

# Pion Form Factor and Factorization to High $Q^2$ E12-19-006



University  
of Regina

Garth Huber

Hall A/C Joint User's Meeting  
July 9, 2021

Supported by:



SAPIN-2021-00026

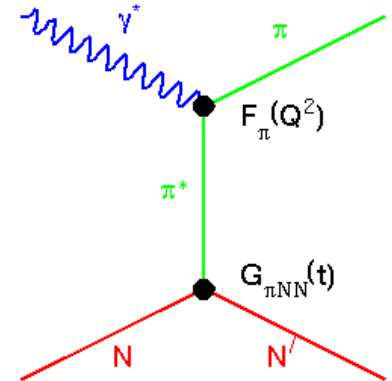
# Motivations of the Experiment

## 1) Determine the Pion Form Factor to high $Q^2$ :

- Indirectly measure  $F_\pi$  using the “pion cloud” of the proton via  $p(e, e'\pi^+)n$

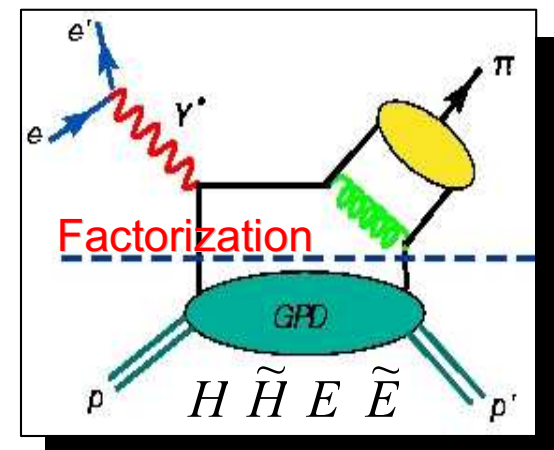
$$|p\rangle = |p\rangle_0 + |n\pi^+\rangle + \dots$$

- The pion form factor is a key QCD observable.**
- The experiment should obtain high quality  $F_\pi$  over a broad  $Q^2$  range. Rated “high impact” by PAC.



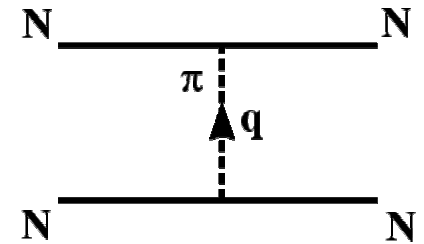
## 2) Study the Hard-Soft Factorization Regime:

- Need to determine region of validity of hard-exclusive reaction mechanism, as GPDs can only be extracted where factorization applies.**
- Separated  $p(e, e'\pi^+)n$  cross sections vs.  $Q^2$  at fixed  $x$  to investigate reaction mechanism towards 3D imaging studies.
- Perform exclusive  $\pi^-/\pi^+$  ratios from  $^2\text{H}$ , yielding insight to hard–soft factorization at modest  $Q^2$ .



# The Pion has Particular Importance

- The pion is responsible for the long-range part of the nuclear force, acting as the basis for meson exchange forces, and playing a critical role as an elementary field in nuclear structure Hamiltonians.



- As the lightest meson, it must be a valence  $q\bar{q}$  bound state, but understanding its structure through QCD has been exceptionally challenging.
  - e.g. Constituent Quark Models that describe a nucleon with  $m_N=940$  MeV as a  $qqq$  bound state, are able to describe the  $\rho$ -meson under similar assumptions, yielding a constituent quark mass of about

$$m_Q \approx \frac{m_N}{3} \approx \frac{m_\rho}{2} \approx 350 \text{ MeV}$$

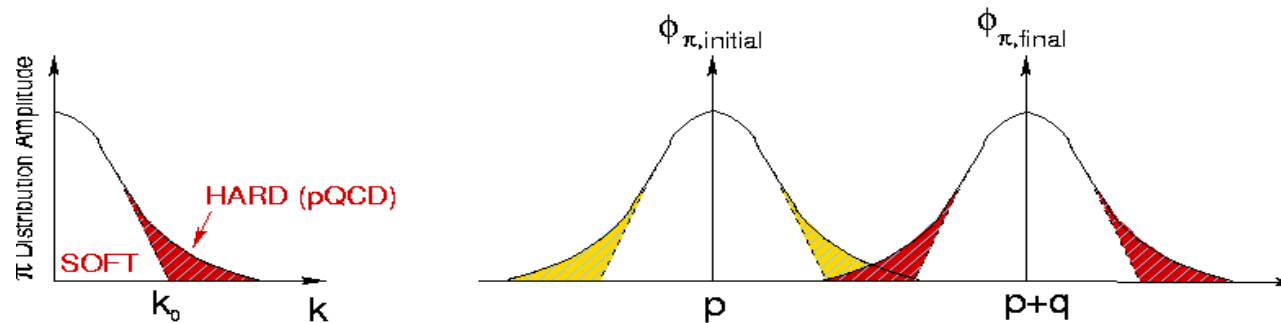
- The pion mass  $m_\pi \approx 140$  MeV seems “too light”.
- **We exist because nature has supplied two light quarks and these quarks combine to form the pion, which is unnaturally light and hence very easily produced.**

# Charged Meson Form Factors

Simple  $q\bar{q}$  valence structure of mesons presents the ideal testing ground for our understanding of bound quark systems.

In quantum field theory, the form factor is the overlap integral:

$$F_{\pi}(Q^2) = \int \phi_{\pi}^*(p) \phi_{\pi}(p+q) dp$$



The meson wave function can be separated into  $\varphi_{\pi}^{soft}$  with only low momentum contributions ( $k < k_0$ ) and a hard tail  $\varphi_{\pi}^{hard}$ .

While  $\varphi_{\pi}^{hard}$  can be treated in pQCD,  $\varphi_{\pi}^{soft}$  cannot.

**From a theoretical standpoint, the study of the  $Q^2$ -dependence of the form factor focuses on finding a description for the hard and soft contributions of the meson wave-function.**

# The Pion in perturbative QCD

At very large  $Q^2$ , pion form factor ( $F_\pi$ ) can be calculated using pQCD

$$F_\pi(Q^2) = \frac{4}{3} \pi \alpha_s \int_0^1 dx dy \frac{2}{3} \frac{1}{xy Q^2} \phi(x) \phi(y)$$

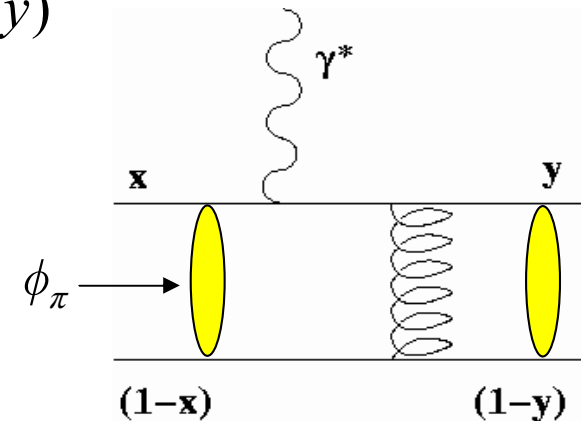
at asymptotically high  $Q^2$ , the pion distribution amplitude becomes

$$\phi_\pi(x) \xrightarrow{Q^2 \rightarrow \infty} \frac{3 f_\pi}{\sqrt{n_c}} x(1-x)$$

and  $F_\pi$  takes the very simple form

$$Q^2 F_\pi(Q^2) \xrightarrow{Q^2 \rightarrow \infty} 16 \pi \alpha_s(Q^2) f_\pi^2$$

$f_\pi = 93$  MeV is the  $\pi^+ \rightarrow \mu^+ \nu$  decay constant.



