PR12-06-101:

Measurement of the Charged Pion Form Factor to High Q^2

Spokespersons:

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Scientific Motivation

- The pion form factor is a topic of fundamental importance to our understanding of hadronic structure.
- Pions are the lightest QCD system, with a relatively simple $q\bar{q}$ valence structure.

At large Q^2 , F_π is presumably given by pQCD

$$F_\pi(Q^2) = \frac{4\pi C_F \alpha_S(Q^2)}{Q^2} \left| \sum_{n=0}^{\infty} a_n \left(\log\left(\frac{Q^2}{\Lambda^2} \right) \right)^{-\gamma_n} \right|^2 \left[1 + O(\alpha_s(Q^2), m/Q) \right]$$

which in the $Q^2 \to \infty$ limit becomes

$$F_\pi(Q^2) \underset{Q^2\to\infty}{\longrightarrow} \frac{16\pi\alpha_s(Q^2) f_\pi^2}{Q^2}$$

where f^2_π=93 MeV is the $\pi^+ \to \mu^+ \nu$ decay constant.

G.P. Lepage, S.J. Brodsky, Phys.Lett. **87B**(1979)359.

F_π is the clearest test case for study of transition between pQCD and non-perturbative regions.

- At what value of Q^2 will the pQCD contributions dominate $F_\pi(Q^2)$?
- A difficult question to answer, as both “hard” and “soft” components (such as gluonic effects) must be taken into account.
  - non-perturbative hard components of higher twist strongly cancel soft components, even at modest Q^2.
    *[Braun et al., PRD **61**(2000)073004]*
  - the situation for nucleon form factors is even more complicated.
- Many model calculations exist, but ultimately...
  - Reliable $F_\pi(Q^2)$ data are needed to delineate the role of hard versus soft contributions at intermediate Q^2.
- A program of study unique to Jefferson Lab.

Theory Review Comments:

- The pion form factor is an object of great theoretical interest, especially at larger values of Q^2, where one can study nonperturbative dynamics of QCD while searching for the transition to the perturbative regime.

- While the merits of studying this observable are clear, the extraction of the pion form factor from data is a non-trivial exercise.

Determination of F_π via Pion Electroproduction

Pion charge radius is well known from $e^+e^- \rightarrow \pi^+\pi^-$ experiments.

$$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

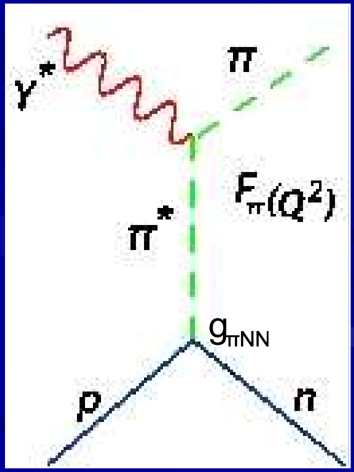
At low Q^2<0.3 GeV^2, the π^+ form factor can be measured exactly using high energy π^+ scattering from atomic electrons.

$\Rightarrow$ 300 GeV pions at CERN SPS. *[Amendolia et al., NP **B277**(1986)168]*

To access higher Q^2, one must employ the $p(e,e'\pi^+)n$ reaction.

- the t-channel process dominates σ_L at small $-t<0.02$ GeV^2.

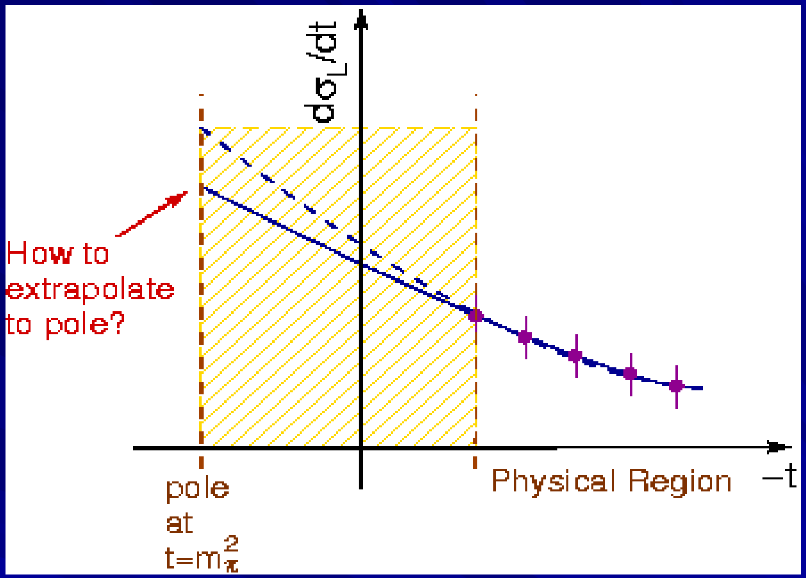
$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t-m_\pi^2)} g_{\pi NN}^2(t) F_\pi^2(Q^2,t)$$



Extraction of form factor from σ_L data

Inelastic $p(e,e'\pi^+)n$ data are obtained some distance from the $e^-\pi^+$ elastic scattering pole.

- How to extrapolate?



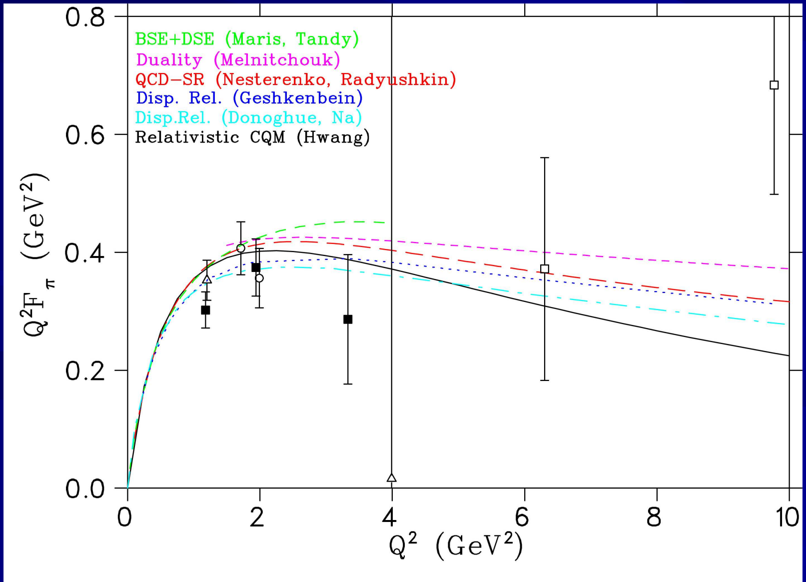
In the actual analysis, a model incorporating the π^+ production mechanism and the `spectator' nucleon is used to extract F_π from σ_L.

Method check:

- It would be of great value to verify that $F_\pi(Q^2)$ values extracted from electroproduction data are in good agreement with those determined from e-π scattering data.
- We propose to take Q^2=0.30 GeV^2 data at very low $-t$=0.005 GeV^2 to rigorously check the electroproduction method.

