Reggeized models for electroproduction of pion at forward angles

Byung Geel Yu,[1,*] Tae Keun Choi,[2] and Kook Jin Kong[1]
[1]*Research Institute of Basic Sciences, Korea Aerospace University, Koyang 412-791, Korea*
[2]*Department of Physics, Yonsei University, Wonju 412-791, Korea*
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I. ELECTROPRODUCTION OF PSEUDOSCALAR MESON

1. Kinematics and differential cross sections

We briefly introduce the kinematics in the exclusive π electroproduction,

$$e(\kappa) + N(p) \to e(\kappa) + N(p') + \pi(q), \tag{1}$$

and denote the notations and definitions of variables. The mandelstam variables for the reaction in the three channel are

$$\begin{aligned} s &= (p+k)^2 = (p'+q)^2, \\ t &= (q-k)^2 = (p-p')^2, \\ u &= (p-q)^2 = (p'-k)^2 \end{aligned} \tag{2}$$

with the virtual photon momentum $k^2 = \kappa'^2 - \kappa^2$ in terms of the initial and the final electron momenta κ and κ'. The virtuality of photon is given by $Q^2 = -k^2$.

In the electroproduction fully expressed in Eq. (1) where the target nucleon is at rest, the z-axis is directed along the three momentum of the virtual photon γ^* which is defined as $\vec{k} = \left(0,\, 0,\, \sqrt{\nu^2 + Q^2}\right)$ with the virtual photon four-momentum $k = \kappa - \kappa' = \left(\nu,\, \vec{k}\right)$, $\nu = E_e - E_e'$ in the laboratory system.

More variables are the Bjorken scaling variable $x_B = Q^2/2M\nu$ as the fraction of the target nucleons momentum carried by the struck quark, and the invariant energy in the γ^*-N system, $W^2 = M^2 + 2M\nu - Q^2 = M^2 + Q^2\left(\frac{1}{x_B} - 1\right)$.

The S-matrix element for electroproduction is[1]

$$S = \frac{1}{(2\pi)^{7/2}} \frac{m_e\sqrt{MM'}}{\sqrt{2E_\gamma q_0 E_e E_e' E E'}} i\delta^4(p+k-p'-q)\mathcal{M}_{fi} \tag{3}$$

$$\frac{d^3\sigma}{dE_e' d\Omega_e' d\Omega_\pi} = \Gamma \frac{d\sigma}{d\Omega_\pi} \tag{4}$$

where the flux of the electron current is given by

$$\Gamma = \frac{\alpha}{2\pi^2}\frac{E_e'}{E_e}\frac{K_H}{Q^2}\frac{1}{1-\epsilon}, \tag{5}$$

with the equivalent photon lab-energy $K_H = \frac{W^2 - M^2}{2M}$.

The ratio of the longitudinal to the transverse polarization of the virtual photon is

$$\epsilon = \left[1 + 2\frac{\nu^2 + Q^2}{4E_e\left(E_e - \nu\right) - Q^2}\right]^{-1} \tag{6}$$

This is rewritten as

$$\epsilon = \frac{1 - y - \frac{Q^2}{4E_e^2}}{1 - y + \frac{y^2}{2} + \frac{Q^2}{4E_e^2}} \tag{7}$$

in terms of $y = \nu/E_e$.

The longitudinal component $\epsilon_L = \left(Q^2/\nu^2\right)\epsilon$. In terms of electron scattering angle, the electron polarization $\epsilon = \left(1 + \frac{2|\vec{k}|^2}{Q^2}\tan\frac{1}{2}\theta_e\right)^{-1}$ measured by the angle θ_e deviated from the reaction plane.

