An introduction to:

A study of the $e^+e^- \to \pi^+\pi^-\pi^0$ process using initial state radiation

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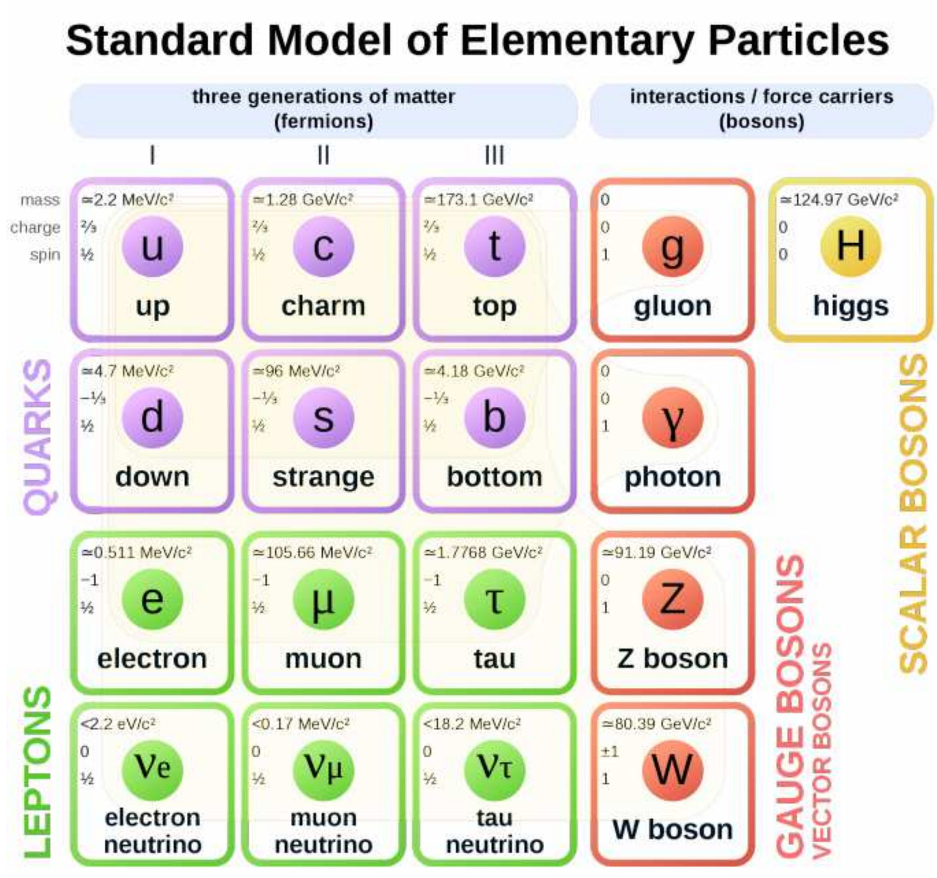
The Standard Model

- **Our understanding the fundamental nature of matter has made huge progress in the last ~150 years.**
- 1897 discovery of the electron by J.J. Thompson provided the first evidence of point-like constituents of matter
- **Electron properties:**
  - Classical properties:
    - Mass: 0.511 MeV/c^2
    - Charge: –1 (1.6×10^{-19} C)
    - Pointlike charge distribution
  - Quantum properties:
    - Intrinsic spin: 1/2hbar
    - Lepton number: +1
    - Electron family number: +1

Many other fundamental constituents have been discovered since 1897



Spin-1/2 quarks and leptons

Spin-1 force carriers

Spin-0 Higgs boson

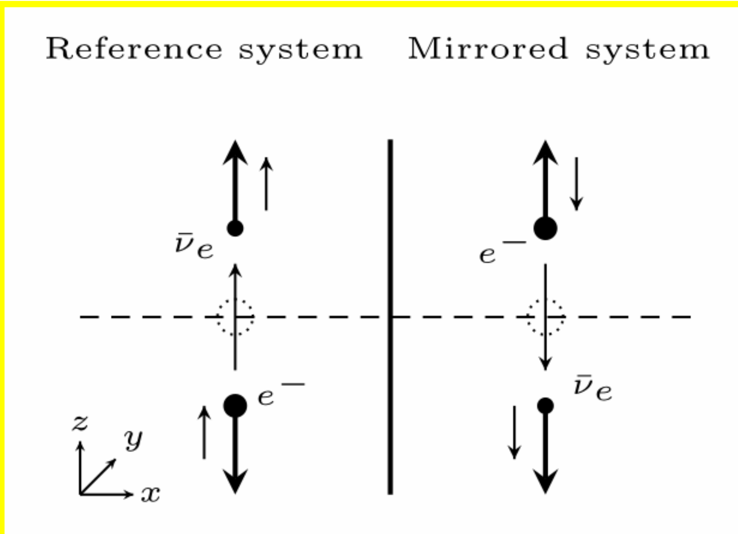
Symmetries of Particle Physics

- **Symmetries and conservation laws are a constant theme in physics, and nowhere more so than particle physics.**
  - If the laws of physics obey a certain symmetry, there must be a corresponding conservation law associated with it.
  - Symmetries provide a powerful framework for understanding why certain physics processes are allowed, while others are forbidden.
- **Parity (P):** reflect nature in a mirror $\mathbf{r}_i \rightarrow \mathbf{r}_i' = -\mathbf{r}_i$
  - Formally: apply the parity operator to a particle state

    $P|\psi\rangle = \eta|\psi\rangle$ where $\eta = \pm 1$ is a multiplicative factor called the eigenvalue
- **Charge Conjugation (C):** replace particles with anti-particles
  - e.g. replace electron e^- with the positron e^+
- **Time Reversal (T):** $t \rightarrow t' = -t$ keeping positions unchanged
- **Question:** Are particle interactions unchanged under P,C,T?