G.P. Lepage, S.J. Brodsky, Phys.Lett. **87B**(1979)359.

This only relies on asymptotic freedom in QCD, *i.e.*  $(\partial \alpha_s / \partial \mu) < 0$  as  $\mu \rightarrow \infty$ .

**$Q^2 F_\pi$  should behave like  $\alpha_s(Q^2)$  even for moderately large  $Q^2$ .**

→ Pion form factor seems to be best tool for experimental study of nature of the quark-gluon coupling constant renormalization.

[A.V. Radyushkin, JINR 1977, arXiv:hep-ph/0410276]

# The pion is the “positronium atom” of QCD, its form factor is a test case for most model calculations

- **What is the structure of the  $\pi^+$  at all  $Q^2$ ?**
  - at what value of  $Q^2$  will the pQCD contributions dominate?
- A difficult question to answer, as both “hard” and “soft” components (such as gluonic effects) must be taken into account.
  - non-perturbative hard components of higher twist strongly cancel soft components, even at modest  $Q^2$ .  
*[Braun et al., PRD 61(2000)073004]*
  - the situation for nucleon form factors is even more complicated.
- **Many model calculations exist, but ultimately...**
  - **Reliable  $F_\pi(Q^2)$  data are needed to delineate the role of hard versus soft contributions at intermediate  $Q^2$ .**
- **A program of study unique to Jefferson Lab** (until the completion of the EIC)

# Measurement of $F_\pi$ via Electroproduction

**Above  $Q^2 > 0.3 \text{ GeV}^2$** ,  $F_\pi$  is measured indirectly using the “pion cloud” of the proton via pion electroproduction  $p(e, e' \pi^+) n$

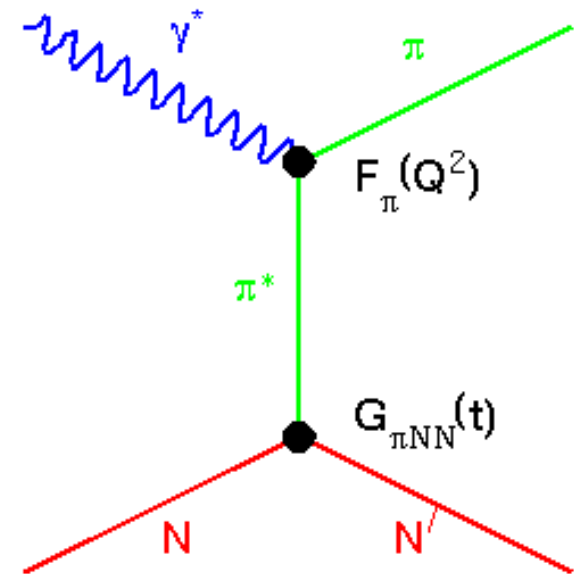
$$|p\rangle = |p\rangle_0 + |n\pi^+\rangle + \dots$$

- At small  $-t$ , the pion pole process dominates the longitudinal cross section,  $\sigma_L$
- In Born term model,  $F_\pi^2$  appears as

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t - m_\pi^2)} g_{\pi NN}^2(t) F_\pi^2(Q^2, t)$$

## Drawbacks of this technique:

1. Isolating  $\sigma_L$  experimentally challenging.
2. The  $F_\pi$  values are in principle dependent upon the model used, but this dependence is expected to be reduced at sufficiently small  $-t$ .

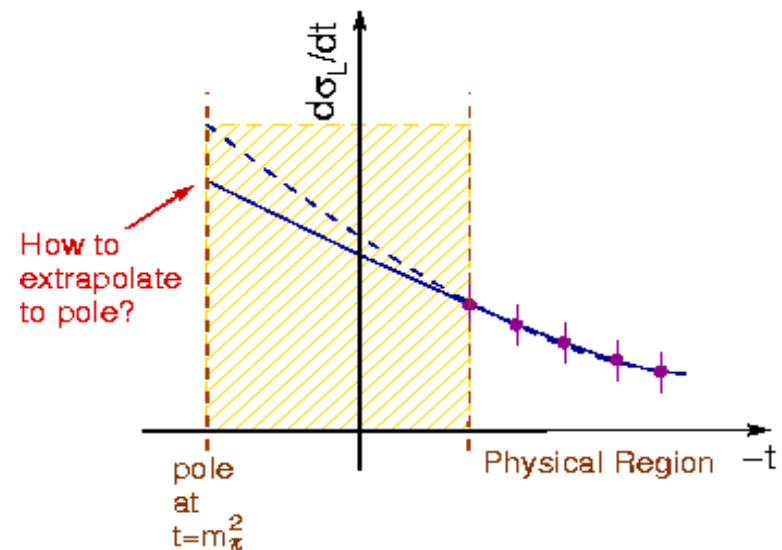


# Extraction of form factor from $\sigma_L$ data

$p(e, e' \pi^+)n$  data are obtained some distance from the  $t=m_\pi^2$  pole.

- No reliable phenomenological extrapolation possible.

A more reliable approach is to use a model incorporating the  $\pi^+$  production mechanism and the 'spectator' nucleon to extract  $F_\pi$  from  $\sigma_L$ .



Our philosophy is to publish our experimentally measured  $d\sigma_L/dt$ , so that updated values of  $F_\pi(Q^2)$  can be extracted as better models become available.



# E12-19-006 Forward Angle Requirements

- This experiment in large part has driven the forward angle requirements of the SHMS+HMS



Test of SHMS at 5.69° in Aug 2018

## Requirements for Fall 2021 Run:

Setting	Beam Energy	$\theta_{\text{SHMS}}$	$\theta_{\text{HMS}}$	$\theta_{\text{OPEN}}$
$Q^2=1.60$ $W=3.08$	9.20	6.28°	<b>12.34°</b>	<b>18.62°</b>
$Q^2=3.85$ $W=3.07$	8.00	<b>5.50°</b>	34.15°	39.65°
$Q^2=5.00$ $W=2.95$	8.00	6.35°	42.91°	49.26°
$Q^2=6.00$ $W=3.19$	9.20	<b>5.50°</b>	46.43°	51.93°
$Q^2=8.50$ $W=2.79$	9.20	<b>5.52°</b>	<b>57.70°</b>	63.22°

- Steve Lassiter has been working on  $\theta_{\text{SHMS}}=5.50^\circ$  requirement
- SHMS+HMS minimum opening angle has also been investigated