Separating the kinematical part of electron scattering in Eq. (3) the full cross section for virtual photoproduction

$$\gamma^*(k) + N(p) \to N(p') + \pi(q) \tag{8}$$

is expressed as follows[2],

$$\frac{d\sigma_{\rm tot}}{d\Omega_\pi} = \frac{d\sigma_U}{d\Omega_\pi} + \epsilon\frac{d\sigma_{TT}}{d\Omega_\pi}\cos 2\Phi_\pi + \sqrt{2\epsilon(\epsilon+1)}\,\frac{d\sigma_{TL}}{d\Omega_\pi}\cos\Phi_\pi, \tag{9}$$

where the unpolarized cross section is given by

$$\frac{d\sigma_U}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \epsilon\frac{d\sigma_L}{d\Omega_\pi}, \tag{10}$$

and $\frac{d\sigma_T}{d\Omega_\pi}$, $\frac{d\sigma_L}{d\Omega_\pi}$, $\frac{d\sigma_{TT}}{d\Omega_\pi}$, and $\frac{d\sigma_{LT}}{d\Omega_\pi}$ are the transverse, longitudinal, transverse-transverse, and longitudinal-transverse cross sections, respectively.

* bgyu@kau.ac.kr

[1] For photoproduction the S-matrix element is given by

$$S = \frac{1}{(2\pi)^2}\sqrt{\frac{MM'}{4E_\gamma q_0 E E'}}\, i\delta^4(p+k-p'-q)\mathcal{M}_{fi}$$

[2] Note that the CEA [?] and the Cornell experiments [? ?] follow the convention for the polarization factor $d\sigma_{LT}$ cross section in Eq. (9),

$$\sqrt{\epsilon(\epsilon+1)/2}\,\frac{d\sigma_{TL}}{d\Omega_\pi}\cos\Phi_\pi.$$

2. Reactions

For completeness we consider in this work the reaction proceeding via four channels as below.

$$\gamma^* p \to \pi^+ n\,, \tag{11}$$
$$\gamma^* n \to \pi^- p\,, \tag{12}$$
$$\gamma^* p \to \pi^0 p\,, \tag{13}$$
$$\gamma^* n \to \pi^0 n\,, \tag{14}$$

II. YCK MODEL

We utilize the effective Lagrangians for the construction of the Born approximation amplitude to one-photon exchange at the tree level. It is, then, extended to reggeize the meson exchange in the t-channel, following the prescription in Ref. [? ?].

For the charged pion process, the charge conjugation C-even exchange in the t channel is allowed with the sign of the photon-meson coupling constant determined from the G-parity counting. Thus, the G-parity-even exchanges $\pi^\pm$, $a_1^\pm$, and $a_1^\pm$ have the opposite signs when coupling to photon between π^+ and π^- reactions. However, in order to avoid confusion caused by the signs of the coupling constants when considering the phase of the Regge propagators between exchange-degenerate pair $\{\rho - a_2\}$, for instance, we will write the signs in absolute values in $g_{\gamma\pi a_1}$ and $g_{\gamma\pi a_2}$, with the sign-opposite only appearing in the pion exchanges in the production amplitudes below. The signs of a_1 and a_2 coupling constants will then be included in the amplitudes in Eqs. (22) and (??) later, which are the final expressions containing the phases desired for the current calculation.

Therefore, the reggeized amplitudes for charged pion electroproduction are written as [?],

$$\mathcal{M}_{\pi^+n} = i\sqrt{2}\, e g_{\pi NN}\bar{u}'(p')\left[\widetilde{F}_\pi(Q^2)\frac{(2q-k)\cdot\epsilon}{t-m_\pi^2}\gamma_5 + \gamma_5\frac{(\not p + \not k + M_p)}{s-M_p^2}\Gamma_{\gamma^* NN}(Q^2)\right]u(p)(t-m_\pi^2)\mathcal{R}^\pi(s,t)\,, \tag{15}$$

$$\mathcal{M}_{\pi^-p} = -i\sqrt{2}\, e g_{\pi NN}\bar{u}'(p')\left[\widetilde{F}_\pi(Q^2)\frac{(2q-k)\cdot\epsilon}{t-m_\pi^2}\gamma_5 - \Gamma_{\gamma^* NN}(Q^2)\frac{(\not p - \not q + M_p)}{u-M_p^2}\gamma_5\right]u(p)(t-m_\pi^2)\mathcal{R}^\pi(s,t)\,, \tag{16}$$