Previously obtained high Q^2 data from Cornell in the 1970's have many problems

- Problematic L/T separation.
  - High and low ε from different experiments used, or only low ε setting taken and a model used to extract F_π.
  - Systematic error?
- Analysis based on assumptions with systematic errors that are difficult to quantify.
  - Data taken far from pole, with $-t_{min}$ *as high as 40* m_π^2.
  - Isoscalar contamination (e.g. t-channel $b_1(1235)$ exchange) an issue.



"[we] question whether F_π has been truly determined for large Q^2."

C.E. Carlson, J. Milana, PRL **65**(1990)1717.

Jefferson Lab can play a major role in clarifying our knowledge of F_π at higher Q^2 over the coming decade.

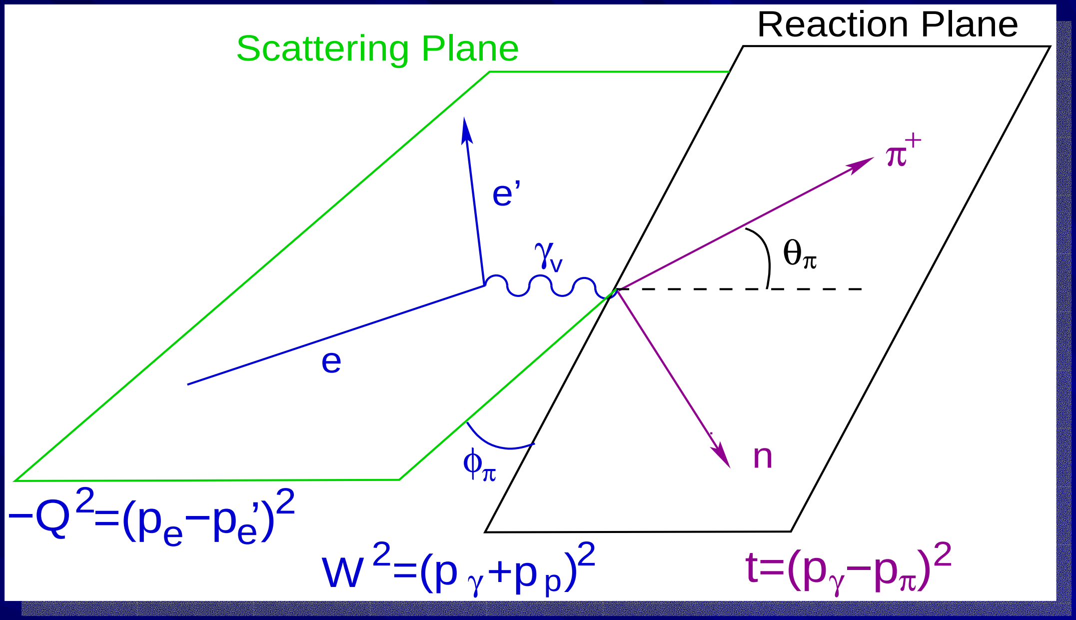
The importance of appropriately-chosen kinematics

- Experiment must access small $-t$ to ensure t-channel dominance.
- Carlson and Milana [PRL **65**(1990)1717] looked at competing non-pole QCD processes complicating the extraction of F_π at large Q^2.
  - background ratio M_{pQCD}/M_{pole} rises dramatically once $-t_{min}>0.20$.
  - "more reliable measurements of F_π at high Q^2 require smaller $|t|$ and thus higher electron energy loss ν."
- 11 GeV upgrade and SHMS small angle capability are crucial for this task.

$\Rightarrow$ large ν $\Rightarrow$ large W $\Rightarrow$ smaller $|t_{min}|$.

$\Rightarrow$ reduced model uncertainty in F_π extraction.

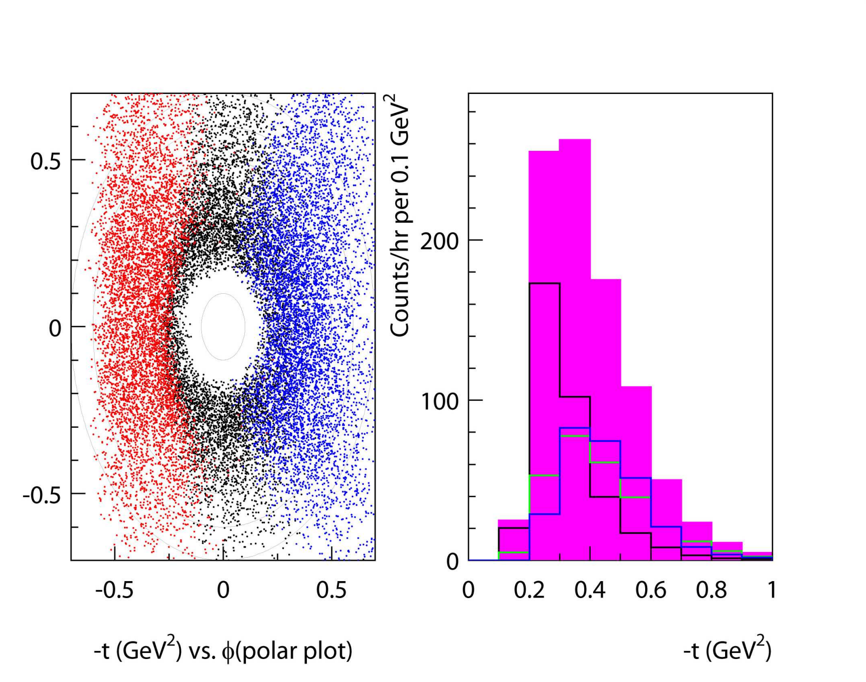
$\Rightarrow$ expected smaller background to π pole diagram.

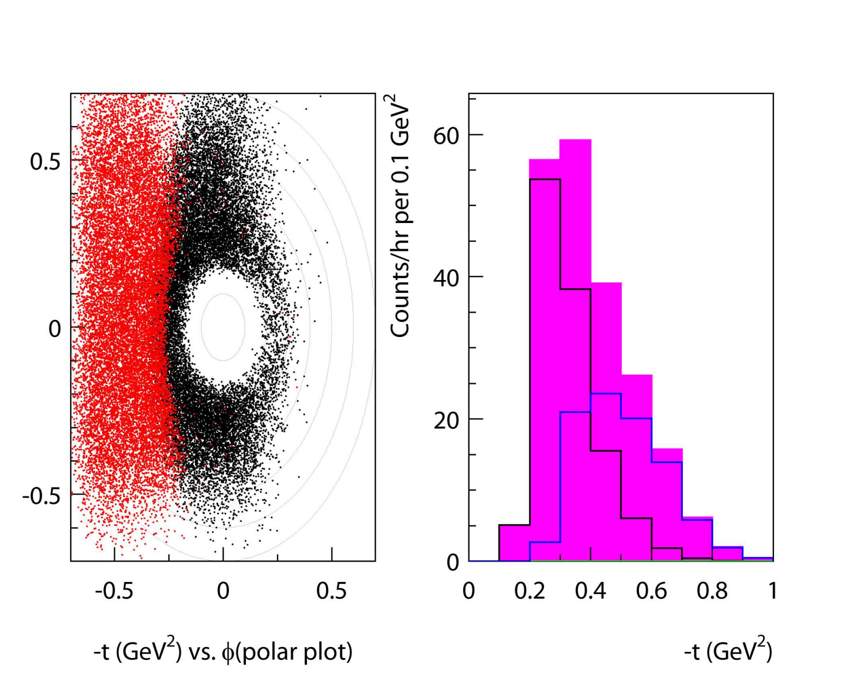


$$2\pi \frac{d\sigma}{dt\,d\phi} = \varepsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon(\varepsilon+1)}\,\frac{d\sigma_{LT}}{dt}\cos\phi + \varepsilon\frac{d\sigma_{TT}}{dt}\cos 2\phi$$

- Extraction of F_π requires t dependence of σ_L to be known.
  - Only three of Q^2, W, t, θ_π are independent.
  - Vary θ_π to measure t dependence.
  - Since non-parallel data needed, LT and TT must also be determined.

