Omega electroproduction simulations

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The installation of the physics simulations necessary for ω analysis into SIMC is rather straightforward. We need to choose the sampling phase space which fits our reaction channel, over a range appropriate to our needs and to carefully calculate the corresponding generation volume. We also require a simple physics model for the appropriate event weighting, and to calculate the necessary Jacobians.

1 Sampling in the Lab system

In our case, we have the reaction $p(e, e'p)X$, with e' in SOS, p in HMS, and X as the undetected recoil. The missing mass M_x spans the ω mass distribution, with the narrow ω sitting on top of a smooth background presumed to consist of $\pi\pi$ production phase space and ρ^0 production.

We choose the electron's xp, yp, $E_{e'}$ sampling parameters in the SOS (as is in the other channels), and choose the recoil mass M_x according to the ω^0 or ρ^0 Breit-Wigner distribution and the proton's xp, yp sampling parameters in the HMS. This gives the most similar sampling style to those already in SIMC for channels such as $p(e, e'\pi^+)n$, with the only exception being the incorporation of the recoil mass width.

For the phase space mechanism, M_x is unconstrained, so we instead choose the proton's xp, yp and E_p, in the same manner as is done for semi-inclusive production. In this case, the missing mass is calculated from the missing momentum and missing energy.

2 Calculation of the Jacobian

2.1 Jacobian for ω (or ρ) production

Because the meson is the recoil particle, the formula for t differs from the usual relation used in $(e, e'\pi)$. Because four momentum is conserved, we have

$$t = (p_\gamma - p_\omega)^2 = (p_{p1} - p_{p2})^2,$$

where p_{p1} and p_{p2} are the proton four momenta before and after the scattering. Working in the lab frame, we then obtain

$$t = 2m_p(m_p - E_p),$$

where E_p is the proton energy detected in the HMS.

For the ω and ρ generators, the cross section model is in the form of

$$\frac{d\sigma}{dt}$$

where t is the four momentum transfer between the photon and the meson.

Since all sampling is in the lab frame, we need a Jacobian (and a factor of Γ_t) to change the cross section model into the lab differential cross section for event weighting. Γ_t is the same as for any other channel, and only the Jacobian for the HMS arm needs to be considered separately here. Since M_x is sampled according to the Breit-Wigner distribution, we only need to change from variable t to $\cos\theta$ in the lab system. (Note that ϕ does not change between the CMS and lab systems, so we ignore it here). This is the same Jacobian as is used in the $p(e, e'\pi^+)n$ case

$$J = \frac{dt}{d\cos\theta_{pq}} = \frac{2qm_p p_p}{m_p + \nu - q\frac{E_p}{p_p}\cos\theta_{pq}} \tag{1}$$

where all quantities are in the lab frame.

2.2 Jacobian calculation for phase space production

For the phase space generator, two body phase space in the $p + X$ system yields [1]

$$\frac{d^2\sigma}{d\Omega_p^* dMx} = \frac{1}{32\pi^2}\frac{M_x}{|\vec{q}^*|}\frac{p_p^*}{W^2}$$

where M_x is the missing mass and the starred quantities are in the CM system. Since E_p is sampled in the lab frame, we need to find the Jacobian between variables (cosθ^*, M_x) and (cosθ, E_p). Using the relation,

$$\frac{\partial t}{\partial \cos\theta^*} = 2q^* p^* \tag{2}$$

we then still require the Jacobian between variables (t, M_x) and (cosθ, E_p) to complete the solution.