$$\mathcal{M}_V = \frac{g_{\gamma\pi V}}{m_0}F^\rho(Q^2)\varepsilon_{\mu\nu\alpha\beta}\epsilon^\mu k^\nu q'^\alpha(-g^{\beta\rho} + q'^\beta q'^\rho/m_V^2)\bar{u}'(p')\left[g^v_{VNN}\,\gamma_\rho + i\frac{g^t_{VNN}}{2M}\sigma_{\lambda\rho}q'^\lambda\right]u(p)\mathcal{R}^V(s,t)\,, \tag{17}$$

$$\mathcal{M}_T = \frac{2g_{\gamma\pi T}}{m_0^2}\,F^T(Q^2)\varepsilon_{\mu\nu\alpha\beta}\,\epsilon^\mu\, k^\nu q'^\alpha q_\rho \Pi^{\beta\rho;\lambda\sigma}(q-k)\bar{u}'(p')\left[\frac{2g^{(1)}}{M}(\gamma_\lambda P_\sigma + \gamma_\sigma P_\lambda) + \frac{4g^{(2)}}{M^2}P_\lambda P_\sigma\right]u(p)\mathcal{R}^T(s,t) \tag{18}$$

$$\mathcal{M}_A = i\frac{g_{\gamma\pi A}}{m_0}F^A(Q^2)(k\cdot q'\,\epsilon_\mu - \epsilon\cdot q'\,k_\mu)(-g^{\mu\nu} + q'^\mu q'^\nu/m_A^2)g^v_{ANN}\,\bar{u}'(p')\gamma_\nu\gamma_5 u(p)\mathcal{R}^A(s,t)\,, \tag{19}$$

$$\mathcal{M}_B = i\frac{g_{\gamma\pi B}}{m_0}F^B(Q^2)(k\cdot q'\,\epsilon_\mu - \epsilon\cdot q'\,k_\mu)(-g^{\mu\nu} + q'^\mu q'^\nu/m_B^2)\bar{u}'(p')i\frac{g^t_{BNN}}{2M}\sigma_{\lambda\nu}q'^\lambda\gamma_5 u(p)\mathcal{R}^B(s,t)\,, \tag{20}$$

where the $\sqrt{2}$ factor is understood in the meson sector in which vector, tensor, and axial mesons are written by the representative symbols for charged reactions. The mass parameter $m_0 = 1$ GeV and the t-channel momentum transfer $q' = q - k$, and momentum sum $P = \frac{1}{2}(p + p')$.

The tensor meson spin-2 projection is given by $\Pi^{\beta\rho;\lambda\sigma}(q') = \frac{1}{2}(\eta^{\beta\lambda}\eta^{\rho\sigma} + \eta^{\beta\sigma}\eta^{\rho\lambda}) - \frac{1}{3}\eta^{\beta\rho}\eta^{\lambda\sigma}$ with $\eta^{\mu\nu} = -g^{\mu\nu} + \frac{q'^\mu q'^\nu}{m_T^2}$.

The Regge propagator for an exchanged meson is written collectively for $\varphi(= V,\, T,\, A\, B)$,

$$\mathcal{R}^\varphi(s,t) = \frac{\pi\alpha'_\varphi \times \text{phase}}{\Gamma(\alpha_\varphi(\text{t}) + 1 - \text{J})\,\sin(\pi\alpha_\varphi(\text{t}))}\left(\frac{s}{s_0}\right)^{\alpha_\varphi(s)-J}, \tag{21}$$

where the Regge phase $\frac{1}{2}((-1)^J + e^{-i\pi\alpha_\varphi(t)})$ in the canonical form is assigned to the spin-J meson for the exchange-nondegenerate case.

The Regge phase in production channels

The Regge phases for the electroproduction amplitudes from Eqs. (15) to (20) for charged reactions are chosen the same as in the case of photoproduction [?] which is expressed in schematic notation as follows,

$$\begin{aligned}\mathcal{M}(\gamma^* N \to \pi^\pm N) = \pm ie\pi &\begin{Bmatrix} e^{-i\pi\alpha_\pi} \\ 1 \end{Bmatrix} + b_1 \begin{Bmatrix} e^{-i\pi\alpha_{b_1}} \\ -1 \end{Bmatrix} \\ &+\rho \begin{Bmatrix} -e^{-i\pi\alpha_\rho} \\ 1 \end{Bmatrix} + a_2 \begin{Bmatrix} -e^{-i\pi\alpha_{a_2}} \\ -1 \end{Bmatrix} \\ &\pm a_1 \frac{1}{2}\left(-1 + e^{-i\pi\alpha_{a_1}}\right),\end{aligned} \tag{22}$$

where the $\sqrt{2}$ is omitted for brevity.