Simulated SHMS+HMS *-t* vs. φ coverage


Q^2=6.0 GeV^2, ε=0.435


Q^2=6.0 GeV^2, ε=0.177

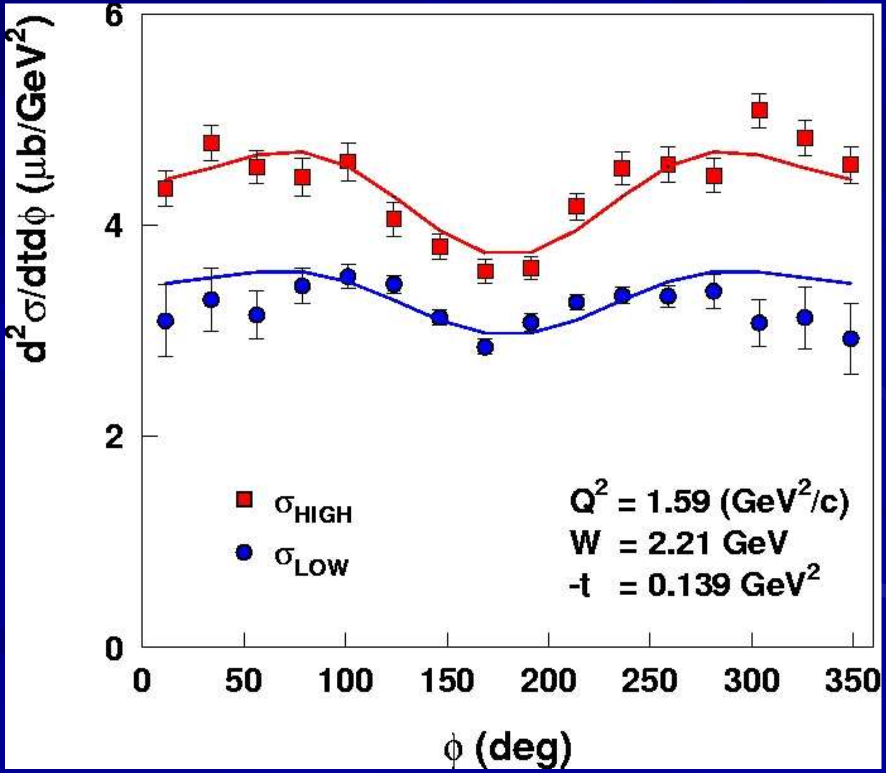
- Multiple SHMS settings ±2° left and right of the *q*-vector are used to obtain good φ-coverage over a range of *–t*.
  - Measurements over $0<\varphi<2\pi$ are required to determine LT, TT contributions versus *-t*.

Radial coordinate (*-t*). Azimuthal coordinate (φ).

The different pion arm settings are combined to yield φ-distributions for each *t*-bin

$$\frac{d^2\sigma}{dt\,d\phi} = \varepsilon \frac{d\sigma_L}{dt d\phi} + \frac{d\sigma_T}{dt\,d\phi} + \sqrt{2\varepsilon(\varepsilon+1)}\,\frac{d\sigma_{LT}}{dt\,d\phi}\cos\phi_\pi + \varepsilon \frac{d\sigma_{TT}}{dt\,d\phi}\cos 2\phi_\pi$$

- Extract all four response functions via a simultaneous fit using measured azimuthal angle (φ_π) and knowledge of photon polarization (ε).
- This technique demands the good knowledge of the magnetic spectrometer acceptances.



Fπ-2 data: nucl-ex/0607005

Magnetic Spectrometer Calibrations

- Similarly to Fπ-2, we propose to use the over-constrained $p(e,e'p)$ reaction and inelastic $e+{}^{12}\mathrm{C}$ in the DIS region to calibrate spectrometer acceptances, momenta, offsets, etc.
  - Fπ-2 beam energy and spectrometer momenta determined to <0.1%.
  - Spectrometer angles ~0.5 mr.
  - Fπ-2 agreement with published $p+e$ elastics cross sections <2%.

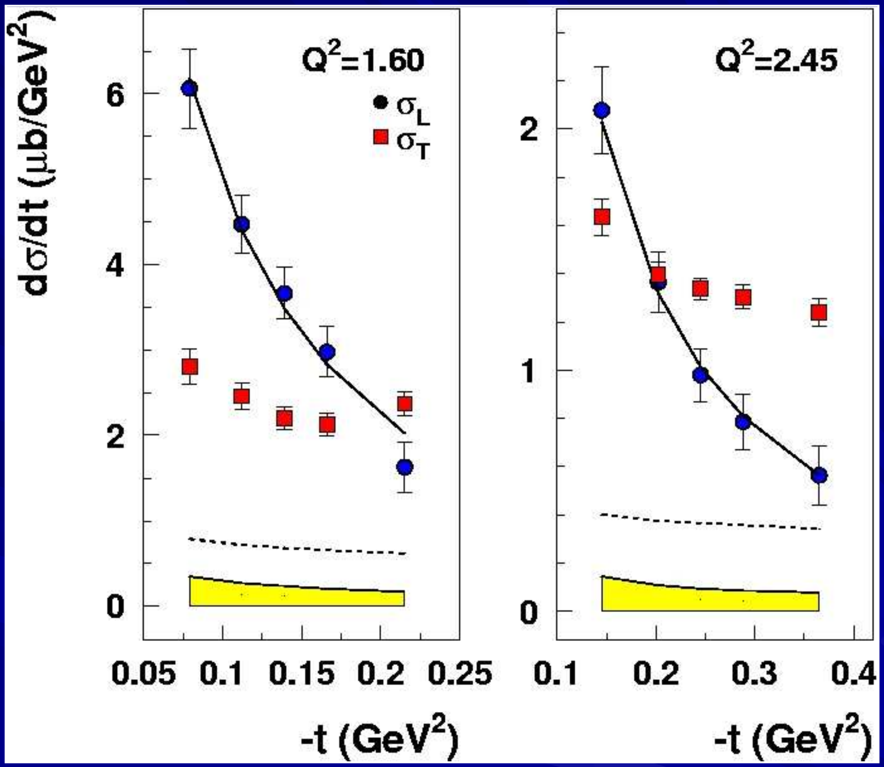
Projected Systematic Uncertainty Source	Pt-Pt ε-random t-random	ε-uncorrelated common to all t-bins	Scale ε-global t-global
Spectrometer Acceptance	0.4%	0.4%	1.0%
Target Thickness		0.2%	0.8%
Beam Charge	-	0.2%	0.5%
HMS+SHMS Tracking	0.1%	0.4%	1.5%
Coincidence Blocking		0.2%	
PID		0.4%	
Pion Decay Correction	0.03%	-	0.5%
Pion Absorption Correction	-	0.1%	1.5%
MC Model Dependence	0.2%	1.0%	0.5%
Radiative Corrections	0.1%	0.4%	2.0%
Kinematic Offsets	0.4%	1.0%	-

- Uncorrelated uncertainties in σ_{UNS} are amplified by $1/\Delta\varepsilon$ in L-T separation.
- Scale uncertainty propagates directly into separated cross section.