Parity Violation in Weak Interactions

- **Parity is not an exact symmetry of nature for all interactions**
- It was discovered in 1967 that Parity is violated in weak interactions, meaning that the mirrored system behaves separately than the reference system



$$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni}^* + e^- + \bar{\nu}_e$$

C.S. Wu experiment:

- β-decay of polarized ^{60}Co
- Electrons emitted parallel or anti-parallel to ^{60}Co spin
- Parity invariance means the two situations can't be distinguished
- Wu's measurements found the transition rates for the two situations are different
- This is now understood to be because the ν is ONLY left handed in nature

- **Parity violation has been confirmed in all weak interactions, such as decay of $\pi^\pm$, which decay preferentially into $\mu^\pm$ instead of $e^\pm$**

$$\pi^\pm \longrightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$$
$$\mu^\pm \longrightarrow e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$$

$$\frac{\Gamma(\pi^+ \rightarrow e^+\nu_e)}{\Gamma(\pi^+ \rightarrow \mu^+\nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.26\times 10^{-4}$$

C and CP Symmetries

- **If C invariance is a valid symmetry, processes with particles and anti-particles should occur with same probability**
- If we apply the C operator to the K^0 particle, we get the anti-particle
  - An "eigenstate of C" would mean we get the same particle back plus the eigenvalue η, so K^0 is NOT an eigenstate of C
  - K^0 is also NOT an eigenstate of Charge+Parity: CP

$$C|K^0\rangle = |\bar{K}^0\rangle \qquad CP|K^0\rangle = -|\bar{K}^0\rangle$$
$$C|\bar{K}^0\rangle = |K^0\rangle \qquad CP|\bar{K}^0\rangle = -|K^0\rangle$$

- **But the superposition of these two states IS an eigenstate of CP**

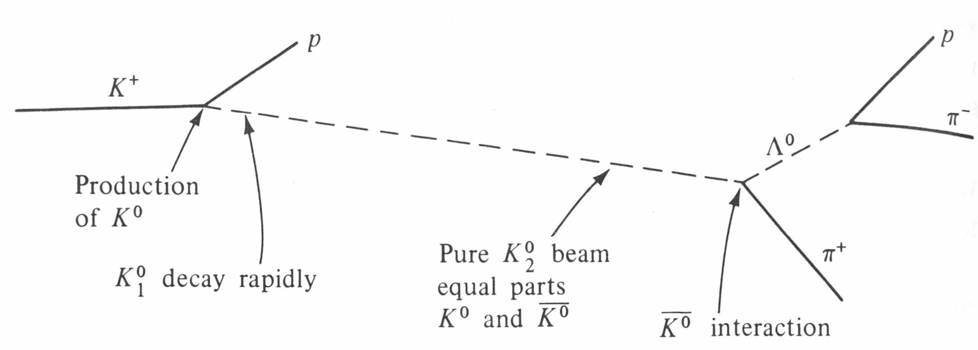
$$|K_1\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}} \qquad |K_2\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}} \qquad CP|K_1\rangle = +|K_1\rangle \qquad CP|K_2\rangle = -|K_2\rangle$$

- **If CP is conserved in weak interactions, we should observe distinct K_1 K_2 particles with unequal masses**
  - Their weak decays should be easy to distinguish experimentally: $K_1 \rightarrow 2\pi$ (CP-even, short lifetime), while $K_2 \rightarrow 3\pi$ (CP-odd, long lifetime)

CP Violation in Weak Interactions

CP Eigenstates: $K_1 \rightarrow 2\pi$ (CP-even) $K_2 \rightarrow 3\pi$ (CP-odd)

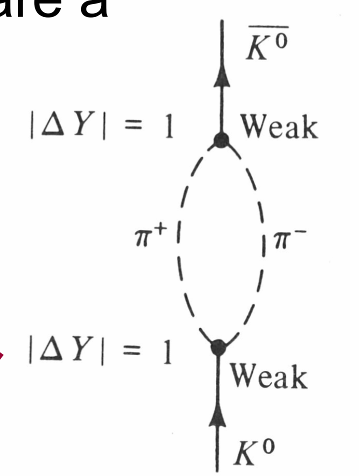
- First tested at Princeton in 1964 by studying K^0 particle decays



- **If CP is conserved in weak interactions, the transition of CP-even (K_1) to CP-odd state (K_2) should not occur**
- But this was observed with small probability, indicating that ~0.3% of the weak decays violated CP symmetry
- This violation implies that the short and long lifetime states are a mixture of the flavor states with unequal balance, ε=0.00223

$$\begin{bmatrix} |K_S\rangle \\ |K_L\rangle \end{bmatrix} \sim \begin{bmatrix} 1+\varepsilon & -(1-\varepsilon) \\ 1+\varepsilon & 1-\varepsilon \end{bmatrix} \begin{bmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{bmatrix}$$

This phenomenon occurs because of virtual second-order weak transitions, such as



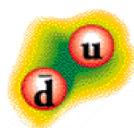
Hadrons are made of quarks

- **Quark model of hadrons** **Gell-Mann, Zweig 1964**

	$Q = -1/3$	$Q = +2/3$
$S = 0$	d	u
$S = -1$	s	

$m_d \cong m_u \cong 0.1$ GeV
$m_s \cong 0.30$ GeV

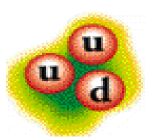
Mesons are bound states of quark-antiquark.