Gross-Riska prescription for nucleon and pion electromagnetic form factors

The nucleon charge form factor in the electromagnetic vertex given by

$$\Gamma_{\gamma^* NN}(k^2) = \widetilde{F}_1^N(k^2)\not\epsilon - \frac{\kappa_N}{4M}F_2^N(k^2)\,[\not\epsilon, \not k] \tag{23}$$

and pion charge form factor are chosen by following the Gross-Riska prescription

$$\widetilde{F}_1^N(k^2)\not\epsilon = F_1^N(k^2)\left(\not\epsilon - \not k\frac{\epsilon\cdot k}{k^2}\right) + F_1^N(0)\not k\frac{\epsilon\cdot k}{k^2}\,, \tag{24}$$

$$\widetilde{F}^\pi(k^2)(2q-k)\cdot\epsilon = F^\pi(k^2)(2q-k)\cdot\left(\epsilon - k\frac{\epsilon\cdot k}{k^2}\right) + F^\pi(0)(2q-k)\cdot k\frac{\epsilon\cdot k}{k^2}\,. \tag{25}$$

In the following, we suggest two versions of the YCK model, where the YCK1 employs the GPDs of the nucleon EM form factors F_1 and F_2, while the YCK2 introduces the dipole type with the cutoff mass $\Lambda_1 = \Lambda_2 = 1.55$ GeV to these form factors.

A. YCK1: The Reggeized model with GPDs for proton EMFFs

For studying the partonic aspect of the reaction in the DIS regime, $Q^2 \geq 1$ GeV^2 and $W \geq 2$ GeV over the resonance region, we consider the case where the nucleon EMFF form factors parameterized by the GPDs in the $\gamma^* NN$ vertex which are written as,

$$F_1^p(Q^2) = \int_0^1 dx\,[e_u u_v(x) + e_d d_v(x)]\, x^{\alpha'_q(1-x)Q^2}, \tag{26}$$

$$F_1^n(Q^2) = \int_0^1 dx\,[e_d u_v(x) + e_u d_v(x)]\, x^{\alpha'_q(1-x)Q^2}, \tag{27}$$

$$F_2^p(Q^2) = \frac{1}{\kappa_p}\int_0^1 dx\left[\frac{e_u\kappa_u}{N_u}(1-x)^{\eta_u}u_v(x) + \frac{e_d\kappa_d}{N_d}(1-x)^{\eta_d}d_v(x)\right]x^{\alpha'_q(1-x)Q^2}, \tag{28}$$

$$F_2^n(Q^2) = \frac{1}{\kappa_n}\int_0^1 dx\left[\frac{e_d\kappa_u}{N_u}(1-x)^{\eta_u}u_v(x) + \frac{e_u\kappa_d}{N_d}(1-x)^{\eta_d}d_v(x)\right]x^{\alpha'_q(1-x)Q^2}\,. \tag{29}$$

Valence quark charge and anomalous magnetic momenta are $e_u = 2/3$, $e_d = -1/3$, $\kappa_u = 1.673$, and $\kappa_d = -2.033$, respectively. $\eta_u = 1.713$ and $\eta_d = 0.566$ from a fit to the nucleon form factor data. We make the slope parameter adjusted to obtain $\alpha'_q = 0.3$ GeV^{-2} which yields the same result from the $\Lambda_1^p = 1.55$ GeV.