Fπ-1 and Fπ-2 used the VGL Regge Model to extract $F_\pi(Q^2)$ from the σ_L data

- Feynman propagator $\left(\frac{1}{t-m_\pi{}^2}\right)$ replaced by π and ρ Regge propagators.
  - Represents the exchange of a <u>series</u> of particles, compared to a <u>single</u> particle.
- Model parameters fixed from pion photoproduction.
- Free parameters: Λ_π, Λ_ρ (trajectory cutoff).

[Vanderhaeghen, Guidal, Laget, PRC **57**(1998)1454]



Fπ-2 data: nucl-ex/0607005

Error bars indicate statistical and random (pt-pt) systematic uncertainties in quadrature.

Yellow band indicates the correlated (scale) and partly correlated (t-corr) systematic uncertainties.

$$F_\pi = \frac{1}{1+Q^2/\Lambda_\pi{}^2}$$

Fit to σ_L to model gives F_π at each Q^2.

$\Lambda_\pi{}^2$=0.513, 0.491 GeV², $\Lambda_\rho{}^2$=1.7 GeV².

The experimental result is not permanently "locked in" to a specific model.

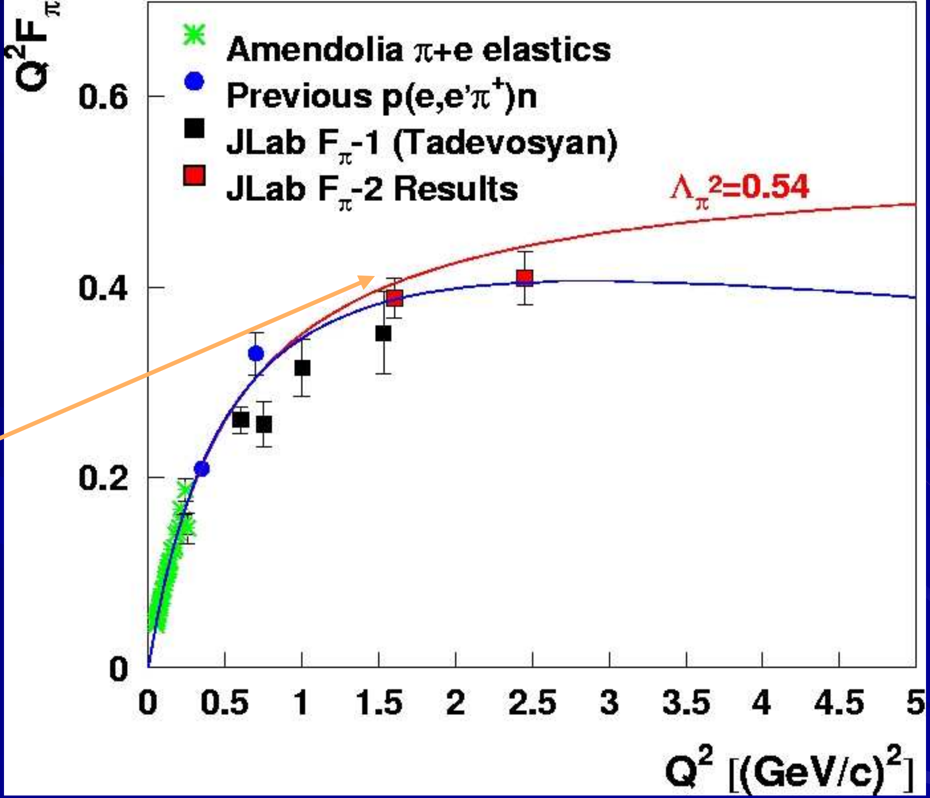
- In principle, the experimentalist would like to use a variety of models to extract $F_\pi(Q^2)$ from the electroproduction data, so that the model dependence can be better understood.
  - Unfortunately, the VGL model is the only reliable model available for our use at present.
- We intend to publish our experimentally measured $d\sigma_L/dt$, so that updated values of $F_\pi(Q^2)$ could be extracted in the future.
- Theory review:

"The fact that the collaboration plans to publish the cross section data is to be applauded, as this would enable any future theoretical advances to subsequently improve on the extracted number."

F_π-1 and F_π-2 Results

- Data point at Q^2=1.60 GeV^2 to check model dependence of form factor extraction.
  - F_π-1 (W=1.95 GeV) and F_π-2 (W=2.22 GeV) agree to ~4%.
  - New point is 30% closer to pion pole, with significantly reduced uncertainties.
- JLab data are consistently below 0.657 fm monopole charge radius curve.
  - A significant deviation would indicate the increased role of π^+ hard wave function components at moderate Q^2.

Fπ-1: nucl-ex/0607007.
Fπ-2: nucl-ex/0607005.



S.R. Amendolia, et al., Nucl. Phys. **B277** (1986) 168.

H. Ackermann, et al, Nucl. Phys. **B137** (1978) 294.

P. Brauel, et al., Z. Phys. **C3** (1979) 101.

Proposed Kinematics (1)

- Q^2=0.30 GeV^2
  - Precision low $-t$ data to test the electroproduction method of F_π.
  - Perform a direct comparison with exact values from π-e elastics.
  - $-t_{min}$=0.005 is 50% smaller than any previous electroproduction data.
  - Measurement requires 5.5° π^+ arm (SHMS), 2.8-4.2 GeV beam.
  - Use 30 μA on 4cm LH_2 target to avoid potentially high accidental coincidence rates.
- Q^2=1.60, 2.45 GeV^2
  - Repeat measurements taken in Fπ-1 and Fπ-2 but at widely different W and t_{min}.
  - Needed to better understand model-dependence of F_π results.

Q^2 (GeV)	W (GeV)	|t| (GeV^2)
1.60	3.00	0.029
	2.22	0.095
	1.95	0.150
2.45	3.20	0.048
	2.22	0.186

Proposed Kinematics (2)

- Q^2=5.25, 6.00 GeV^2
  - Constraints: $|t_{min}|\approx 0.2$, $\Delta\varepsilon\approx 0.3$, 10.9 GeV beam, 5.5^o π^+ arm.

    $\Rightarrow$ maximum Q^2 near 6.0 GeV^2.
  - Take Q^2=5.25 GeV^2 "nearby" where expected precision is better.