π^+

$$\pi^+ = u\bar{d},\quad \pi^- = d\bar{u},\quad \pi^o = \frac{1}{\sqrt{2}}(u\bar{u} - d\,\bar{d}),\quad k^+ = d\bar{s},\quad k^o = d\bar{s}$$

Baryons are bound states of three quarks.



p

proton = (uud), neutron = (udd), Λ= (uds)
anti-baryons are bound states of 3 anti-quarks:

$$\bar{p} = \bar{u}\,\bar{u}\,\bar{d} \qquad \bar{n} = \bar{u}\,\bar{d}\,\bar{d} \qquad \bar{\Lambda} = \bar{u}\,\bar{d}\,\bar{s}$$

These quark objects are:

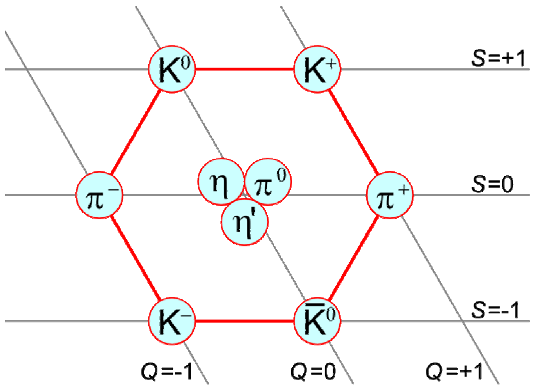
- point-like.
- spin 1/2 fermions.
- parity = +1
  (-1 for anti-quarks).
- two quarks are in isospin doublet (u and d),
  s is an iso-singlet (=0).
- For every quark there is an anti-quark.
- Quarks feel all interactions (have mass, electric charge, etc).

Making mesons and baryons with quarks

Making Mesons (orbital angular momentum L=0)

The properties of SU(3) tell us how many mesons to expect: $3 \otimes \overline{3} = 1 \oplus 8$

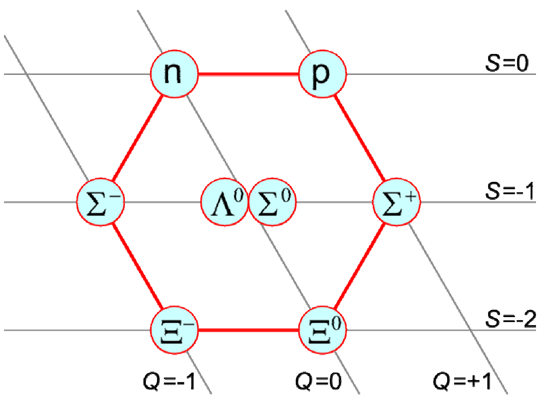
Thus we expect an octet with 8 particles and a singlet with 1 particle.

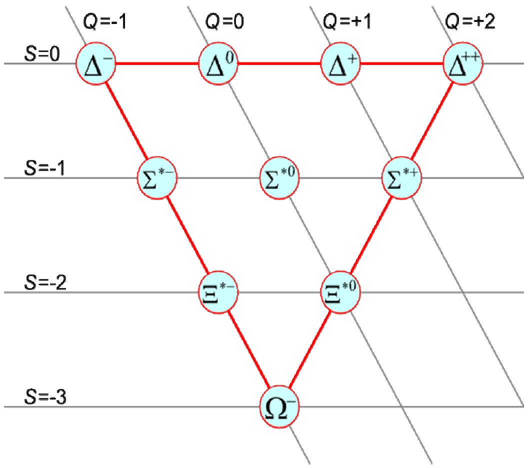


Making Baryons (orbital angular momentum L=0).

Now must combine 3 quarks together: $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$

Expect a singlet, 2 octets, and a decuplet (10 particles) $\Rightarrow$ 27 objects total.

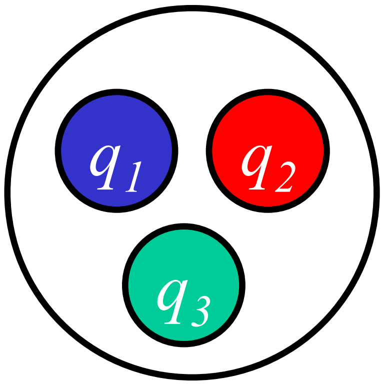




If SU(3) were a perfect symmetry then all particles in a multiplet would have the same mass.

Quarks carry color charge

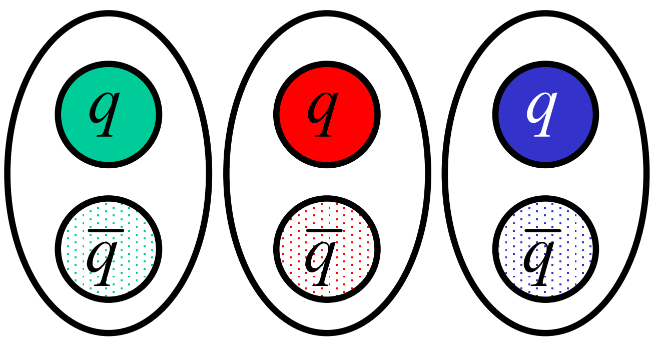
- **Without color, identical quarks would be forbidden from occupying the same space-spin state, and the Standard Model would be unable to explain the large number of hadron states observed by experiment.**
- Hadron wave function: $\Psi(hadron) = \psi(spin)\ \Phi(space)\ \chi(flavor)\ \varphi(color)$



BARYONS

RED + BLUE + GREEN = "WHITE"
or "COLORLESS"

MESONS



GREEN + ANTIGREEN = "COLORLESS"
RED + ANTIRED = "COLORLESS"
BLUE + ANTIBLUE = "COLORLESS"

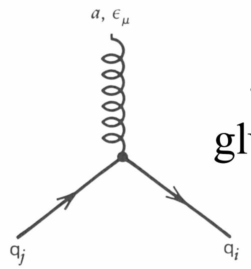
- Hadron color neutrality provides a natural explanation for why only baryons and mesons are observed in nature

Quantum Chromodynamics (QCD)