We evaluate

$$\frac{\partial(t, M_x)}{\partial(\cos\theta, E_p)} = \begin{vmatrix} \partial t/\partial\cos\theta & \partial t/\partial Ep \\ \partial Mx/\partial\cos\theta & \partial Mx/\partial Ep \end{vmatrix} = \frac{\partial t}{\partial\cos\theta}\left(\frac{\partial Mx}{\partial Ep}\right) - \frac{\partial t}{\partial Ep}\left(\frac{\partial Mx}{\partial\cos\theta}\right).$$

Since $t = 2m_p(m_p - E_p)$, we have

$$\frac{\partial t}{\partial\cos\theta} = 0, \qquad \frac{\partial t}{\partial E_p} = -2m_p.$$

In the lab system, we also have

$$M_x = \sqrt{-Q^2 + 2(m_p + \nu)(m_p - E_p) + 2qp_p\cos\theta},$$

so

$$\frac{\partial M_x}{\partial\cos\theta} = \frac{-qp_p}{M_x}.$$

Thus

$$\frac{\partial(t, M_x)}{\partial(\cos\theta, E_p)} = \frac{2m_p q p_p}{M_x}. \tag{3}$$

Dividing (3) by (2) gives the needed Jacobian

$$J = \frac{\partial(\cos\theta^*, M_x)}{\partial(\cos\theta, E_p)} = \frac{m_p q p_p}{M_x q^* p^*} \tag{4}$$

where the starred quantities are in the CMS.

3 Cross section models

For calculating the cross section in lab system, we have three subroutines: `physics_omega.f`, `physics_rho_recoil.f`, and `physics_Xphasespace.f`.

Because the mechanism of producing the ω (or ρ) changes drastically from one kinematics to another, we may need to use a variety of different physics models or theoretical formulas. Each model is in a separate subroutine so that in the future, if the users want to change physics model, they only need to change the call to the appropriate routine. Depending on the form of the model formula, one needs only to choose the correct Jacobian and "insert" the desired model.

3.1 One Pion Exchange model for ω Electroproduction

This model is based upon ω photoproduction results and was extended for electroproduction and compared to the first DESY ω electroproduction experiment [2]. The implementation of this model in SIMC was based directly upon the code by Pawel Ambrozewicz [1], and is found in the subroutine `sig_joos`.

The model cross section consists of transverse and longitudinal parts

$$\frac{d^2\sigma}{dtd\phi} = \frac{1}{2\pi}\Big(\frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt}\Big),$$

each of which are complicated functions of Q^2, W, t and u. This is then multiplied by the normalized relativistic Breit-Wigner factor

$$a_l^2 = \frac{2}{\pi}\frac{m_\omega^2\Gamma_\omega}{(M_x^2 - m_\omega^2)^2 + m_\omega^2\Gamma_\omega^2}$$

to yield the M_x weighted triple differential cross section $d^3\sigma/dt/d\phi/dM_x$. This model was used in the cross section extraction of the Hall C experiment at $Q^2 \sim 0.5$ GeV^2, $W = 1.785$ GeV. A comparison of this model with the Hall C and DESY experimental cross sections is shown in Fig. 1.

In our SIMC implementation, the Jacobian factor in Eqn. 1 and the transverse photon flux factor Γ_t are applied to give the 6-fold lab differential cross section $d^6\sigma/d\Omega_p/dE_p/d\Omega_{e'}/dE_{e'}$.

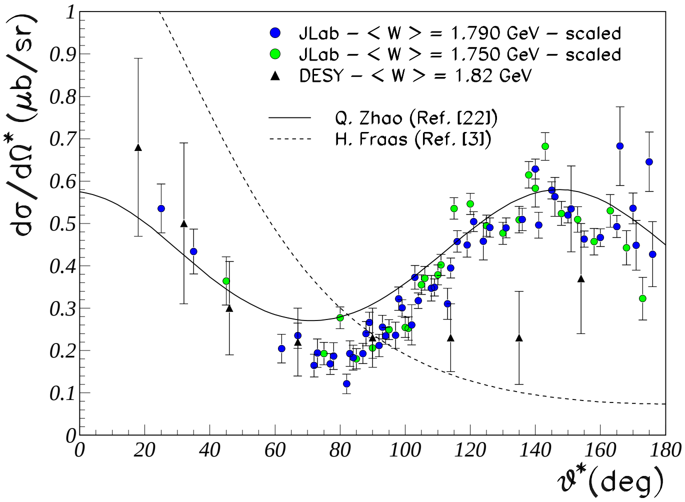

Figure 1: Comparison of the Fraas One Pion Exchange model (dotted curve) with JLab Hall C and DESY ω electroproduction data. This model was used by the Hall C experiment to extract the cross sections from their data. The Zhao model (solid curve) provides a better description of the data, but is unpublished and so is not incorporated in any MC cross section models. This figure is from Ref. [3].