Q^2 (GeV)	W (GeV)	$\Delta\varepsilon$	|t| (GeV^2)
5.25	3.20	0.31	0.17
6.00	3.20	0.26	0.21

- Q^2=3.50, 4.50 GeV^2
  - These points are crucial if highest Q^2 points suggest a "turnover" in Q^2F_π and pQCD limit being reached.

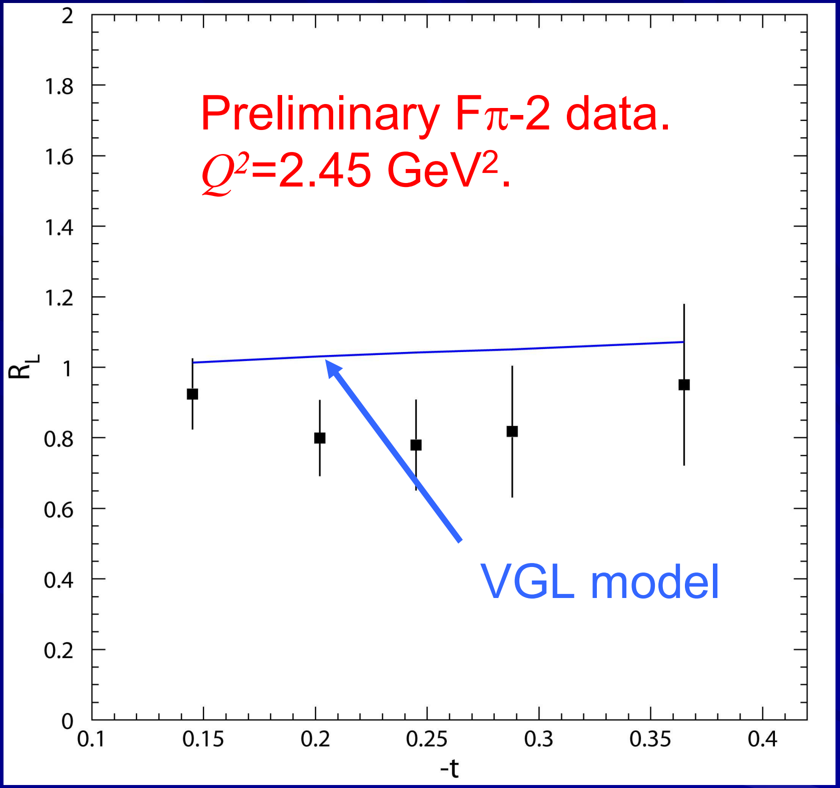
Q^2 (GeV)	W (GeV)	$\Delta\varepsilon$	|t| (GeV^2)
3.50	3.10	0.37	0.10
4.25	3.28	0.30	0.12

How to verify that σ_L is dominated by the *t*-channel process

- *t*-channel diagram is purely isovector.
- measure

$$R_L = \frac{\sigma_L[n(e,e'\pi^-)p]}{\sigma_L[p(e,e'\pi^+)n]} = \frac{|A_V - A_S|^2}{|A_V + A_S|^2}$$

  using a deuterium target.
- isoscalar backgrounds (such as resonant contributions to the *t*-channel) will dilute the ratio.
- We propose the same tests at Q^2=1.60 and 3.50 GeV^2.



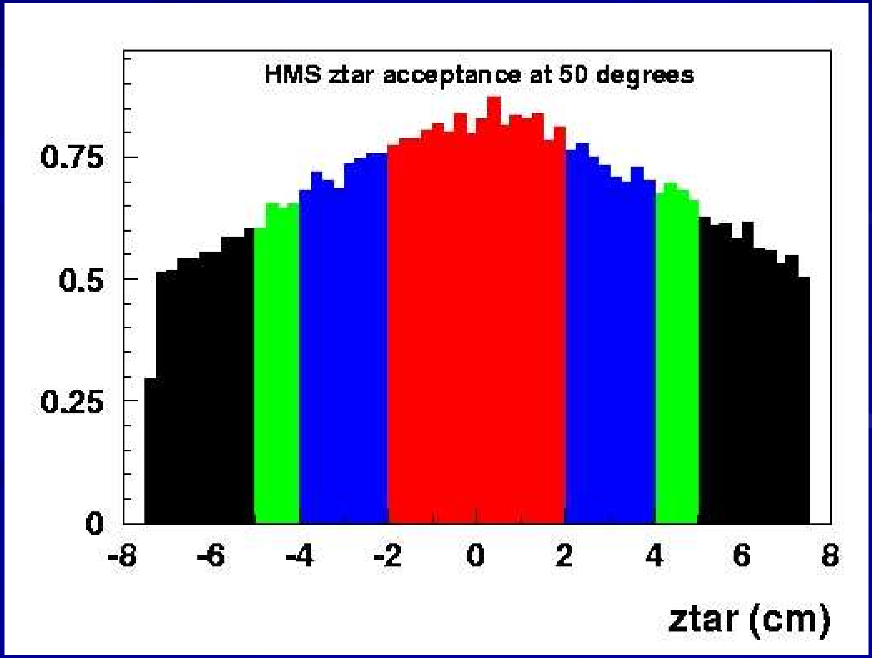
Because one of the many problems encountered by the historical data was isoscalar contamination, allocation of beamtime to this test will increase the confidence in the extraction of $F_\pi(Q^2)$ from our σ_L data.

Target Cell Length

Technical Review:

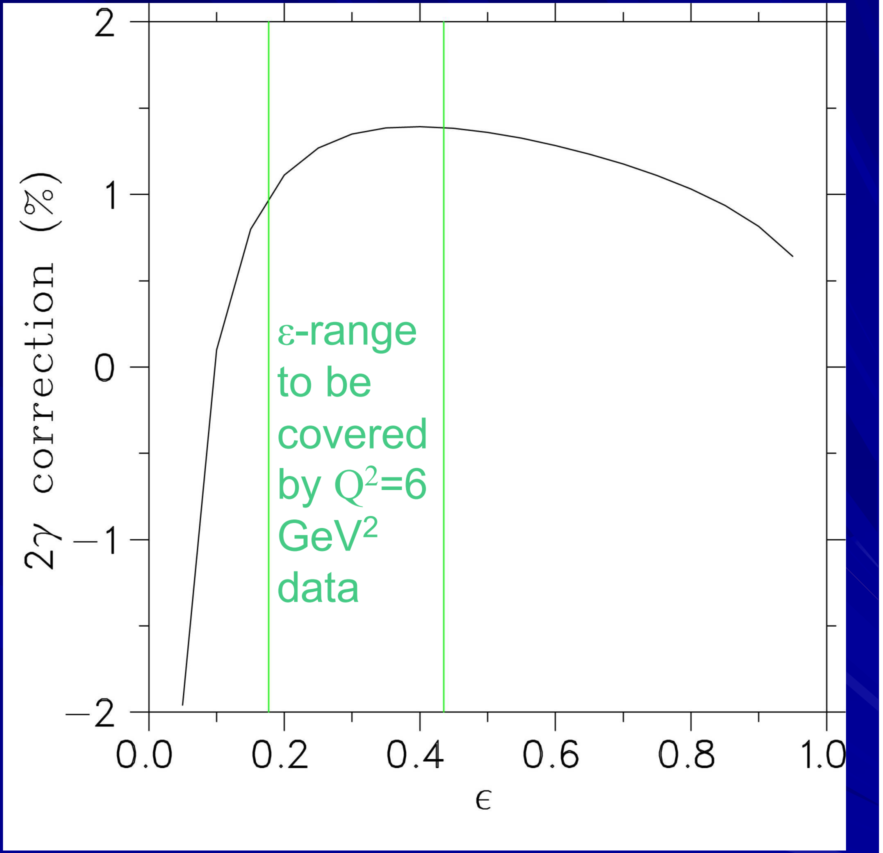
“The experiment assumes beyond the standard 4 cm targets also non-standard 8 cm long targets. Although certainly possible, it would be useful to have an allowable range for ease of scheduling and cost reasons.”