- **QCD: the fundamental theory of quark-gluon interactions in the Standard Model**

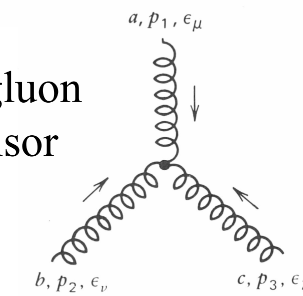
$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{\psi}_{q,a}\left(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu T^C_{ab} \mathcal{A}^C_\mu - m_q \delta_{ab}\right)\psi_{q,b} - \frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu}$$

$\Psi_{q,a}$ = quark spinor with flavor q and color index a

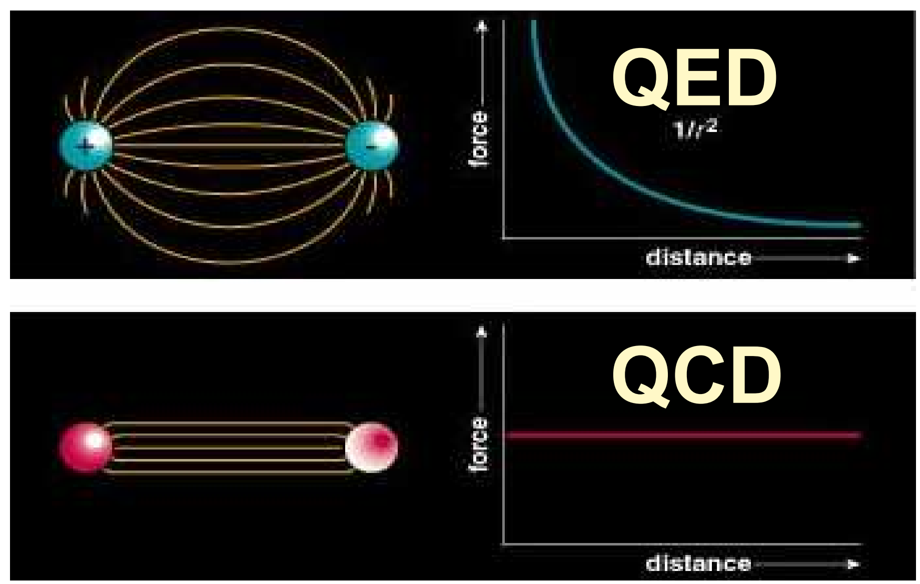


$\mathcal{A}_\mu^{\ C}$ = gluon field

$F^A_{\ \mu\nu}$ = gluon field tensor

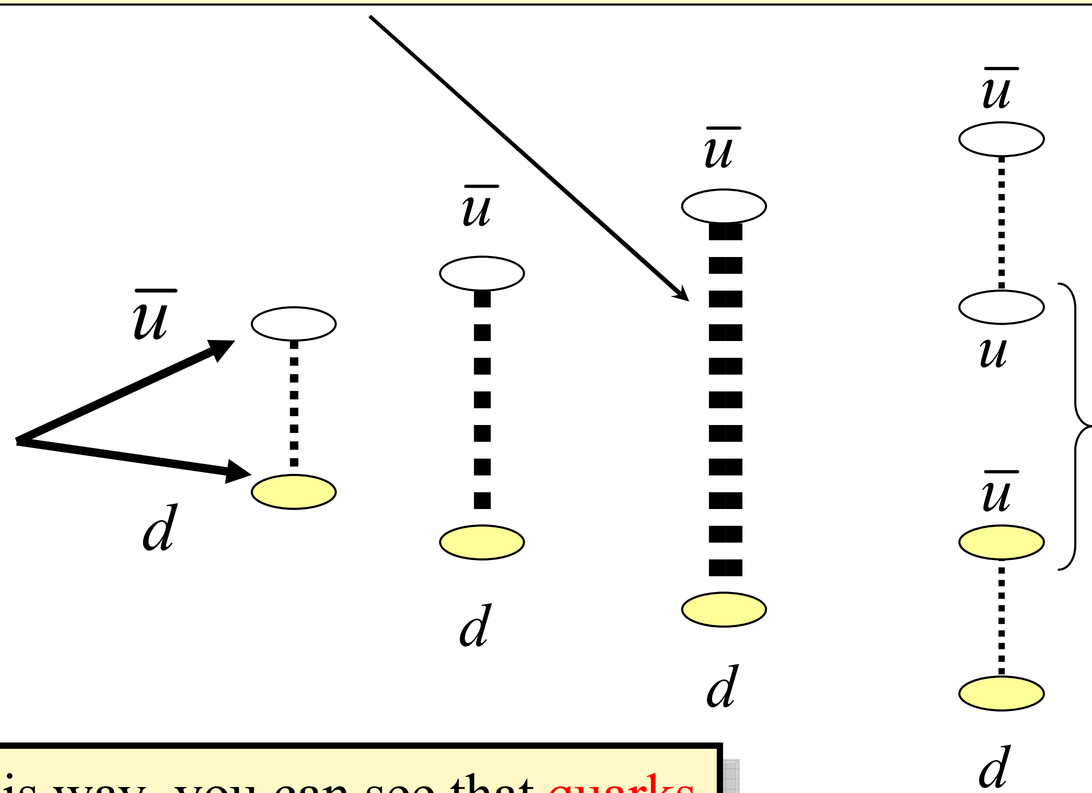


Because the gluons of QCD carry color charge, they interact strongly and the force between quarks does not drop rapidly with distance