The unpolarized valence quark densities are from Ref. Martin2002;

$$u_v(x) = 0.262x^{-0.69}(1-x)^{3.5}(1+3.83x^{0.5}+37.65x), \tag{30}$$

$$d_v(x) = 0.061x^{-0.65}(1-x)^{4.03}(1+49.05x^{0.5}+8.65x) \tag{31}$$

at the input scale $\mu^2 = 1$ GeV^2. For simplicity, the zero-skewness GPDs are implemented in the $\gamma^* NN$ vertex form factors throughout the work.

- In epion-tch-v2.exe, the slope $\alpha'_q = 0.3$ is given as an input with the F_2 able to be turned on with the same slope α'_q chosen automatically. The pion cutoff $\Lambda_\pi = 775$ MeV. The positive sign of a_1 exchange is chosen. The output is read as the BSA.dat for both the t-dependence and Q^2-dependence

B. YCK2: The conventional Reggeized model

The proton electromagnetic form factors (EMFF) are considered in charged pion processes, which are expressed as

$$F_1^p(Q^2) = \frac{1+\tau\,\mu_p}{1+\tau}\left(1+Q^2/\Lambda_1^2\right)^{-2}, \tag{32}$$

$$\kappa_p F_2^p(Q^2) = \frac{\mu_p - 1}{1+\tau}\left(1+Q^2/\Lambda_2^2\right)^{-2} \tag{33}$$

with $\tau = Q^2/4M_P^2$ and $\Lambda_2 = \Lambda_1$.

In the description of the reaction cross sections model prediction with the magnetic term is worse than without it. Thus, it is usually neglected.

- In epion-tch-v2.exe, the cutoff $\Lambda_1 = 1.55$ GeV is given as an input with the F_2 turned on with cutoff $\Lambda_2 = \Lambda_1$ chosen automatically. The pion cutoff $\Lambda_\pi = 775$ MeV. The positive sign of a_1 exchange is chosen. The output is read as the BSA.dat for both the t-dependence and Q^2-dependence

C. Meson sectors

In both approaches the monopole type of charge form factor is shared for the meson exchange in common,

$$F_\varphi(Q^2) = \left(1+Q^2/\Lambda_\varphi^2\right)^{-1} \tag{34}$$

for $\varphi = \pi$, V, T, A, B and h_1,

For the pion exchange, the cutoff mass

$$\Lambda_\pi = 0.775 \text{ GeV} \tag{35}$$

is chosen for the pion charge form factor. In the Particle Data Group, it is reported that mesons of high spin are decaying to $a_1 \to \rho\pi$, $b_1 \to \omega\pi$, and $h_1 \to \rho\pi$, respectively. According to the Vector Meson Dominance, $a_1 \to \rho\pi \to \gamma\pi$, for instance, this leads to

$$\Lambda_{a_1} = 0.775 \text{ GeV}\,,\ \Lambda_{b_1} = 0.782 \text{ GeV}\,,\ \Lambda_{h_1} = 0.775 \text{ GeV}\,, \tag{36}$$

respectively. For the tensor meson a_2 exchange, we assume $\rho(1450) \to a_2\pi$ rather than the $\rho(775)$ and take the cutoff mass $\Lambda_{a_2} = 1.45$ GeV in order to assign its role similar to that in the case of photoproduction [**?**].

The role of the axial meson $a_1(1260)$ affects the SSA as well as the ratio of the cross sections $d(\pi^-p)/d(\pi^+n)$, so that the change of its sign leads to the different results. We once predicted $d\sigma_{LT'}/d\sigma_U$ with the sign of the a_1 positive. But the case of a_1 with the sign negative results in the better description of $d\sigma_{LT'}/d\sigma_U$ and the cross section ratio as well.

The LT' component of the cross section relevant to the beam polarization asymmetry for the virtual photon is calculated as

$$\frac{d\sigma_{LT'}}{d\sigma_U}, \tag{37}$$

and the results are compared with the data measured from the KaonLT BSA experiment.

In the next section, the KM and VR models are presented with our model BTK for comparison. In the epion-km-vr-btk-bsa.f90, only the nucleon charge form factor F_1 is taken in three model calculations with the F_2 excluded in common. The KM and VR models reproduce the BSA in this code to yield BSA_t.dat and BSA_QQ.dat as the outputs for the t- and Q^2-dependences of the BSA. But, the btk model in this code is simply an extension of the JKPS model, and hence, results in the failure of the BSA, because of the absence of the nucleon F_2 FF. For the case of reproducing the BSA in our model, only the epion-tch-v2 model is valid for both nucleon form factors F_1 and F_2 parameterized by GPDs (YCK1), or by the dipole form factors (YCK2).