- The SHMS has a very large y_{tar} acceptance and sits at forward angles, so is not an issue.
- The HMS is at angle up to 47° and poses the main limitation, although not as extreme as SOS.
- Simulations indicate a cell length up to 10 cm is probably okay.



Two γ exchange?

- In the Rosenbluth separation of the proton electric form factor, 2γ contributions may be important because one is trying to separate a small cross section (electric) from a much larger (magnetic) one.
- 2γ exchange is not expected to be a significant issue in the extraction of σ_L in pion electroproduction.



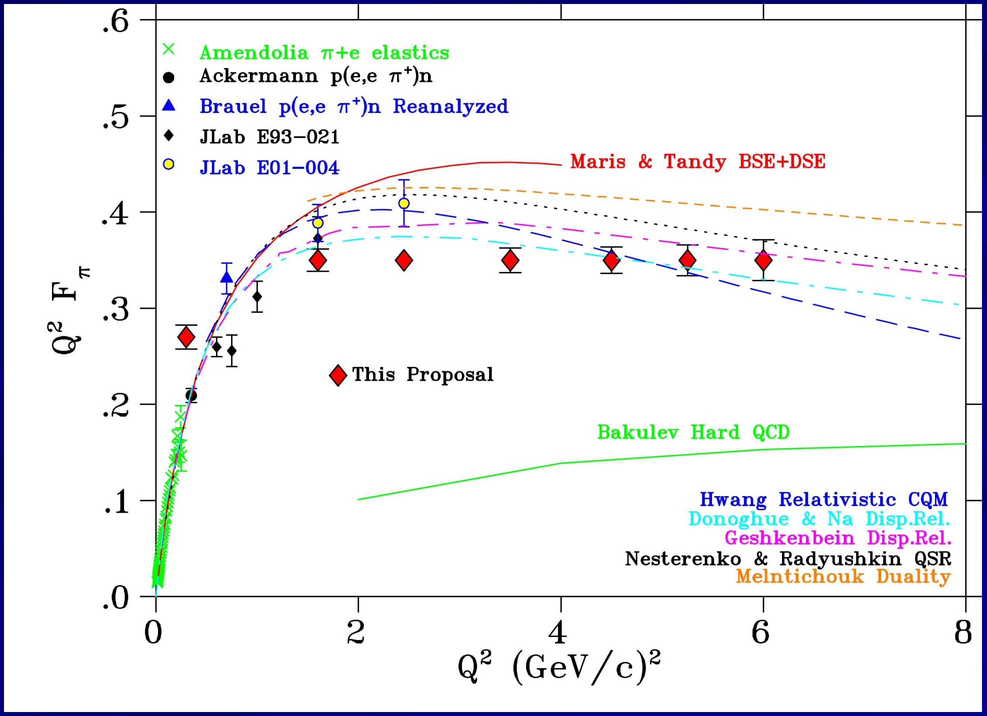
Q^2=6 GeV^2 calculation performed by Tjon and Melnitchouk.

Correction=δ_{FULL}-$\delta_{Mo\&Tsai}$

Projected Error Bars

- Rates and uncertainties are based on an empirical parameterization of existing electroproduction data.
  - conservative assumptions used when extrapolating to poorly measured Q^2.
- Error is amplified by Δε and potentially large *r*.

Q^2 (GeV)	*W* (GeV)	Projected $r=\sigma_T/\sigma_L$	Δε	$\Delta F_\pi/F_\pi$ (%)
0.30	2.20	0.63	0.41	5.2
1.60	3.00	0.18	0.38	3.6
2.45	3.20	0.19	0.44	3.0
3.50	3.20	0.32	0.37	4.0
4.50	3.28	0.38	0.30	4.3
5.25	3.20	0.56	0.31	5.0
6.00	3.20	0.73	0.26	6.6
Fπ-2 Final Errors				
1.60	2.22	0.48	0.27	4.9
2.45	2.22	0.80	0.28	6.0



Beam Time Estimate

Q^2 (GeV^2)	ε settings	LH+ Hours	LD+ Hours	LD- Hours	Over-head	Total Hours
6.00	3	376	--	--	12	388
5.25	3	231	--	--	12	243
4.50	3	125	--	--	12	137
3.50	3	39	31	153	20	243
2.45	4	43	--	--	16	59
1.60	3	24	16	21	20	81
0.30	3	24	--	--	12	36
Subtotals		862	47	174	104	
^{1}H(*e,e'p*) + Optics						80
9 Beam Changes						72
Grand Total: 1339 hrs (56 days)						

- Calibration measurements detailed in the proposal indicate that a useful set of ^{1}H(*e,e'p*) coincidence data can be compiled with a reasonable investment of beam time.
- The LD-, Q^2=0.30 and some of the optics runs are unaffected if the maximum beam current is <90 μA.