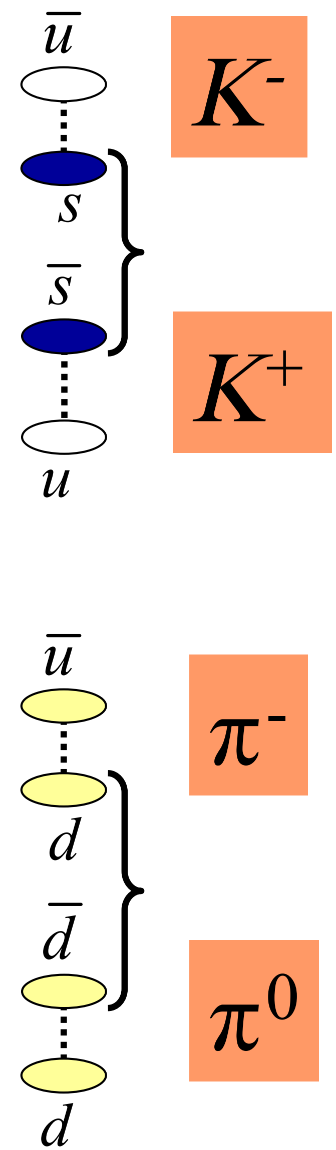


Color confinement within Hadrons

As quarks move apart, the potential energy associated with the "gluon spring"increases, until it is large enough to convert into mass energy (qq pairs).



Hadrons!



In this way, you can see that quarks are always confined inside hadrons (that's **CONFINEMENT**) !

Physics Beyond the Standard Model

- **Despite its many successes, there are big open questions in particle physics which point to where our knowledge is incomplete:**
  - The nature of leptons:
    - **How are $\mu^{\pm}$ different from $e^{\pm}$?**
    - Is the ν its own anti-particle, and how is this tied to the lack of right handed ν?
  - The nature of color confinement in QCD
    - Can we use QCD to predict the observed properties of hadrons?
    - Exactly how does baryon number originate, and can it be violated?
  - Why is there so little anti-matter in the universe?
  - The nature of Dark Matter?
    - What is it, and how many types are there?
  - The nature of Dark Energy?
    - Does it correspond to an unknown piece of particle physics, or is it instead due to the nature of space-time itself?

Muon Anomalous Magnetic Moment

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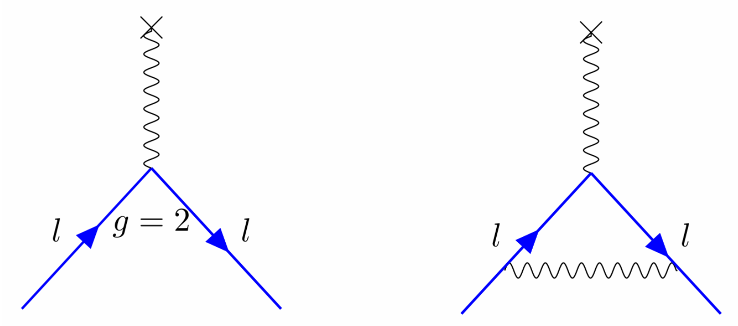
- A charged spin-1/2 lepton or quark with charge *q* and mass *m* has an intrinsic magnetic moment $\vec{\mu} = g \frac{q}{2m} \vec{s}$

- To leading order in quantum mechanics, *g*=2
- Quantum corrections like at right make *g*>2



- The anomalous magnetic moment is the deviation of *g* from 2

$$a_l = \frac{(g-2)_l}{2}$$

- Many types of particles can be exchanged in the loop giving rise to (g-2), from electromagnetic (QED), electroweak (EW) and hadronic (had) processes

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{had}}$$

- **A measurement of the anomalous magnetic moment of a charged lepton is one of our most reliable probes of Physics Beyond the Standard Model**

Muon (g-2) QED contributions

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- **Because the electron and muon are relatively low mass and have long lifetimes, they are the particles of choice for (g-2) studies**
- **Due to their electric charge, the electromagnetic contribution to the anomalous magnetic moment dominates**
- It is calculated as a perturbative expansion in terms of the fine structure constant α

$$a_\mu^{\mathrm{QED}} = \sum_{n=1}^{5} \left(\frac{\alpha}{\pi}\right)^n a_\mu^{(2n)}$$

- Explicitly for terms n=1-5 we have

$$a_\mu^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\mu/m_e) + A_2^{(2n)}(m_\mu/m_\tau) + A_3^{(2n)}(m_\mu/m_e, m_\mu/m_\tau)$$

- The total QED contribution to the $\mu^\pm$ magnetic moment has been calculated up to 12th order as

$$a_\mu^{QED}(a(ae)) = 116\,584\,718.842\ (7)\ (17)(6)(100)\ (28)\ [106] \times 10^{-11}$$

Uncertainty due to τ lepton mass

Uncertainties in 8th 10th 12th QED terms

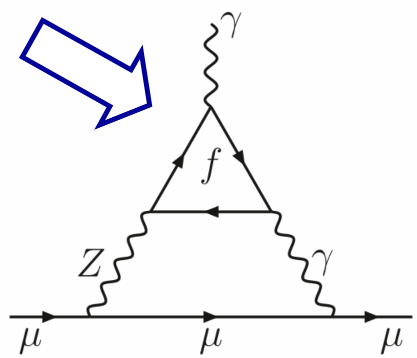
Uncertainty in α

Sum of uncertainties in quadrature

Muon (g-2) ElectroWeak Contributions

- Electroweak effects contribute through processes such as

$$a_\mu^{\rm EW} = \underbrace{a_\mu^{\rm EW(1)}}_{\text{1-loop}} + \underbrace{a_{\mu;\rm bos}^{\rm EW(2)}}_{\text{Boson 2-loop}} + \underbrace{a_{\mu;\rm ferm}^{\rm EW(2)}}_{\text{Fermion 2-loop}} + \underbrace{a_\mu^{\rm EW(\geq 3)}}_{\geq\text{3-loop}}$$