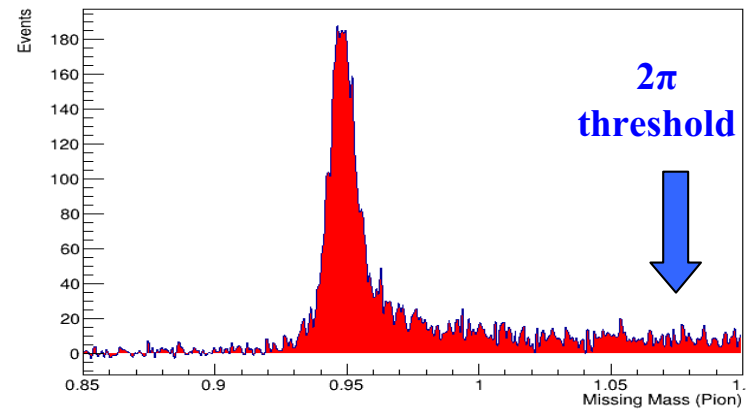
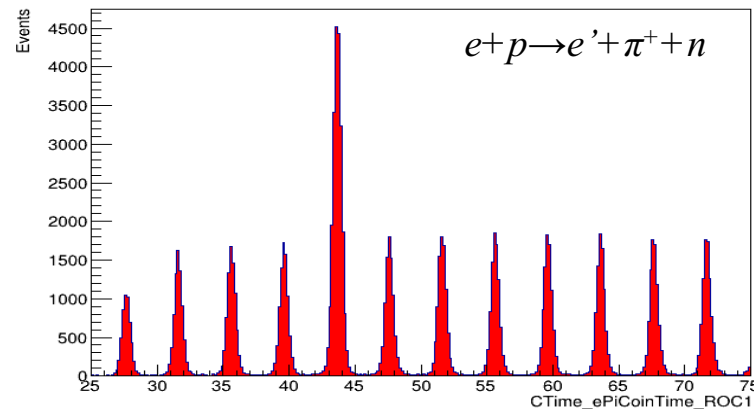
# $p(e, e'\pi^+)n$ Event Selection

## Coincidence measurement between charged pions in SHMS and electrons in HMS.

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Easy to isolate  
exclusive channel

- Excellent particle identification
- CW beam minimizes “accidental” coincidences
- Missing mass resolution easily excludes 2-pion contributions



Sample data from Kaon-LT experiment E12-09-011

$Q^2=3.0$ ,  $W=3.14$ ,  $x=0.25$ , low  $\varepsilon$  Run: 8045

$E_{\text{beam}}=8.186$  GeV,  $P_{\text{SHMS}}=+6.0530$  GeV/c,  $\theta_{\text{SHMS}}=6.910^\circ$

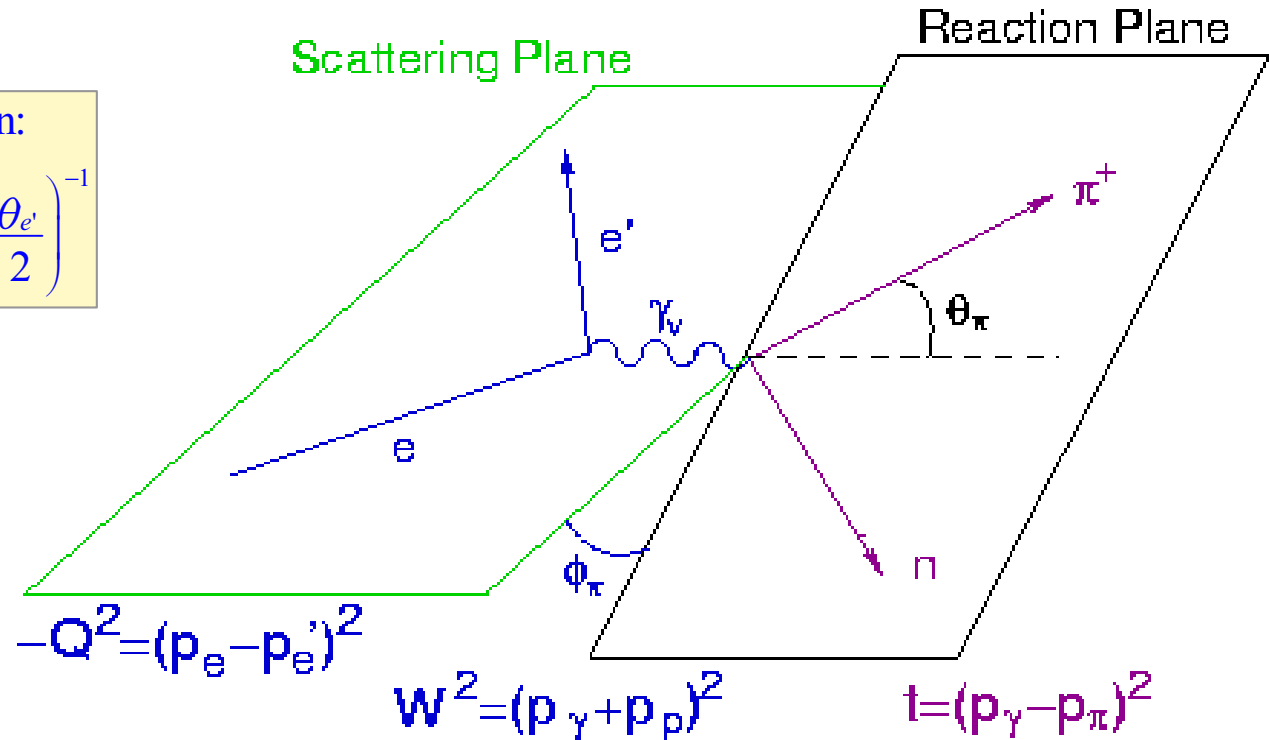
Plots by Vijay Kumar

$$2\pi \frac{d^2\sigma}{dtd\phi} = \varepsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon(\varepsilon+1)} \frac{d\sigma_{LT}}{dt} \cos\phi + \varepsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi$$



Virtual-photon polarization:

$$\varepsilon = \left( 1 + 2 \frac{(E_e - E_{e'})^2 + Q^2 \tan^2 \frac{\theta_{e'}}{2}}{Q^2} \right)^{-1}$$



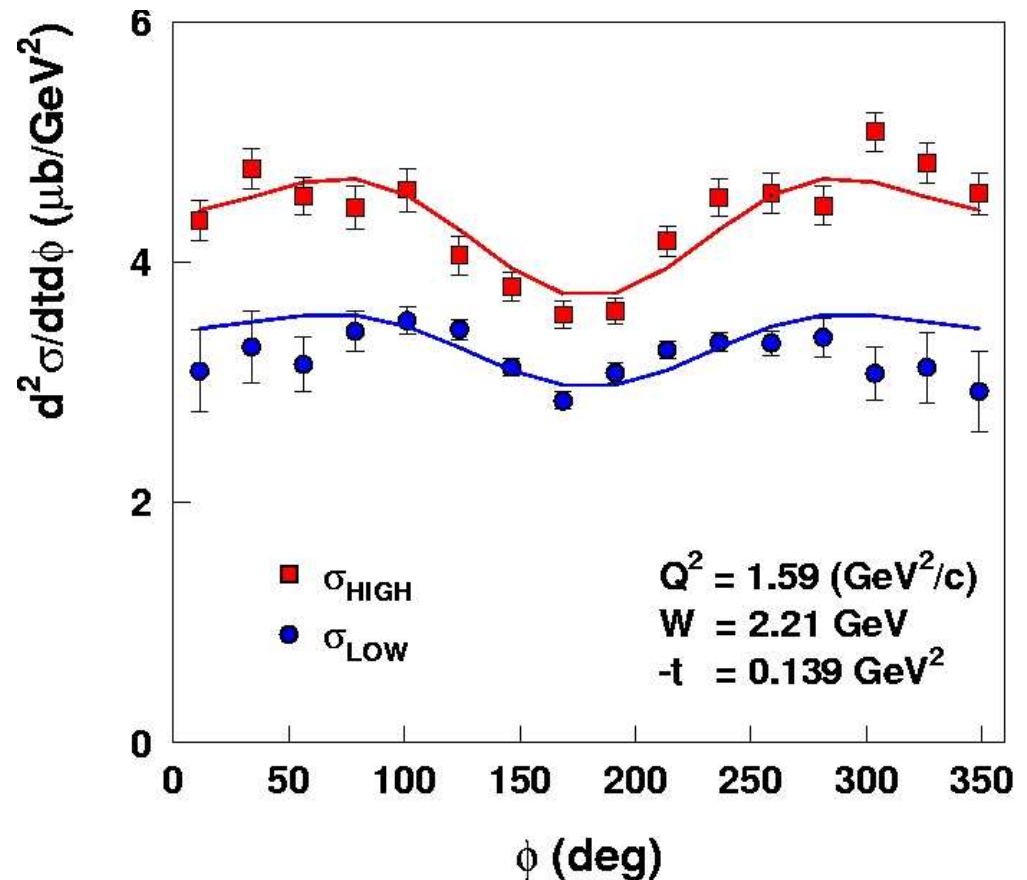
**Extraction of  $F_\pi$  requires  $t$  dependence of  $\sigma_L$  to be known.**