Summary

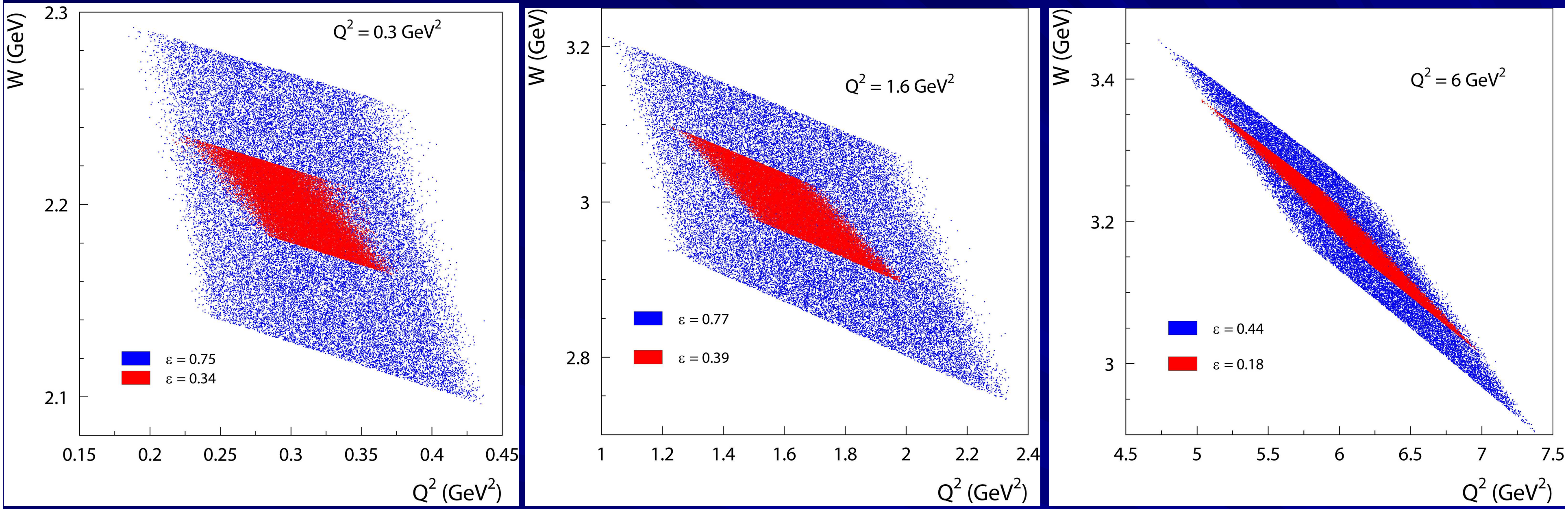
- To reliably extract F_π at higher Q^2, get closer to the pole than Bebek et al. did.
  - Measure $d\sigma_L$ at $-t<0.2$ GeV^2, $W>3$ GeV.
  - 11 GeV beam and forward angle capability of SHMS are essential.
- Use best available model(s) for σ_L to extract F_π.
  - $d\sigma_L/dt$ vs $-t$ to test reliability of model.
  - Non parallel kinematics used. ⇒ *LT, TT*.
- Take $\pi^\pm$ data to verify t-channel dominance.
  - Deuterium target.
- Extraction of F_π to Q^2=6 GeV^2 would challenge QCD-based calculations in the most rigorous manner.
- Test electroproduction method by taking low Q^2 data very close to the pole.

A unique opportunity for JLab to dramatically improve the $F_\pi(Q^2)$ database.

→ An essential part of the Hall C program 18-24 months after the start of SHMS commissioning.

Appendix

Simulated SHMS+HMS Q^2-W coverage



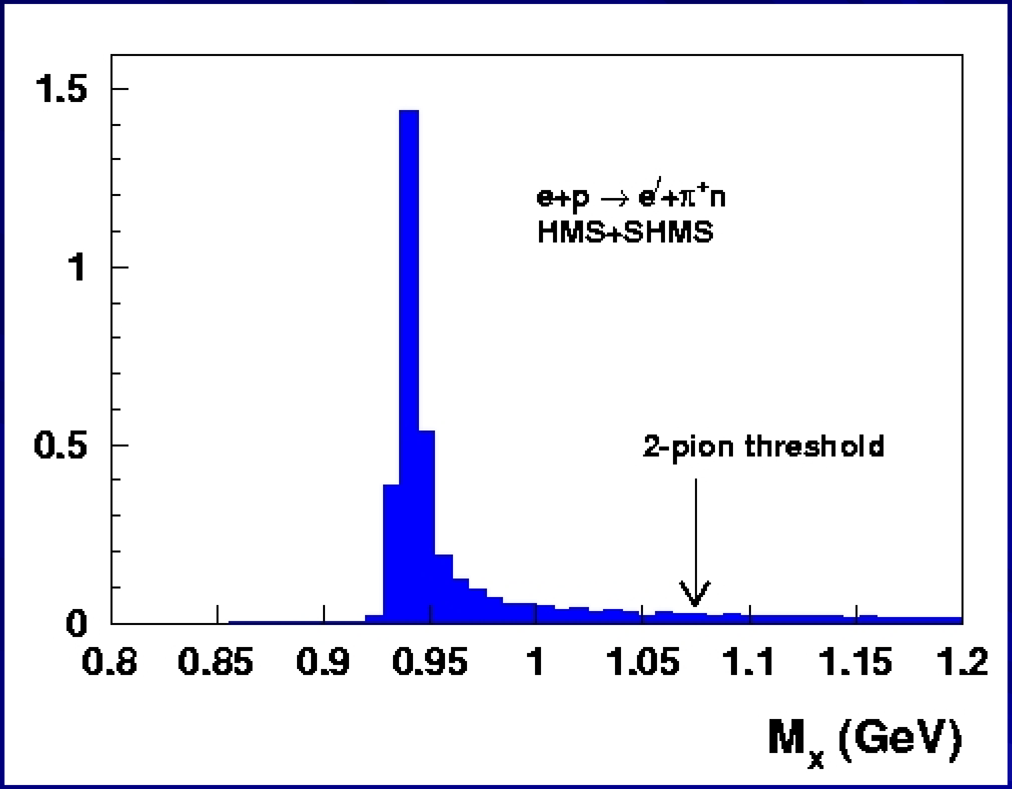
- Cuts are placed on the data to equalize the Q^2-W range measured at the different ε-settings.
- Fπ-2 experiment was most similar to tilted diamond at right.
- Squarer acceptance at center and left is desirable, as it allows the Q^2- and W-dependences of the cross section to be more easily disentangled.

Particle Identification

- SHMS will sit at very forward angles (5.5°-11°) throughout entire experiment.
- Positive SHMS polarity runs:
  - Good π^+/K^+ separation can be accomplished with a heavy gas Cerenkov, and accidental coincidence subtraction.
  - Decay products from real $e^- \bullet K^+$ coincidences cannot be eliminated in this manner, but are projected to be <0.1%.
- Negative SHMS polarity runs:
  - Sometimes expect e^-/K^- ratio >1000:1, K^-/π^- >10:1.
  - Gas Cerenkov (20:1 rejection) + EM calorimeter (225:1 rejection) should provide good 4500:1 electron rejection.
  - Singles rates could be as high as 1 MHz.

    → Need to be careful determining tracking efficiency.
- HMS sits at larger angles (10°-45°).
  - rates are expected to be low and will be well within present operating parameters of HMS experiments.

Expected Missing Mass Resolution



Q^2=6.0 GeV^2, high ε simulation.

“Near Parallel” Kinematics

- Central kinematics only.
- Middle ε not shown.