3.2 Regge model inspired ω cross sections

This model is inspired by the Regge model calculation of J.M. Laget [4]. Since we do not have access to his ω electroproduction code, the model is based upon a parameterization of his published response functions at $Q^2 = 2.35$ GeV^2, $W = 2.47$ GeV (Fig. 2), and is found in the subroutine `sig_gmh`.

In his model, Laget has replaced the usual pion Regge trajectory with one which saturates at large negative t (Fig. 3). This means that instead of the usual Q^2 scaling,

$$F_{\pi\omega\gamma}(Q^2) = \frac{1}{1 + \frac{Q^2}{\Lambda_0^2}}$$

with $\Lambda_0^2 = 0.462$ GeV^2, we instead have a more complicated scaling as a function of both Q^2 and t

$$F_{\pi\omega\gamma}(Q^2, t) = \frac{1}{1 + \frac{Q^2}{\Lambda_\pi^2(t)}},$$

where

$$\Lambda_\pi^2(t) = \Lambda_0^2 \times \left(\frac{1 + \alpha_\pi(0)}{1 + \alpha_\pi(t)}\right)^2.$$

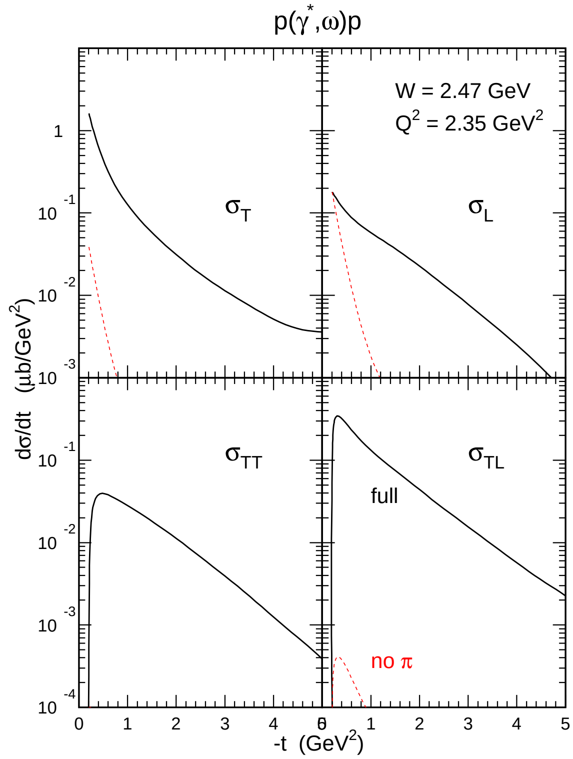

Figure 2: J.M. Laget's ω electroproduction cross sections. The dashed curves are the contribution of two gluon and f_2 meson exchange amplitudes only. The solid curves are the full calculation, including π meson exchange contributions. This figure is from Ref. [4].

$F_{\pi\omega\gamma}(Q^2, t)$ becomes independent of Q^2 at large $-t$.