- Only three of  $Q^2$ ,  $W$ ,  $t$ ,  $\theta_\pi$  are independent.
- Vary  $\theta_\pi$  to measure  $t$  dependence.
- Since non-parallel data needed, LT and TT must also be determined.

# The different pion arm (SHMS) settings are combined to yield $\phi$ -distributions for each $t$ -bin

$$2\pi \frac{d^2\sigma}{dtd\phi} = \varepsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon(\varepsilon+1)} \frac{d\sigma_{LT}}{dt} \cos\phi + \varepsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi$$

- Extract all four response functions via a simultaneous fit using measured azimuthal angle ( $\phi_\pi$ ) and knowledge of photon polarization ( $\varepsilon$ ).
- **This technique demands good knowledge of the magnetic spectrometer acceptances.**
- **Control of point-to-point systematic uncertainties crucial due to  $1/\Delta\varepsilon$  error amplification in  $\sigma_L$**
- Careful attention must be paid to spectrometer acceptance, kinematics, efficiencies, ...



*T. Horn, et al, PRL 97 (2006)192001*

# Magnetic Spectrometer Calibrations

Uncertainties from  $F_\pi$  Proposal (E12-06-101)

- Similarly to  $F\pi$ -2, we plan to use the over-constrained  $p(e, e'p)$  reaction and inelastic  $e+^{12}\text{C}$  in the DIS region to calibrate spectrometer acceptances, momenta, offsets, etc.
  - $F\pi$ -2 beam energy and spectrometer momenta determined to  $<0.1\%$ .
  - Spectrometer angles  $<0.5$  mr.
  - $F\pi$ -2 agreement with published  $p+e$  elastics cross sections  $<2\%$ .

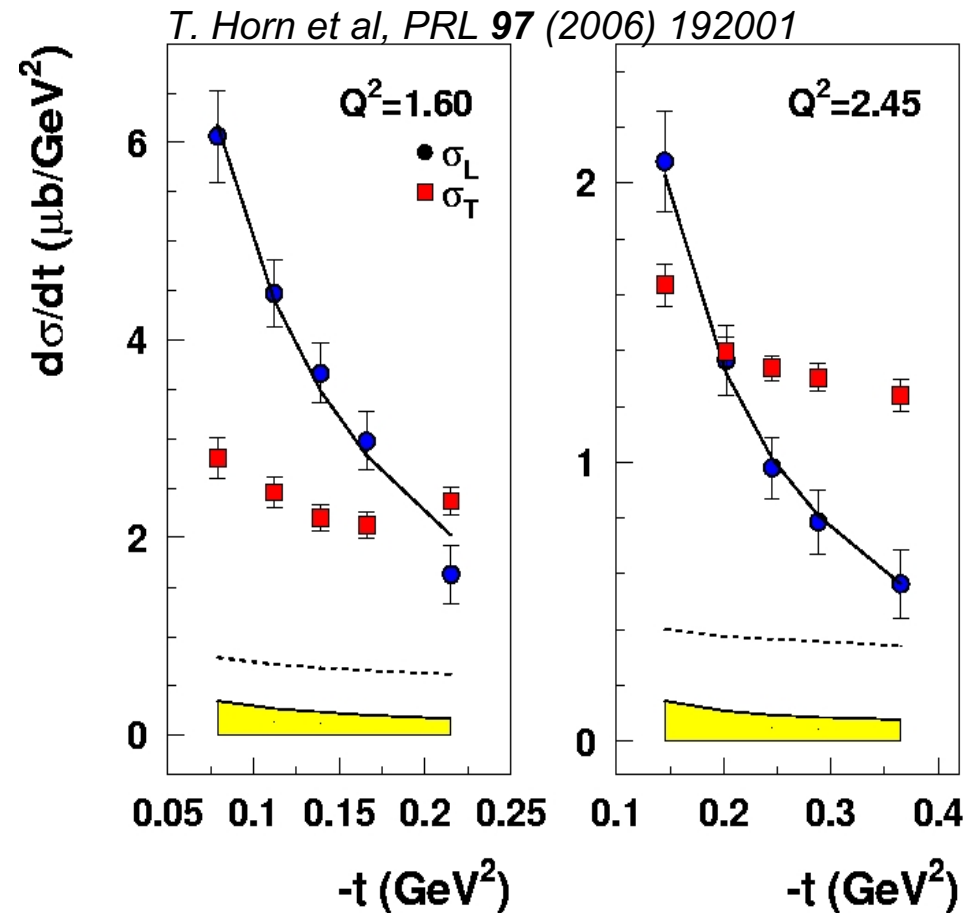
Projected Systematic Uncertainty Source	Pt-Pt $\epsilon$ -random t-random	$\epsilon$ -uncorrelated common to all t-bins	Scale $\epsilon$ -global t-global
Spectrometer Acceptance	0.4%	0.4%	1.0%
Target Thickness		0.2%	0.8%
Beam Charge	-	0.2%	0.5%
HMS+SHMS Tracking	0.1%	0.4%	1.5%
Coincidence Blocking		0.2%	
PID		0.4%	
Pion Decay Correction	0.03%	-	0.5%
Pion Absorption Correction	-	0.1%	1.5%
MC Model Dependence	0.2%	1.0%	0.5%
Radiative Corrections	0.1%	0.4%	2.0%
Kinematic Offsets	0.4%	1.0%	-

- Uncorrelated uncertainties in  $\sigma_{UNS}$  are amplified by  $1/\Delta\epsilon$  in L/T separation.
- Scale uncertainty propagates directly into separated cross section.