Q^2 (GeV^2)	W (GeV)	-t (GeV^2)	ε	Beam (GeV)	P_{HMS} (e’)	θ_{HMS} (e’)	P_{SHMS} (π)	θ_q
0.30	2.20	0.005	0.34	2.80 (2D)	0.53	25.97	2.26	-5.71
			0.75	4.20 (3D)	1.93	11.04	2.26	-9.11
1.60	3.00	0.029	0.38	6.60 (3A)	1.42	23.84	5.16	-6.18
			0.77	9.90 (5B)	4.72	10.61	5.16	-9.39
2.45	3.20	0.048	0.27	7.40 (4C)	1.11	31.73	6.27	5.15
			0.70	10.90 (5A)	4.61	12.68	6.27	-8.97
3.50	3.10	0.099	0.31	7.90 (4B)	1.38	32.87	6.46	-6.36
			0.67	10.90 (5A)	4.38	15.56	6.46	-9.98
4.50	3.28	0.122	0.22	8.80 (4A)	1.14	39.16	7.59	-5.19
			0.52	10.90 (5A)	3.24	20.57	7.59	-8.23
5.25	3.20	0.171	0.19	8.80 (4A)	1.02	45.08	7.69	-5.08
			0.50	10.90 (5A)	3.12	22.68	7.69	-8.51
6.00	3.20	0.21	0.18	9.20 (5C)	1.02	47.24	8.07	-5.01
			0.44	10.90 (5A)	2.72	26.02	8.07	-8.01

Projected Rates

Q^2 (GeV^2)	W (GeV)	ε	Beam & Target	SHMS LH+ Singles (kHz) π^+ K^+ p	SHMS LD- Singles (kHz) e^- π^- K^-	HMS LH+ Singles (kHz) π^+ K^+ p	Random LH+ Coinc (Hz)	Real Coinc (Hz)
0.30	2.20	0.34	30μA / 4cm	22 2 6	--	2 26 2	3.7	1.3
		0.75		35 5 10	--	52 26 6	108	30
1.60	3.00	0.38	+: 90μA / 8cm	32 12 10	1130 4 0.1	6 54 22	20	3
		0.77	-: 15μA / 8cm	24 10 8	90 4 0.3	110 200 1.8	210	64
2.45	3.20	0.27	90μA / 8cm	14 8 5	--	1.0 38 1.2	2.8	0.5
		0.70		10 4 4	--	40 8 0.8	29	20
3.50	3.10	0.31	+: 90μA / 8cm	10 5 4	900 2 0.3	0.8 12 0.6	1.0	0.4
		0.67	-: 15μA / 8cm	3 1.6 1.4	80 0.6 0.1	14 2.6 0.4	3.8	7.3
4.50	3.28	0.22	90μA / 8cm	6 4 2.4	--	0.4 12 0.4	0.4	0.1
		0.52		3 1.6 1.2	--	3.4 3 0.2	0.8	1.5
5.25	3.20	0.19	90μA / 8cm	6 3 2.2	--	0.3 8 0.2	0.2	0.06
		0.50		1.6 1.0 0.8	--	2.0 1.6 0.2	0.3	0.9
6.00	3.20	0.18	90μA / 8cm	3.6 2.6 1.8	--	0.2 6 0.2	0.2	0.05
		0.44		1.4 1.0 0.8	--	0.5 1.2 0.2	0.1	0.5

$^{1}\mathrm{H}(e,e'p)$ Coincidence Elastic Runs

- A large body of $^{1}\mathrm{H}(e,e')p$ elastic data can be amassed to help constrain the momentum and angle offsets of the spectrometers.
- The over-constrained $^{1}\mathrm{H}(e,e'p)$ reaction is also quite useful.
  - Reconstruct proton, missing energy, and 3 components of missing momentum.
- The table demonstrates that a useful set of coincidence data can be compiled with a reasonable investment of beam time.

“Reverse setting”: Protons detected in SHMS.

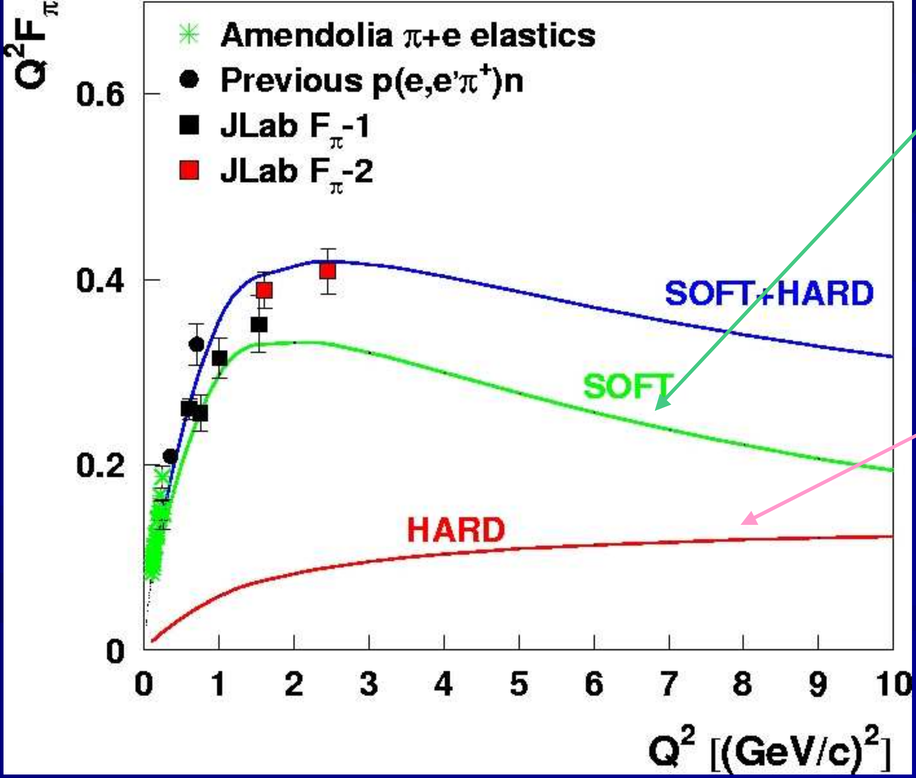
Beam (GeV)	Q^2 (GeV^2)	$\theta_{e'}$ (deg)	$P_{e'}$ (GeV)	θ_p (deg)	P_p (GeV)	Coinc Rate (Hz)
10.9	8.18	19.5	6.54	24.76	5.21	2.4
10.9	2.31	8.5	9.67	46.87	1.96	820
10.9	3.33	10.5	9.13	40.78	2.46	210
9.2	2.43	10.5	7.90	45.21	2.03	680
8.8	0.94	6.5	8.30	59.49	1.09	16000
7.4	0.49	5.5	7.14	66.88	0.74	92000
7.4	1.62	10.5	6.54	50.77	1.54	2800
7.4	2.79	14.5	5.91	41.49	2.24	400

The role of Soft and Hard terms in F_π

QCD Sum Rules:

[V.A. Nesterenko and A.V. Radyushkin, Phys.Lett. B115(1982)410]

- Interpolation between perturbative and non-pertubative sectors using dispersion relation methods in combination with the Operator Product Expansion.
- Not rigorously derived from QCD, but an intuitive bridge between low and high energy properties of QCD.



SOFT: QCD Sum Rules used to give a local quark-hadron duality estimate with no free parameters.

$$F_\pi^{soft} = 1 - \frac{1+6s_0/Q^2}{(1+4s_0/Q^2)^{3/2}}$$

HARD: simple model based on the interpolation between the Q^2=0 value (related by Ward identity to $O(\alpha_s)$ term of 2-point correlator) and the asymptotic behavior.

$$F_\pi^{hard} = \frac{\alpha_s}{\pi}\frac{1}{(1+Q^2/2s_0)}$$

Duality Interval: $s_0 = 4\pi^2 f_\pi^2 \approx 0.7\ \mathrm{GeV}^2$