III. KM MODEL

The gauge prescription for the charged pion exchange with the nucleon pole term by the KM model is the same with ours. The full description of the model is given in the paper, Murat M. Kaskulov and Ulrich Mosel, *Deep exclusive charged electroproduction above the resonance region*, Phys. Rev. C **81**, 045202 (2010).

The model is featured by the nucleon form factor

$$F_s(Q^2,s) = \frac{s\,\ln\left[\frac{\xi Q^2}{M_p^2}+1\right]\frac{2\xi Q^2+s}{\xi^2 Q^4} - \frac{s(\xi Q^2+s)}{\xi Q^2(\xi Q^2+M_p^2)} + \ln\left[\frac{s-M_p^2}{M_p^2}\right] - i\pi}{\left(\frac{\xi Q^2}{s}+1\right)^2\left(\frac{s^2+2sM_p^2}{2M_p^4} + \ln\left[\frac{s-M_p^2}{M_p^2}\right] - i\pi\right)} \tag{38}$$

$$F_u(Q^2,u) = \frac{u\,\ln\left[\frac{\xi Q^2}{M_p^2}+1\right]\frac{2\xi Q^2+u}{\xi^2 Q^4} - \frac{u(\xi Q^2+u)}{\xi Q^2(\xi Q^2+M_p^2)} + \ln\left[\frac{M_p^2-u}{M_p^2}\right]}{\left(\frac{\xi Q^2}{u}+1\right)^2\left(\frac{u^2+2uM_p^2}{2M_p^4} + \ln\left[\frac{M_p^2-u}{M_p^2}\right]\right)} \tag{39}$$

with $\xi = 0.4$ for the nucleon charge form factor F_1 in the s- and u-channels, respectively.

For the pion exchange, the cutoff mass

$$\Lambda_\pi = 0.68 \text{ GeV} \tag{40}$$

is chosen for the monopole form factor in Eq. (34), averaging over the scattered values $\Lambda_\pi = 0.768$ GeV for $Q^2 < 0.4$ GeV^2, $\Lambda_\pi = 0.632$ for $0.6 < Q^2 < 1.5$ GeV^2 and $\Lambda_\pi = 0.678$ GeV at high Q^2. To account for the anti-shrinkage effect further, due to the decrease of the slope of the parton contribution with increasing Q^2, the slope of the pion trajectory is modified to be

$$\alpha'_\pi = \frac{0.74}{1+2.4\frac{Q^2}{s}} \text{ GeV}^{-2}\ . \tag{41}$$

The beam spin azimuthal (BSA) moment is defined as

$$A_{LU}(\phi) = \sqrt{2\epsilon(1-\epsilon)}\,\frac{d\sigma_{LT'}}{d\sigma_U}, \tag{42}$$

where the virtual photon polarization for the ratio of the longitudinal to the transverse component ϵ is given in Eq. (6).

The $A_{LU}(\phi)$ is calculated with the energy of the electron beam energy for the input, $E_e = 5.77$ GeV in Eq. (6) for the JLab experiment at 5.7 GeV.

IV. VR MODEL

The VR model is based on the paper, T. Vrancx and J. Ryckebusch, *Charged-pion electroproduction above the resonance region,* Phys. Rev. C **89**, 025203 (2014)