# $F_\pi$ Extraction from JLab data

- Model is required to extract  $F_\pi$  from  $\sigma_L$
- JLab  $F_\pi$  experiments used the VGL Regge model
  - Propagator replaced by  $\pi$  and  $\rho$  Regge trajectories
  - Most parameters fixed by photoproduction data
  - 2 free parameters:  $\Lambda_\pi, \Lambda_\rho$
  - At small  $-t$ ,  $\sigma_L$  only sensitive to  $\Lambda_\pi$

$$F_\pi(Q^2) = \frac{1}{1 + Q^2 / \Lambda_\pi^2}$$



New model by R. Perry, A. Kizilersu, A.W. Thomas [PLB 807 (2020) 135581] may allow a second way to extract  $F_\pi$  from  $\sigma_L$  data

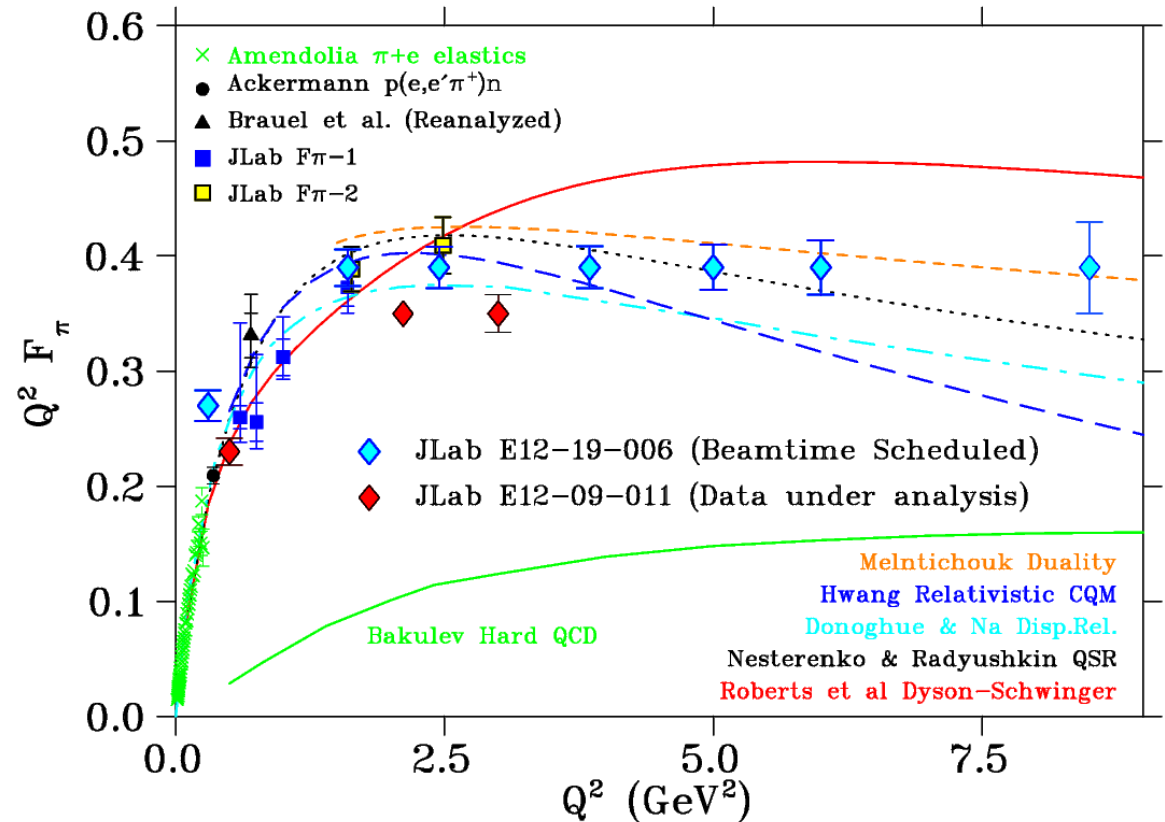
# Current and Projected $F_\pi$ Data

SHMS+HMS will allow measurement of  $F_\pi$  to much higher  $Q^2$ .

**No other facility worldwide can perform this measurement.**

New overlap points at  $Q^2=1.6, 2.45$  will be closer to pole to constrain  $-t_{min}$  dependence.

New low  $Q^2$  point (data acquired in 2019) will provide comparison of the electroproduction extraction of  $F_\pi$  vs. elastic  $\pi+e$  data.

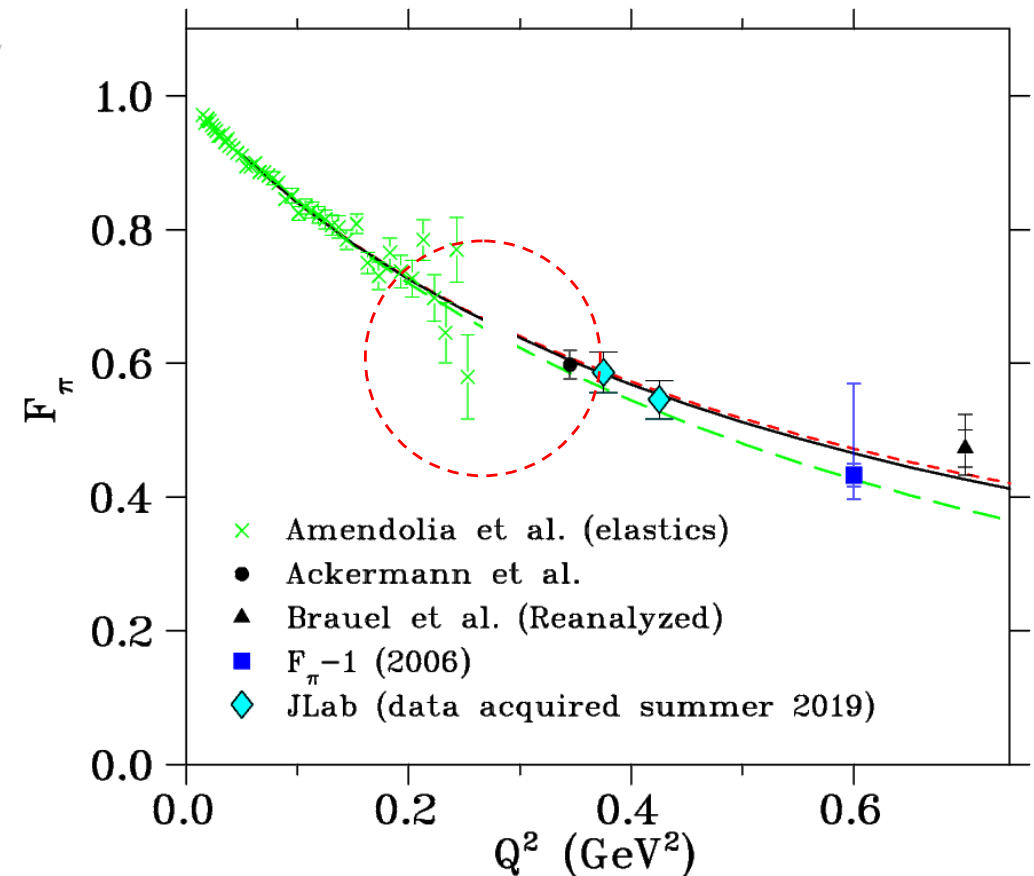


The  $\sim 10\%$  measurement of  $F_\pi$  at  $Q^2=8.5 \text{ GeV}^2$  is at higher  $-t_{min}=0.45 \text{ GeV}^2$

**The pion form factor is the clearest test case for studies of QCD's transition from non-perturbative to perturbative regions.**

# Check of Pion Electroproduction Technique

- Does electroproduction really measure the on-shell form-factor?
- Test by making  $p(e, e' \pi^+) n$  measurements at same kinematics as  $\pi^+ e$  elastics.
- **Can't quite reach the same  $Q^2$ , but electro-production appears consistent with extrapolated elastic data.**