Unfortunately, we do not know the function Laget used to saturate the Regge trajectory, but the following seems to reproduce his plot in Ref. [5]:

$$\alpha_\pi(t) = \begin{cases} 0.7(t - m^2_{\pi^o}) & t \geq m^2_{\pi^o}, \\ tanh(0.7(t - m^2_{\pi^o})) & t < m^2_{\pi^o}. \end{cases} \tag{5}$$

After dividing the σ_{TT} and σ_{TL} in Fig. 2 by $\sin\theta^*$ and $\sin^2\theta^*$, respectively, all four response functions were then divided by $F^2_{\pi\omega\gamma}(Q^2 = 2.35, t)$ and individually fitted

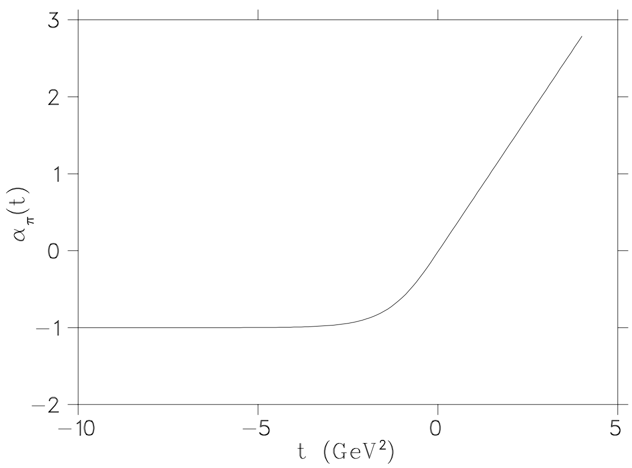

Figure 3: Saturated π^0 Regge trajectory as given by Eqn. 3. $\alpha(t) \rightarrow -1$ as $t \rightarrow -\infty$.

with the following functions.

$$\begin{cases} \sigma_X = ae^{-bt'} + ce^{-d\sqrt{t'}} + fe^{-gt'^2} & X = T \\ \sigma_X = ae^{-bt'} + ce^{-d\sqrt{t'}} + fe^{-gt'^{1/4}} & X = L \\ \sigma_X = ae^{-bt'} + ce^{-d\sqrt{t'}} & X = TT, LT \end{cases}$$

where $t' = abs(t) - abs(t_{min})$.

To construct the differential cross section at the desired kinematics, we scale in Q^2 according to $(F_{\pi\omega\gamma}(Q^2,t)/F_{\pi\omega\gamma}(2.35,t))^2$ and in W according to an assumed factor of $((2.47^2 - m_p^2)/(W^2 - m_p^2))^2$. They are then combined to form[1]

$$\frac{d^2\sigma}{dtd\phi} = \frac{1}{2\pi}\Big(\sigma_T + \epsilon\ \sigma_L + \epsilon\ \cos 2\phi\ \sigma_{TT}\ \sin\theta^* + \sqrt{2\epsilon(1+\epsilon)}\ \cos\phi\ \sigma_{LT}\ \sin^2\theta^*\Big).$$

After multiplication by the normalized relativistic Breit-Wigner factor, the Jacobian factor in Eqn. 1 and the transverse photon flux factor Γ_t, we obtain the 6-fold lab differential cross section $d^6\sigma/d\Omega_p/dE_p/d\Omega_{e'}/dE_{e'}$.

The resulting cross sections closely resemble the curves in Fig. 2. Comparing to the DESY data and Laget's model calculation at $Q^2 = 0.84$ GeV^2, $W = 2.30$ GeV, this model is 20% high at $-t = 0.1$, and 300-500% high between $-t = 1$ and 3 GeV^2. Thus, this subroutine is optimized for $Q^2 > 2$, $W > 2$.

[1]Note that Laget uses $-\sqrt{\epsilon(1+\epsilon)}$ in front of his σ_{LT} so his cross sections are corrected accordingly.

3.3 Background ρ^0 model

In addition to the phase space model, we also implement the diffractive ρ^0 model used by P. Ambrozewicz [1] to account for the smooth background underneath the ω peak. The differential cross section is of the form

$$\frac{d\sigma}{dtdM_x} = \left(\frac{m_\rho}{M_x}\right)^n De^{bt'}.$$

The factor $(\frac{m_\rho}{M_x})^n$ is a Soding-model skewing correction to the ρ^0 Breit-Wigner mass distribution, due to the interference between resonant and non-resonant pion pair production.

4 Acknowledgements

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