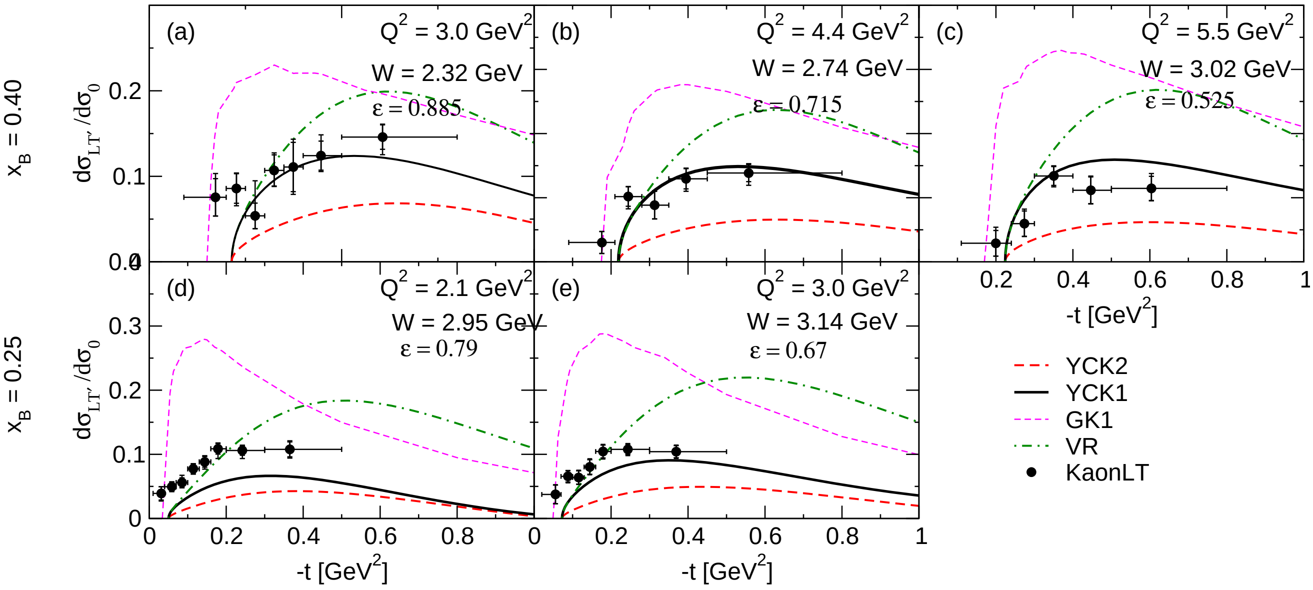

FIG. 1. t-dependence of $d\sigma_{LT'}/d\sigma_U$ for the exclusive reaction $p(e, e'\pi^+)n$ at JLab KaonLT experiment. The solid curve is from the epion-tch-v2, where we choose the GPDs for F_1 and F_2 with α'_q=0.3 GeV^{-2} with Λ_π=775 MeV. a_1 with the sign positive is shown. The red dashed curve comes from the case of choosing dipole FF for F_1 and F_2, $\Lambda_1 = \Lambda_2$ =1.55 GeV with the same cutoff Λ_π. The output is BSA.dat. **The green dash-dotted curve is obtained from the VR model in the epion-km-vr-btk-bsa.f90 with BSA_t.dat as an output.**

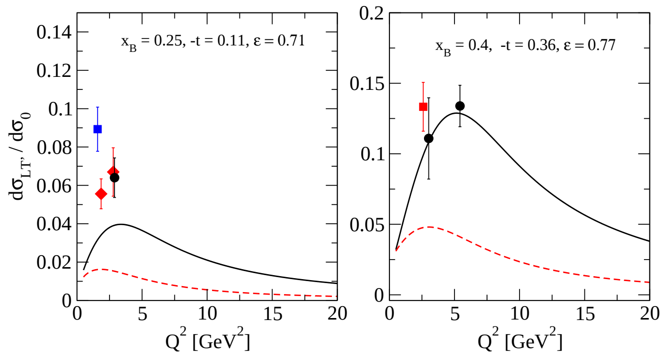

FIG. 2. Q^2-dependence of $d\sigma_{LT'}/d\sigma_U$ for the exclusive reaction $p(e, e'\pi^+)n$ from JLab. **In epion-tch-v2, we choose the GPDs for F_1 and F_2 with α'_q=0.3 GeV^{-2} with Λ_π=775 MeV. The solid curve is from the GPDs for nucleon form factors with the a_1 exchange of the sign positive. The dashed results from the dipole FF for the BSA. The output is BSA.dat.**