**Data for new test acquired in Summer 2019:**

- **small  $Q^2$  (0.375, 0.425) competitive with DESY  $Q^2=0.35$**
- **$-t$  closer to pole ( $=0.008$   $\text{GeV}^2$ ) vs. DESY 0.013**

***A similar test for  $K^+$  form factor is part of Kaon-LT***



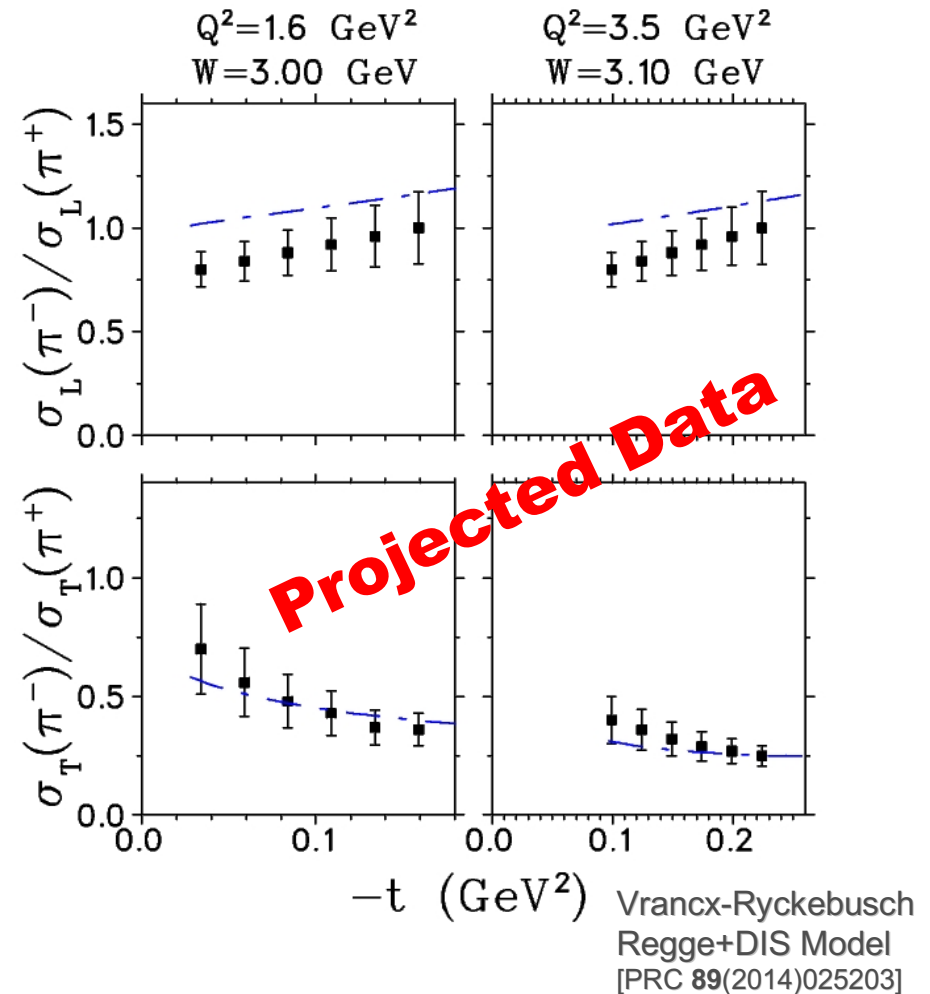
# Verify that $\sigma_L$ is dominated by $t$ -channel process

- $\pi^+$   $t$ -channel diagram is purely isovector.
- Measure

$$R_L = \frac{\sigma_L[n(e, e' \pi^-) p]}{\sigma_L[p(e, e' \pi^+) n]} = \frac{|A_V - A_S|^2}{|A_V + A_S|^2}$$

using a deuterium target.

- Isoscalar backgrounds (such as  $b_1(1235)$  contributions to the  $t$ -channel) will dilute the ratio.
- We will do the same tests at  $Q^2=1.60, 3.85, 6.0 \text{ GeV}^2$ .

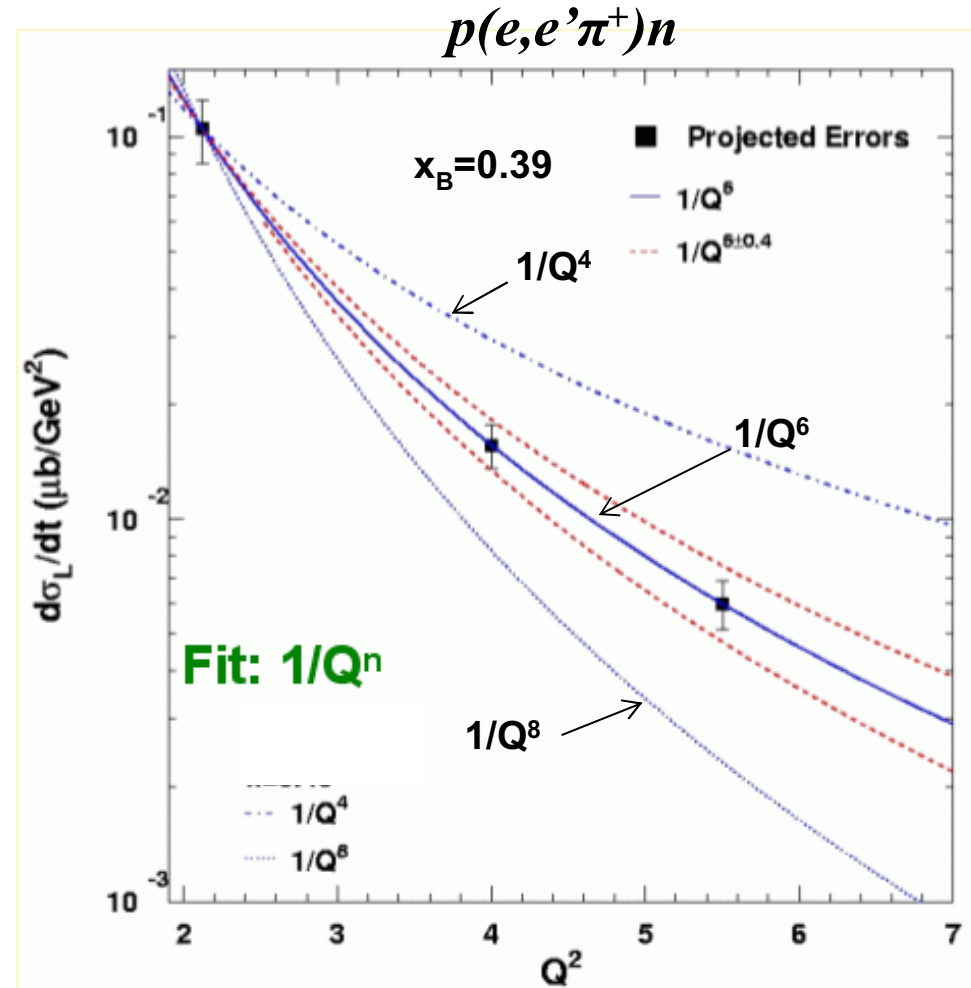


Because one of the many problems encountered by the historical data was isoscalar contamination, this test will increase the confidence in the extraction of  $F_\pi(Q^2)$  from our  $\sigma_L$  data.