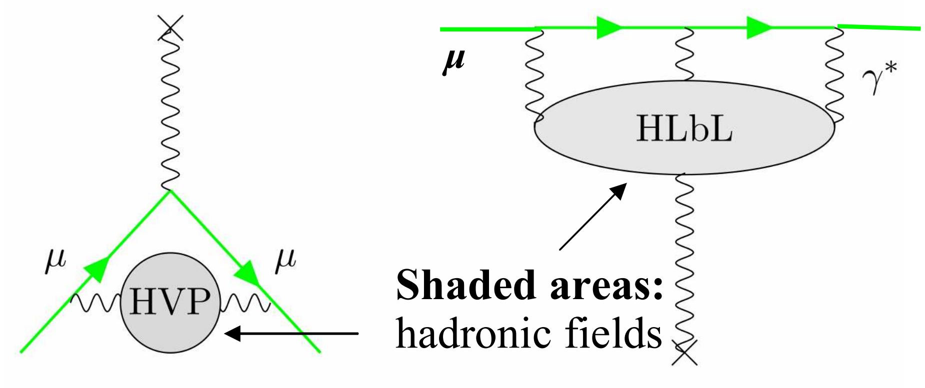


- **Fermion 2-loop diagrams like upper right (where *f=u,d,s*) is by far the dominant source of uncertainty in EW contribution**
  - An issue is that *u,d,s* quark masses, due to color confinement, are not well-defined, and this leads to significant non-perturbative corrections to the calculation
  - Uncertainty from unknown 3-loop contributions, and neglected 2-loops suppressed by M_Z^2/m_t^2 and $(1-4s_W^2)$, is significantly smaller

	EW contributions	$\times 10^{-11}$
1-loop	$a_\mu^{\rm EW(1)}$	194.79(1)
Boson 2-loop	$a_{\mu;\rm bos}^{\rm EW(2)}$	−19.962(3)
Fermion 2-loop	$a_\mu^{\rm EW(2)}(e,u,d;\mu,c,s)$	−6.22(28)
	$a_\mu^{\rm EW(2)}(\tau,t,b)$	−8.12(1)
	$a_{\mu;\text{f-rest,H}}^{\rm EW(2)}$	−1.500(2)
	$a_{\mu;\text{f-rest, no H}}^{\rm EW(2)}$	−4.58(10)
≥3-loop	$a_\mu^{\rm EW(\geq 3)}$	0.00(20)
Total EW contribution	$a_\mu^{\rm EW}$	154.4(4)

Muon (g-2) Hadronic Contributions

- **Two main types:** $a_\mu^{\text{had}} = a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}}$
  - **HVP** = hadronic vacuum polarization
    - Includes predominant Leading Order part
  - **HLbL** = hadronic light-by-light
    - One of the largest uncertainties in $(g-2)_\mu$

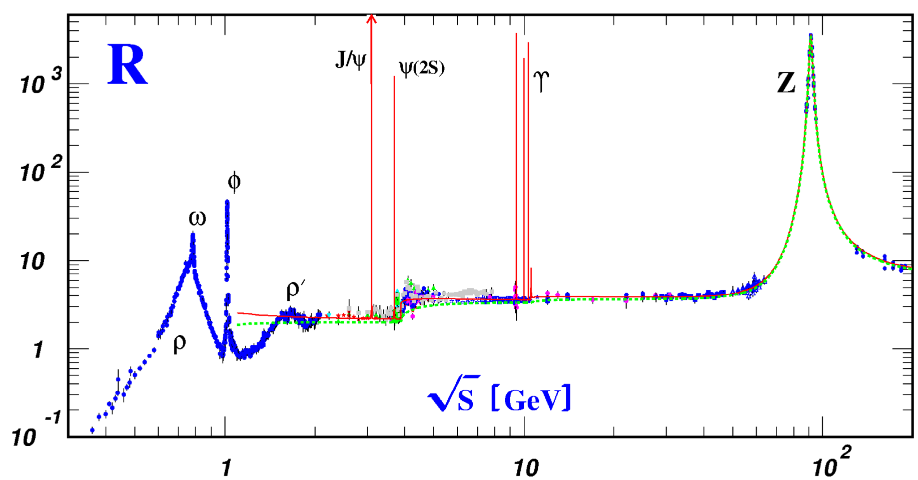


- **Neither can be calculated in perturbation theory, need either lattice QCD or data-driven evaluations of hadronic contributions**
  - In the data-driven approach, the leading-order requires a dispersive calculation needing experimental input such as the *hadronic R-ratio*

$$a_\mu^{\text{HVP,LO}} = \frac{\alpha^2}{3\pi^2}\int_{s_{\text{thr}}}^{\infty} \frac{K(s)}{s} R_{\text{had}}(s)\,\mathrm{d}s$$

$$R_{\text{had}}(s) = \frac{\sigma_0^{\text{had}}(e^+e^- \to \gamma^* \to hadrons)}{\sigma_0^{\mu\mu}}$$

Tree-level calculation for $e^+e^- \to \gamma^* \to \mu^+\mu^-$

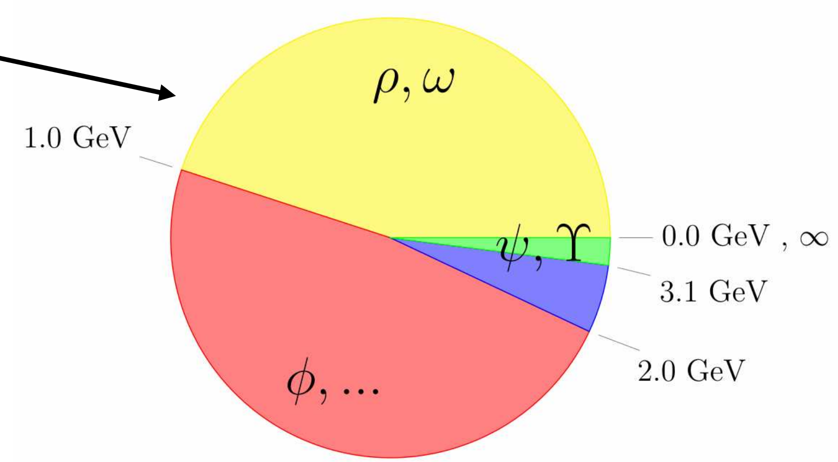


Uncertainties in Data-Driven approach

$$R_{\text{had}}(s) = \frac{\sigma_0^{\text{had}}(e^+e^- \to \gamma^* \to hadrons)}{\sigma_0^{\mu\mu}}$$

