absorbed in the load. To see this, we consider the power dissipation in a terminating resistor, R_L. When there is a time-dependent voltage v across a resistor R_L, the power dissipated in the resistor is just

$$P = \frac{\langle v \rangle^2}{R_L}$$

where the brackets denote an RMS average. For the harmonically driven transmission line,

$$v = v(d,t) = A(1+r)e^{-ikd}e^{i\omega t}$$

which we can compute from our general solutions once we have an expression for the amplitude A. This is found by solving the loop equation for the input loop of Fig. 3. For $Zs = Zc$, which is a good approximation for our apparatus,

$$A = \frac{v_s}{2}.$$

From this we get

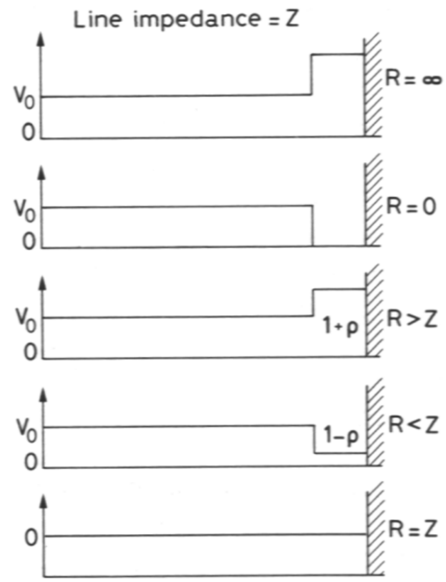
$$P = \frac{v_s^2}{2}\frac{R_L}{(R_L + Z_c)^2}.$$

This function approaches zero as R_L approaches zero or infinity, and has a broad maximum at $R_L = Z_C$. To achieve maximum power transfer from source to load, the load (and source) must be matched to the characteristic impedance of the line.

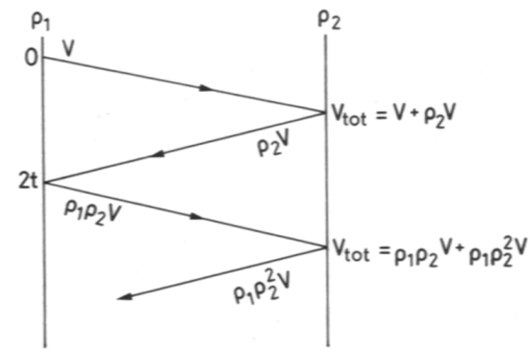
Many devices are designed to use cables with a fixed Z_C, with some standards being $Z_C = 50\ \Omega$ or $75\ \Omega$. When both the input and output devices are matched with a cable and with each other, then there are no reflected pulses to worry about. One common exception to this practice is the oscilloscope, which has an input impedance of typically 1 MΩ, far higher than any standard coax cable. When using an oscilloscope to look at a signal at the end of a cable, one can *terminate* the cable by adding an additional 50 Ω resistor (if $Z_C = 50\ \Omega$) in parallel with the oscilloscope. The oscilloscope hook-up then does not send reflections back to the source to cause trouble. (Note, a 50 Ω terminator is typically only used on an oscilloscope when one is in the high frequency/long distance regime, in which case a cable acts like a transmission line with 50 Ω impedance. For low-speed/short distance work, a cable acts like just a wire, and hooking a 50 Ω resistor to one's electronic device could cause a lot of trouble all by itself.)

REFLECTIONS UPON REFLECTIONS.

The different cases of pulse reflection off the end of a transmission line are shown in Figure 4, where we see the voltage at the input end of a cable (the left side in Figure 3) as a function of time. At $t = 0$, a step function signal was generated at this end of the cable, with amplitude V_0. While the voltage step propagates down the cable the first time, there are no reflections, so the voltage stays fixed at V_0. After a time $t = 2\tau$, where τ is the time for a pulse to travel the length of the cable, we see the effects of the reflected signal. When the load impedance is $R = \infty$ (top graph), the step reflects without changing sign, so after $t = 2\tau$ the reflected signal adds to the input voltage, giving $V \approx 2V_0$ (slightly less if the signal is attenuated in the cable).