# $p(e, e' \pi^+) n$ $Q^{-n}$ Hard–Soft Factorization Test

- QCD counting rules predict the  $Q^{-n}$  dependence of  $p(e, e' \pi^+) n$  cross sections in Hard Scattering Regime:
  - $\sigma_L$  scales to leading order as  $Q^{-6}$ .
  - $\sigma_T$  scales as  $Q^{-8}$ .
  - As  $Q^2$  becomes large:  $\sigma_L \gg \sigma_T$ .

$x$	$Q^2$ (GeV <sup>2</sup> )	$W$ (GeV)	$-t_{min}$ (GeV/c) <sup>2</sup>
0.31	1.45–3.65	2.02–3.07	0.12
0.39	2.12–6.0	2.05–3.19	0.21
0.55	3.85–8.5	2.02–2.79	0.55

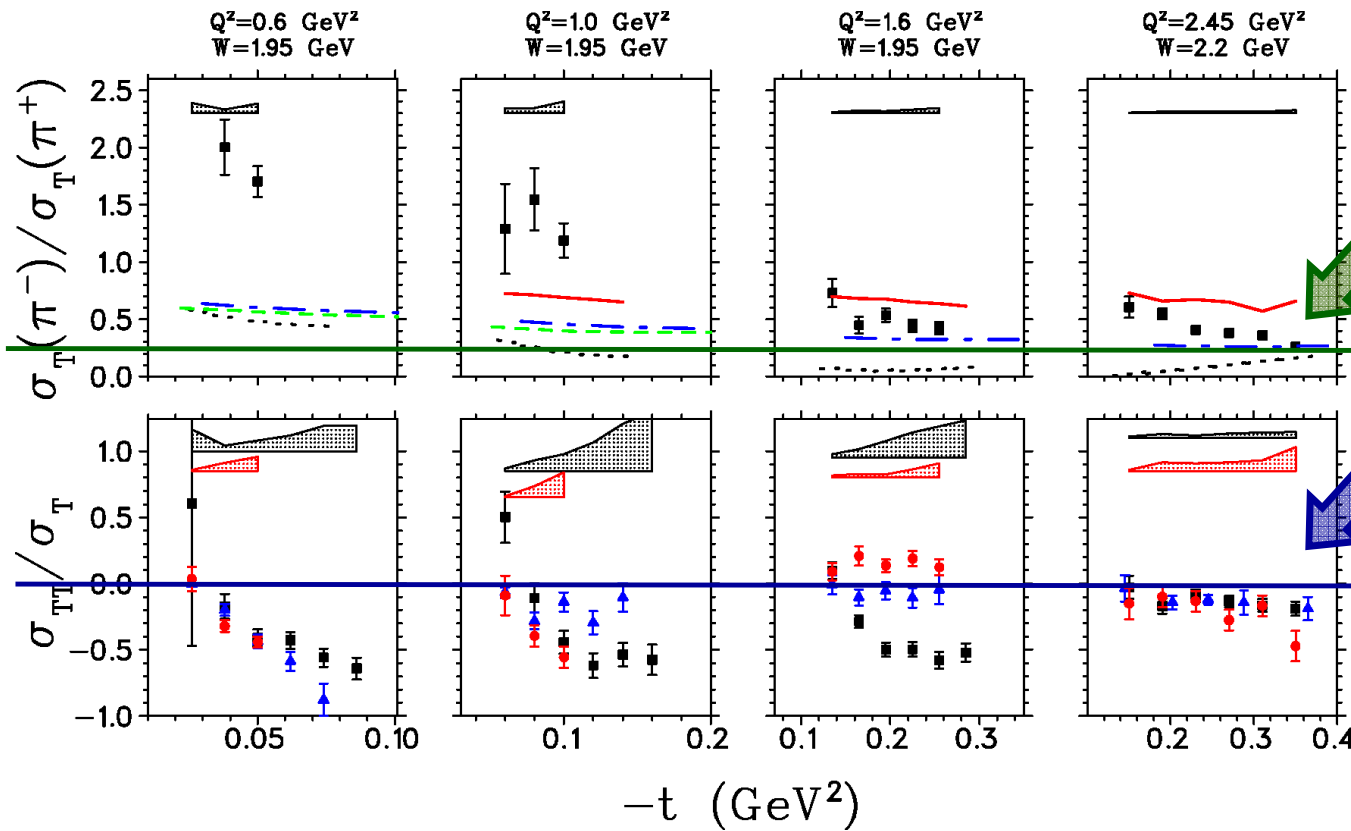


- **Experimental validation of onset of hard scattering regime is essential for reliable interpretation of JLab GPD program results.**
  - If  $\sigma_L$  becomes large, it would allow leading twist GPDs to be studied.
  - If  $\sigma_T$  remains large, it could allow for transversity GPD studies.

# $\pi^-/\pi^+$ Hard-Soft Factorization Test

- **Transverse Ratios tend to  $1/4$  as  $-t$  increases:**
  - Is this an indication of Nachtmann's quark charge scaling?
- **$-t=0.3 \text{ GeV}^2$  seems too low for this to apply. Might indicate the partial cancellation of soft QCD contributions in the formation of the ratio.**

A. Nachtmann, Nucl.Phys.B115 (1976) 61.



$$R_T \rightarrow \frac{2Q_d^2}{2Q_u^2} = \frac{1}{4}$$

- Another prediction of quark-parton mechanism is the suppression of  $\sigma_{TT}/\sigma_T$  due to  $s$ -channel helicity conservation.
- Data qualitatively consistent with this, since  $\sigma_{TT}$  decreases more rapidly than  $\sigma_T$  with increasing  $Q^2$ .

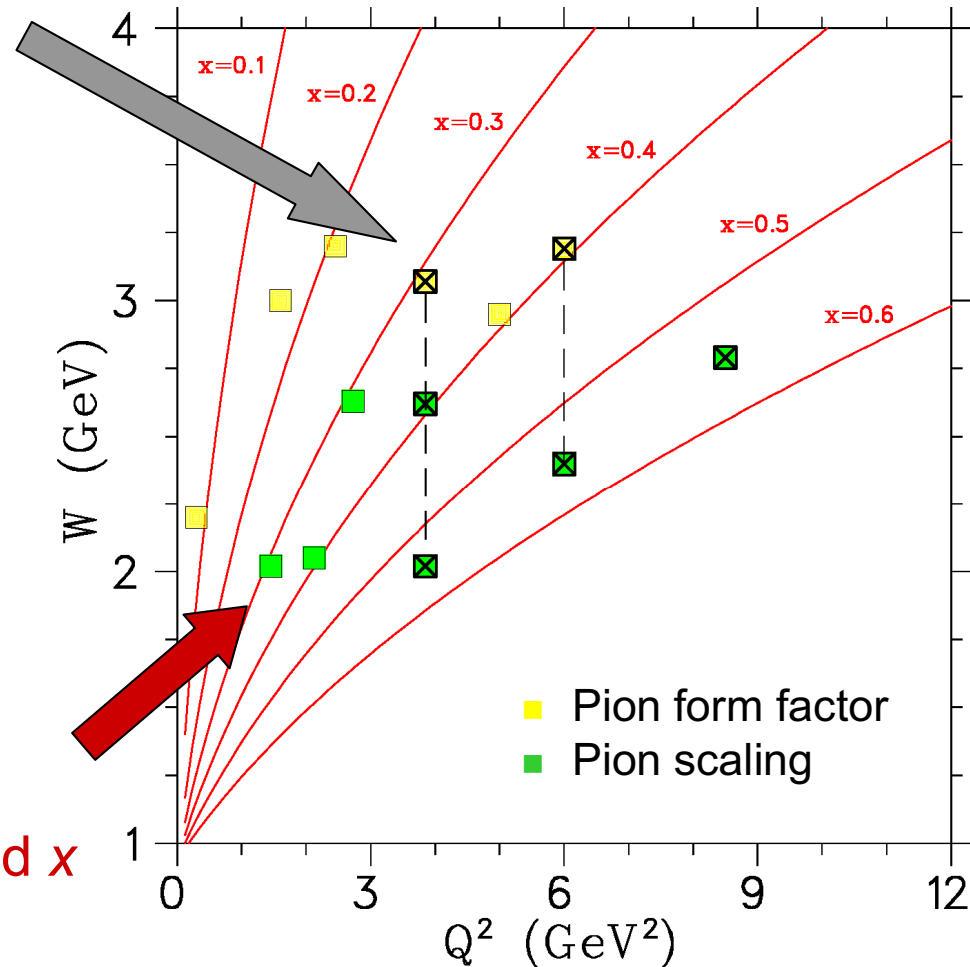
$^2\text{H}(e,e'\pi^+)n$     $^2\text{H}(e,e'\pi)pp$     $^1\text{H}(e,e'\pi^+)n$