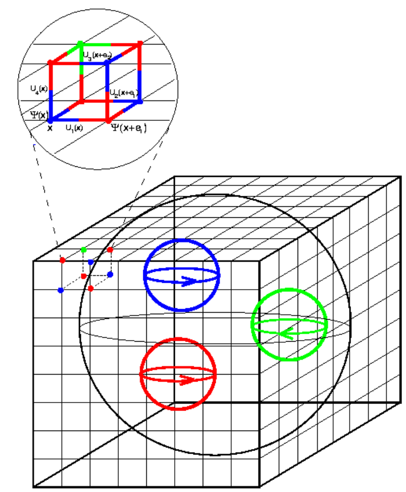
$$\sigma_0^{\text{had}} = \left|\frac{\alpha}{\alpha(s)}\right|^2 \sigma_{\text{had}}$$

- **σ_{had} refers to the "undressed" $e^+e^- \to hadrons$ cross section**
  - Measured after subtracting radiative corrections (vacuum polarization and initial-state radiation), but keeping final-state radiation
  - Relies on precise knowledge of higher-order radiative corrections, which vary with the hadronic final state
  - Largest effect is observed in 2π channel, due to low mass threshold
- **The largest theoretical uncertainty in a^{had}_{μ} comes from the Leading Order Hadronic Vacuum Polarization (LO-HVP) contribution**
  - **Uncertainty^2 in LO-HVP over various energy ranges**
  - **Experimental challenge:** measure >30 exclusive channels in 1.2-2.0 GeV energy range
  - Contribute 20% of LO-HVP value, but 46% of the total uncertainty



Muon (g-2) Hadronic Contributions

- **Lattice QCD approach is an alternate way to estimate Leading Order HVP and HLbL contributions**
  - Allows first principles QCD calculations to be performed within a discrete space-time grid
  - Much progress has been made in the accuracy of these calculations
  - **This led to a significant shift in the Standard Model prediction for a_μ in the 2025 evaluation**

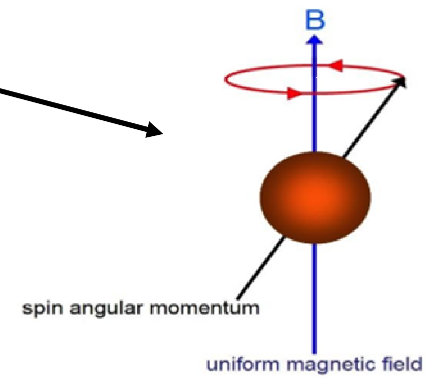


- Nonetheless, the data-driven approach will continue to play a crucial role in understanding the underlying physics
- Improving the measurements of Exclusive Channels will remain crucial for reducing uncertainties in Data-Driven approach

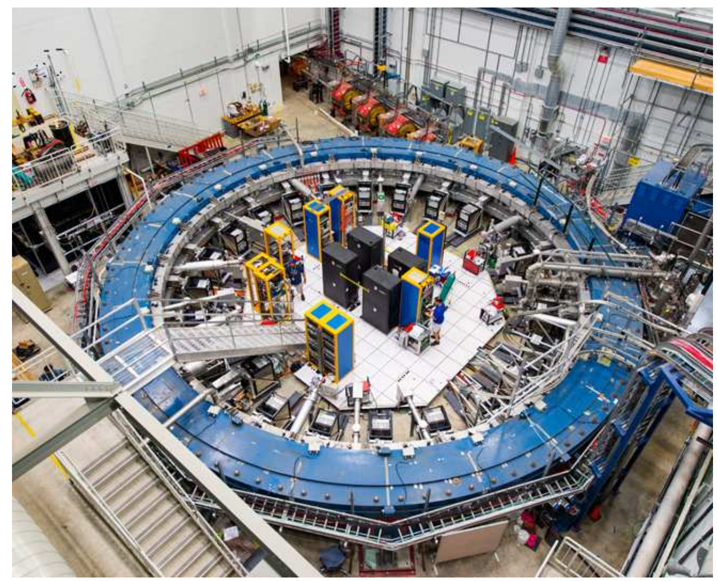
Hadronic contributions	Value $\times 10^{-11}$
HVP LO (lattice)	7132 ± 61
HVP NLO	-99.6 ± 1.3
HVP NNLO	12.4 ± 0.1
HVP (LO+NLO+NNLO)	7045 ± 61
HLbL	115.5 ± 9.9