◄ Reflections along a transmission line. Note how the voltage level is changed due to interference from the reflection



Lattice diagram for multiple reflections

Figure 4. On the left we see different cases of a step function voltage signal propagating down a line and reflecting from its end just after the edge reflected. With multiple reflections (right) the resulting signal can be quite complex.

When $R = 0$, the step changes sign upon reflection (remember Phys 112), and so the reflected signal destructively interferes with the input voltage, so after $t = 2\tau$ we have $V = 0$ (second graph). When $R = Z_C$ (bottom graph) there is no reflected step. If the reflected step reflects again off the left side of the cable and makes another trip, then the voltage after $t = 4\tau$ will change yet again. All these effects can be seen in the lab.

II. LABORATORY EXERCISES.

The first step in the lab is to checkout the signal generator and the oscilloscope to make sure that they work the way that you think they should (see the instrument manuals) in the absence of any complications coming from transmission lines.

Exercise 1. Use the *BK PRECISION* Model 4070A waveform generator to make a square wave with amplitude of 1 Volt and a frequency of 2 MHz. Send the output directly to the oscilloscope using a short (less than 0.5 m) RG-62/U BNC cable. Determine the signal period, the amplitude, and the rise time at the leading edge of the square wave (see Brophy, 4rd edition, pp. 211-213). Record a copy of the signal displayed on the oscilloscope. Given that the signal generator only goes up to 21.5 MHz, how does the measured rise time compare with expectations? Next, connect the signal generator to the oscilloscope with the spool of RG-174 cable to observe the degradation of the signal. Record a copy of this signal.

The next step is to add a transmission line using the connection panel provided (see Fig. 5). Make a 500 kHz square wave signal with 2V amplitude and 25% duty cycle, and connect the signal generator to the P0 input connector on the panel. Connect P1 and P2 with a ~5 m length of RG-62/U cable. Use the 10X oscilloscope probe to look at point P2 by clipping onto the wires on the back of the panel (see Appendix I).

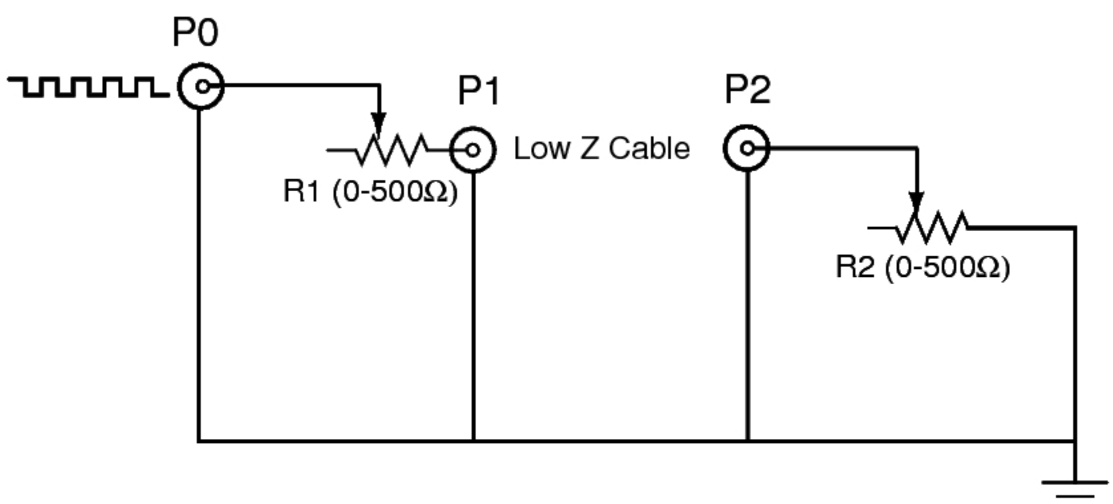


Figure 5. Schematic diagram of the connection panel.