# E12-19-006 Optimized Run Plan

Points along vertical lines allow  $F_\pi$  values at different distances from pion pole, to check the model properly accounts for:

- $\pi^+$  production mechanism
- spectator nucleon
- off-shell ( $t$ -dependent) effects.

Points along red curves allow  $1/Q^n$  scaling tests at fixed  $x$



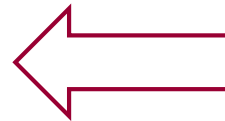
For more details, visit Pion-LT RedMine: <https://redmine.jlab.org/projects/hall-c/wiki/>

# Strong Endorsement in many Reviews

Report to PAC18, 12 GeV Session:  
Measuring  $F_\pi$  at Higher  $Q^2$

G.M. Huber, H.P. Blok, D.J. Mack  
on behalf of the Exclusive Reactions Working Group

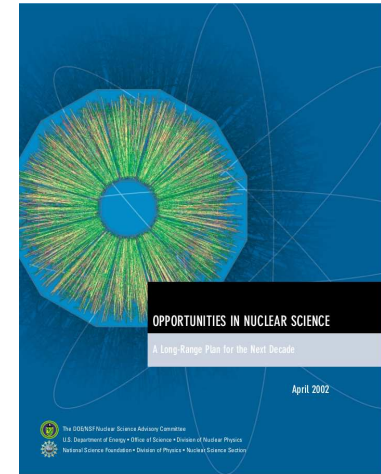
July 6, 2000



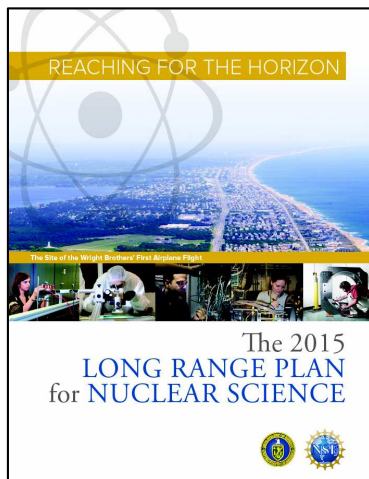
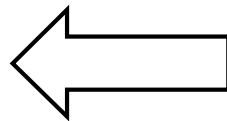
$F_\pi$  first proposed to JLab PAC in 2000!

$F_\pi$  Rated “Early  
High Impact” by  
PAC35 in 2010

$F_\pi$  endorsed by NSAC  
in 2002, as one of the  
key motivations for the  
12 GeV Upgrade.



$F_\pi$  endorsed again by NSAC in 2015,  
“as one of the flagship goals of the  
JLab 12 GeV Upgrade”.



**PAC47 (2019) Theory Report:**

**“Since the proposals were originally reviewed,  
the physics motivations for BOTH studies have  
only increased.”**

**“A” rating reaffirmed by PAC for BOTH studies.**

# 2021 Run Plan Outline

- **Aug 23 – Oct 16: 9.2 GeV  
(5 pass @ 1.82 GeV/pass)**

Q <sup>2</sup>	W	Target & SHMS polarity
1.6	3.08	LH+, LD+, LD-
6.0	3.19	LH+
8.5	2.79	LH+

- **Nov 28 – Dec 11: 9.9 GeV  
(5 pass @ 1.96 GeV/pass)**

Q <sup>2</sup>	W	Target & SHMS polarity
3.85	3.07	LH+
5.0	2.95	LH+
6.0	3.19	LH+

- **Oct 20 – Nov 27: 8.0 GeV  
(4 pass @ 1.96 GeV/pass)**

Q <sup>2</sup>	W	Target & SHMS polarity
6.0	2.40	LH+, LD+, LD-
2.45	3.20	LH+
3.85	3.07	LH+, LD+, LD-
5.0	2.95	LH+

- **Dec 12 – 14: 6.0 GeV  
(3 pass @ 1.96 GeV/pass)**

Q <sup>2</sup>	W	Target & SHMS polarity
3.85	2.02	LH+

- **Dec 14 – 21: Schedule  
Contingency**

These are primarily the low  $\epsilon$  settings

Additional 9 weeks of beam for high  $\epsilon$  data scheduled in Fall 2022

# E12-19-006 Important Group Members

- Spokespersons: Dave Gaskell, Tanja Horn, GH
- Graduate Students on the Experiment:



Jacob Murphy  
Ohio U.



Muhammad Junaid  
U. Regina



Nathan Heinrich  
U. Regina

- Run Coordinators (2021):
  - DG, Mark Jones, Simona Malace (2x), Stephen Kay, Douglas Higinbotham, Wenliang Li, Carlos Yero, Holly Szumilla-Vance, Arun Tadepalli, Gabriel Niculescu, Ciprian Gal, Dave Mack, Vladimir Berdnikov
- If interested in joining the team, please contact DG, TH, or GH

# A 12 GeV Flagship Experiment



Garth Huber, huberg@uregina.ca

- **E12–19–006 is expected to provide the definitive  $p(e, e' \pi^+)n$  L/T–separation data set, and will remain important for decades to come**
- **$F_\pi -1$  and  $F_\pi -2$  experiments were very productive, and are among JLab’s top cited results (top 4 listed):**
  - Volmer et al, PRL 2001 ( $F_\pi -1$ ) 333 citations
  - Horn et al, PRL 2006 ( $F_\pi -2$ ) 273 citations
  - Tadevosyan et al, PRC 2007 ( $F_\pi -1$ ) 224 citations
  - Huber et al, PRC 2007 ( $F_\pi -2$ ) 217 citations
- **WE REALLY NEED YOUR ASSISTANCE TO MAKE THE EXPERIMENT A SUCCESS!!**
  - Fall 2021: 756 person shifts needed @ 2 workers/shift
  - We would like co-authors to take shifts in both 2021 and 2022 runs. 2021 requirement: 10 shifts. 2022: ~5 shifts.  
Otherwise, please contact DG, TH, GH for alternate arrangements
  - Shift sign up now open at:  
<https://misportal.jlab.org/mis/apps/physics/shiftSchedule/index.cfm?experimentRunId=HALLC-PIONLT>