Simplifying the KM form factors for the nucleon charge form factor in Eqs. (38) and (39), VR consider the phenomenological s-dependence of the dipole form-factor

$$F_1(Q^2, x) = \left(1 + \frac{Q^2}{\Lambda^2_{\gamma pp*}(x)}\right)^{-2}, \quad (x = s, u) \quad (43)$$

where

$$\Lambda^2_{\gamma pp*}(s) = \Lambda_{\gamma pp} + (\Lambda_\infty - \Lambda_{\gamma pp})\left(1 - \frac{M_p^2}{s}\right), \quad (44)$$

$$\Lambda^2_{\gamma pp*}(u) = \Lambda_{\gamma pp} + (\Lambda_\infty - \Lambda_{\gamma pp})\left(1 - \frac{M_p^2}{2M_p^2 - u}\right) \quad (45)$$

which assume $\Lambda_\infty = 2.194$ and $\Lambda_{\gamma pp} = 0.84$ GeV in numerical calculations.

For the pion exchange, the cutoff mass

$$\Lambda_\pi = 0.655 \text{ GeV} \quad (46)$$

is chosen for the monopole form factor in Eq. (34) with the slope of the pion trajectory given in Eq. (41).

The LT' component of the cross section relevant to the beam polarization asymmetry for the virtual photon is calculated as

$$\frac{d\sigma_{LT'}}{d\sigma_U}, \quad (47)$$

and the results are compared with the data measured from the $KaonLT_BSA$ experiment.

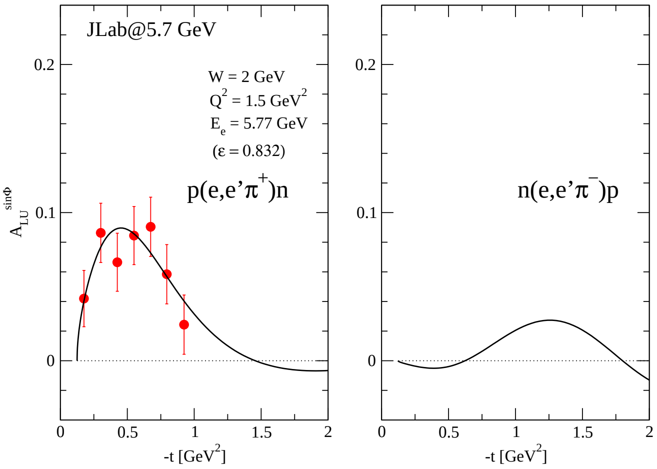

FIG. 3. t-dependence of A_{LU} for the exclusive reaction $p(e, e'\pi^+)n$ at JLab 5.7 GeV electron beam. **The program epion-km-vr-btk-bsa.exe requires the electron beam energy 5.77 as an input to yield the BSA_t.dat.**

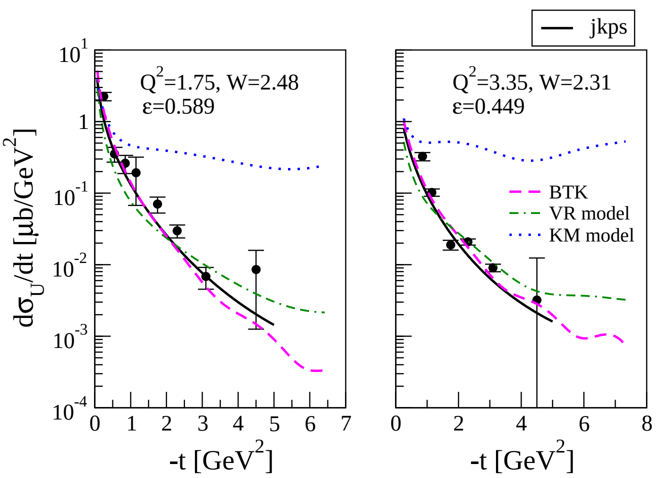

FIG. 4. t-dependence of $d\sigma_U$ for the exclusive reaction $p(e, e'\pi^+)n$. **The program epion-km-vr-btk-bsa.exe yields the dSU.dat.**