Exercise 2. Short the far end of the cable at P2 by setting R2 to 0 Ω. (You can check any of the resistance values with an ohm-meter provided that you disconnect the input signal first.) Observe the signal at P1 as you vary the input impedance R1. Record copies of the signal for R1 set to 0, 20, 100, 250, 500 Ω, making sure that the voltage and time scales are clearly identified. Explain the different waveforms qualitatively. (You may be *brief* in your explanations, start by explaining the amplitude and sign of the voltage when $0<t<2\tau$ for each of the plots. Then explain the voltages when $2\tau<4\tau$, etc.)

Exercise 3. With the far end of the cable still shorted, set R1 so that there are minimal reflections at the input, i.e. the input impedance matches Z_c. (Since the characteristic impedance of the RG-62/U cable is only about twice of the signal generator, reflections at the input may be very small and hard to detect.) Measure the time between successive peaks of the short pulse and derive the pulse transmission speed from the known length of the cable. Measure the baseline signals before and after the short pulse, and from this estimate the attenuation constant α for the cable. In order to better compare your measurements to the manufacturer specifications, you may wish to repeat these measurements at several higher frequencies, adjusting the signal generator parameters as required to avoid overlap of successive pulses.

Note: the attenuation coefficient (see the Radiotron Designer's Handbook, 4th edition, RCA, 1952) is defined by

$$\ell\alpha = 20\log\left(\frac{V_{OUT}}{V_{IN}}\right),$$

where ℓ is the cable length, V_{IN} is the output voltage that would be delivered by the signal generator across an infinite impedance, and $V_{OUT}=V_{IN}-V_{LOAD}$ is the voltage delivered at the end of the cable. Therefore, the attenuation coefficient can be expressed as

$$\alpha = \frac{20}{\ell}\log\left(1-\frac{V_{LOAD}}{V_{IN}}\right).$$

Exercise 4. Disconnect the far end of the cable at P2 and observe the signal at P1 as you vary R1. Record the signal at roughly the same five values as in Exercise 2, and again qualitatively explain the observed signals.

Exercise 5. Reconnect the cable at P2 and observe the signal as both R1 and R2 are changed. Play with the settings in order to get a good impedance match at P2, and then measure the impedance-matched R2 value, which is a good estimate of Z_c. Compare this with the impedance-matched value of R1 from Exercise 3. With care, the results should be in reasonable agreement, but you may see the difference that comes about because R1 is in series with the output impedance of the signal generator. If the two values don't agree well, try both measurements again, but this time increase the gain on the oscilloscope to zoom in on the relevant part of the signal. Although there will be some ambiguity in what constitutes a zero reflected pulse, you can estimate the uncertainty in your measurement by measuring resistor values that are clearly a bit too high and clearly a bit too low, in addition to the impedance-matched value.

As a final step with this cable, look at the signal at P2 as R1 and R2 are varied. Note that a good pulse shape is obtained if reflections are suppressed at either end of the cable, as you would expect.

Exercise 6. Do similar measurements using RG-58/U and RG-174 cables, trying a variety of lengths of cable to check for systematic errors and improve the precision of your results. Use all your measurements to determine the characteristic impedance, the transmission speed, and the attenuation coefficient for each type of cable. Refer to the Belden Electronic Wire and Cable Catalog for the complete technical specifications of the cables.

REPORT. Your report should include answers to the questions in the text, the data and plots requested and a physical explanation of your results where appropriate.

III. REFERENCES.

J.J. Brophy, *Basic Electronics for Scientists*, 4th Edition, 1983, McGraw-Hill.
W.R. Leo, *Techniques for Nuclear and Particle Physics Experiments*, 2nd Edition, 1994, Springer-Verlag.
D.W. Preston and E.R. Dietz, *The Art of Experimental Physics*, 1991, John Wiley & Sons.