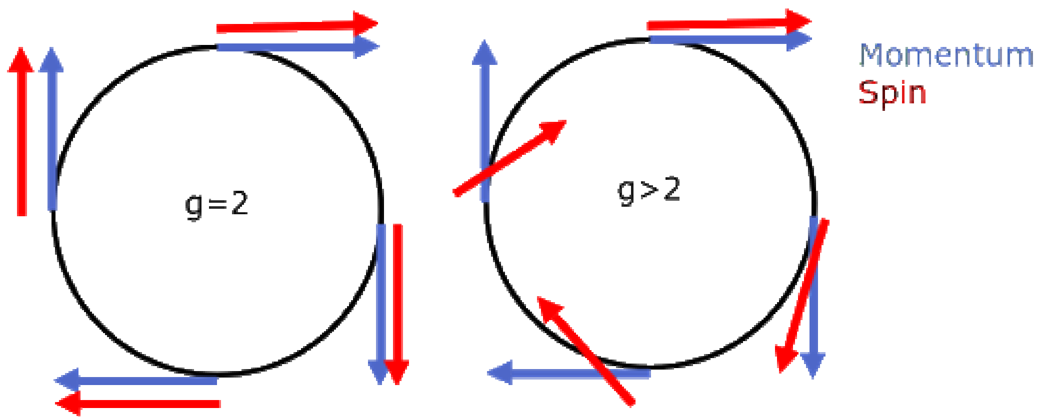
Fermilab Muon (g-2) Experiment

- **In external B-field, muon spin precesses at frequency determined by *g***
- μ^+ circulate in a magnetic storage ring with very uniform vertical B-field
- Difference between spin precession frequency and cyclotron frequency directly proportional to a_μ



$$\omega_a := \omega_s - \omega_c = \frac{qB}{m_\mu c} a_\mu$$





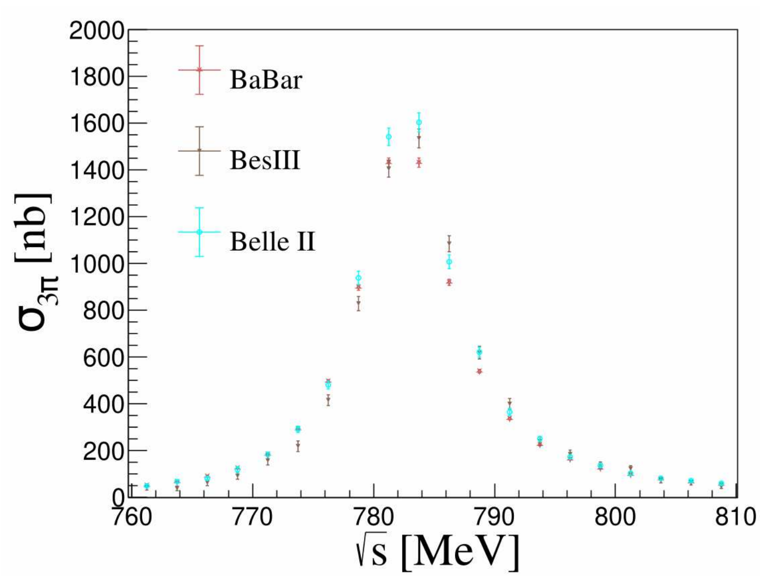
Theory and Experiment consistent within uncertainties

Expt world average a_μ^{exp} =116 592 071.1 (14.5) x 10^{-11}
2025 SM prediction a_μ^{SM}=116 592 033 (62) x 10^{-11}

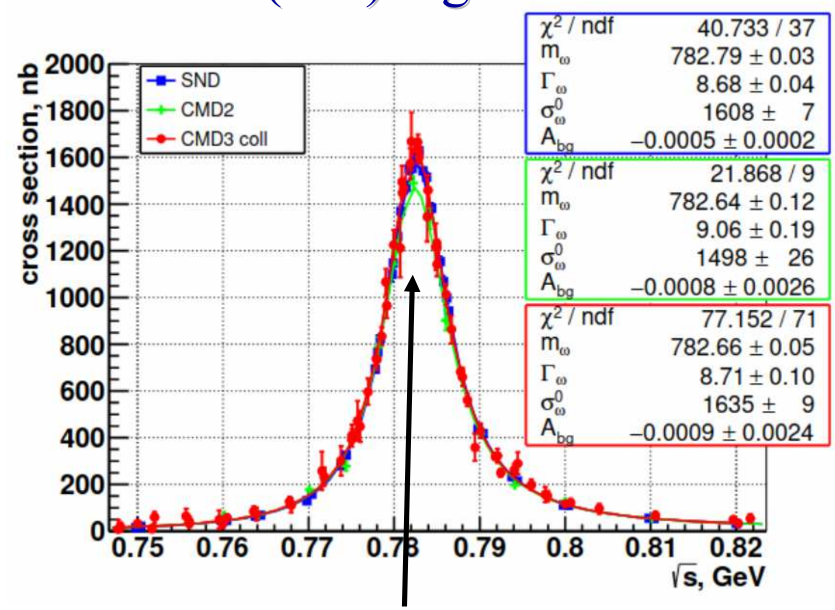
Status of $e^+e^- \to$ *hadrons* data sets

- Tensions between different experiments persist, leading to uncertainties in data-driven approach to HVP and HLbL calculations
- While the tension originates in dominant $\pi^+\pi^-$ channel, there are fewer data sets available in 3π channel, and these experimental uncertainties also play a role in the theory prediction

$e^+e^- \to \pi^+\pi^-\pi^0$ results for $\omega(770)$ region



Initial state radiation method:
2021 BaBar (SLAC) results with 5x larger dataset than previous expt



Energy scan method:
At ω peak, CMD3 results higher than CMD2 by ~2.2σ

Summary motivation of the thesis

- The successful theory prediction of the experiment a_μ value rested in large part on advancements in calculating the hadronic HVP and HLbL contributions
  1. Recent improvements in Lattice QCD calculations
  2. Controlled uncertainties in data-driven approach
- Further improvements in the data-driven approach require more precise $e^+e^-\rightarrow hadrons$ data, in particular for exclusive reactions such as the dominant $e^+e^-\rightarrow\pi^+\pi^-$ and $e^+e^-\rightarrow\pi^+\pi^-\pi^0$
- A new analysis of $e^+e^-\rightarrow\pi^+\pi^-\pi^0$ data from KLOE at Frascati will be presented