APPENDIX I - THE OSCILLOSCOPE PROBE

When using an oscilloscope to look at an electronic signal one often has to worry about whether the 'scope unintentionally affects the signal - an effect called *circuit loading*. A typical oscilloscope has an input resistance of about 1 MΩ and an input capacitance of about 20 pF, as shown in Figure 6. In other words, the minimum effect of hooking up a 'scope will be roughly like attaching the monitored circuit point to a 1 MΩ resistor to ground, in parallel with a 20 pF capacitor. When working with very high speed digital electronics, even this load could change the circuit performance, in which case you can't hook up a 'scope directly. But, for a great many applications 1 MΩ and 20 pF together don't draw much current, and the oscilloscope gives a fairly accurate reading of whatever signal is present in the circuit. (Occasionally, the author has found when debugging a circuit that the darn thing works when the 'scope is attached looking at the signal, but then stops working when the 'scope is removed. Replacing the scope with a small capacitor can then sometimes fix the circuit! About the only time this works is when the circuit gets stuck in some unwanted high-frequency oscillation since even a small capacitor to ground will suck the energy out of a high-frequency signal.)

Circuit loading can get much worse when a cable is inserted between the circuit and the 'scope since a typical BNC cable has a capacitance of around 50150 pF/m. When working with low frequency circuits, <1 MHz, circuit loading may not matter even with a good length of cable since the DC resistance is still equal to the 1 MΩ resistance of the oscilloscope and the impedance of the cable capacitance ($Z = 1/i\omega C$) might still be several kiloOhms, and this is maybe not a problem. But if your signals are at frequencies above 1 MHz, chances are a 'scope with a simple BNC cable will cause trouble. For such circumstances one uses an *oscilloscope probe*, which is a device designed to minimize unintentional circuit loading.

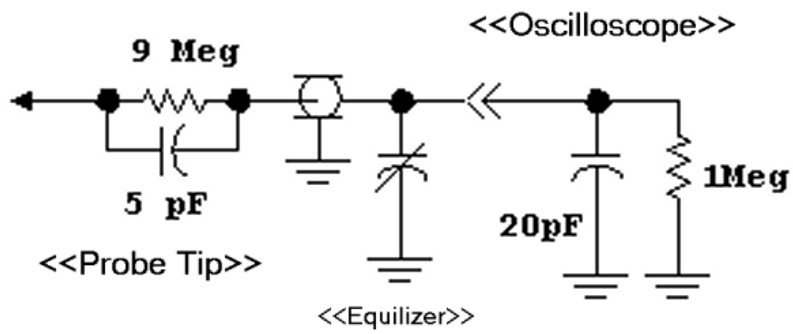


Figure 6. Schematic diagram of a typical oscilloscope *10X* probe. The probe tip contains a large resistor in parallel with a small stray capacitance. The oscilloscope (right part of drawing) looks to the outside world like a resistor and capacitor in parallel.

One of the most common oscilloscope probes is the *10X passive probe,* which we use in this lab and a circuit diagram for it is shown in Fig. 6. The basic idea of this kind of ‘scope probe is to put a large resistor, typically 9 MΩ, right at the tip of the probe so the cable capacitance is isolated from the circuit. There is always some capacitance at the probe tip, but some effort goes into making it small. Then, after a fixed length of low-capacitance cable, there is an equalizer box which connects to the 'scope input. Note, the probe resistance and the 'scope resistance make a resistor divider that reduces the signal amplitude by a factor of ten, which is why it s called a *10X* probe. By reducing the signal amplitude, one greatly reduces the circuit loading.

The equalizer box contains a small capacitor and perhaps a resistor (there are different versions), and is designed to work together with the cable and the 'scope to produce a constant *10X* reduction in signal amplitude, independent of frequency. The details of the design depend on the cable and the 'scope. Some probes are designed to work only with specific 'scopes, some probes can be adjusted for different 'scopes. A properly adjusted probe is said to be *frequency compensated*. Without compensation, a square wave signal (which contains lots of different frequencies) would appear as a distorted square wave on the oscilloscope.

In addition to the common *10X* passive probe, one also runs across *1X* passive probes. These are basically the same as *10X* probes but with a much smaller tip resistor and a low-capacitance cable. This probe provides essentially no reduction in signal, but of course there is greater circuit loading (but less loading than from a bare BNC cable). Finally, you can buy *active probes*, which as their name implies have high input impedance amplifiers in their tips. Active probes can give much better performance than passive probes, which is needed for very demanding applications (e.g. cell phone receivers and associated electronics, etc.).