Pion Form Factor Physics

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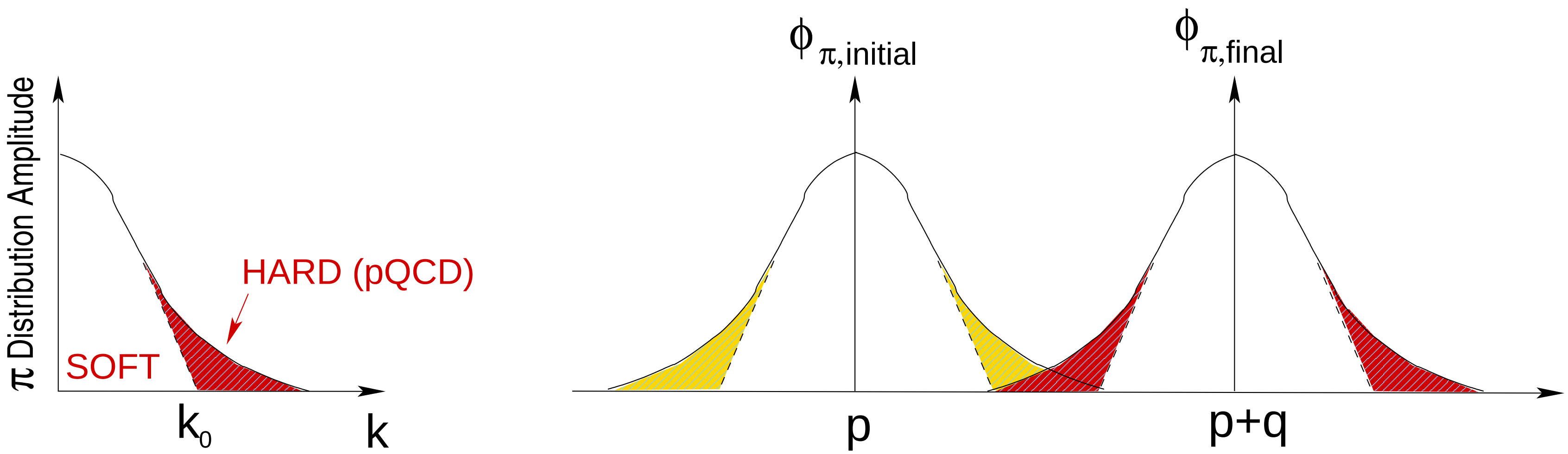
The Pion Form Factor in QCD

F_{π^+} has a unique place in our quest to understand hadronic structure, as the $\bar{q}q$ valence structure of the π^+ is relatively simple.

In quantum field theory, the pion form factor is given as the overlap integral

$$F_{\pi^+}(Q^2) = \int \phi_\pi^*(p)\phi_\pi(p+q)dp$$



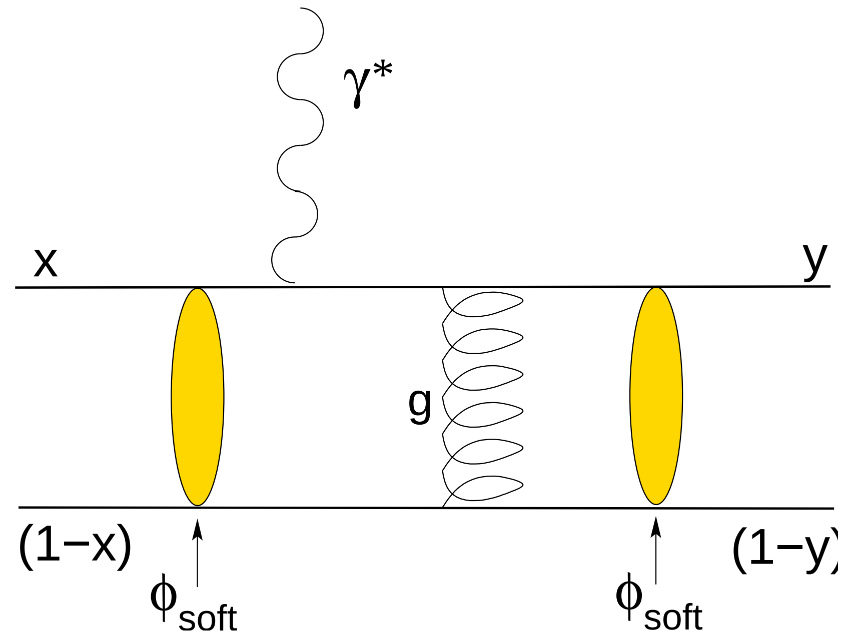
The pion wave function can be separated into a ϕ_π^{soft} part with only low-momentum contributions ($k < k_0$) and a hard tail ϕ_π^{hard}. While ϕ_π^{hard} can be treated in pQCD, ϕ_π^{soft} cannot.

From a theoretical standpoint, the study of the Q^2 dependence of F_π focusses on finding a description for the soft and hard contributions to the pion wave function.

QCD Hard Scattering Picture:

At very high Q^2, perturbative QCD (pQCD) can be used.

$$F_{\pi^+}(Q^2) = \int_0^1 dx \int_0^1 dy \frac{2g^2}{3xyQ^2} \phi(x)\phi(y)$$



$g^2 = \frac{4}{3}\pi\alpha_s$ $(q-g$ coupling const$)^2$

xyQ^2 virtuality of exchanged gluon.

As $Q^2 \to \infty$, only the hard portion of the wave function remains

$$\phi_\pi(x) \to 6 f_\pi x(1-x)$$

where f_π determines the asymptotic normalization of the wave function from $\pi^+ \to \mu^+\nu_\mu$ decay.

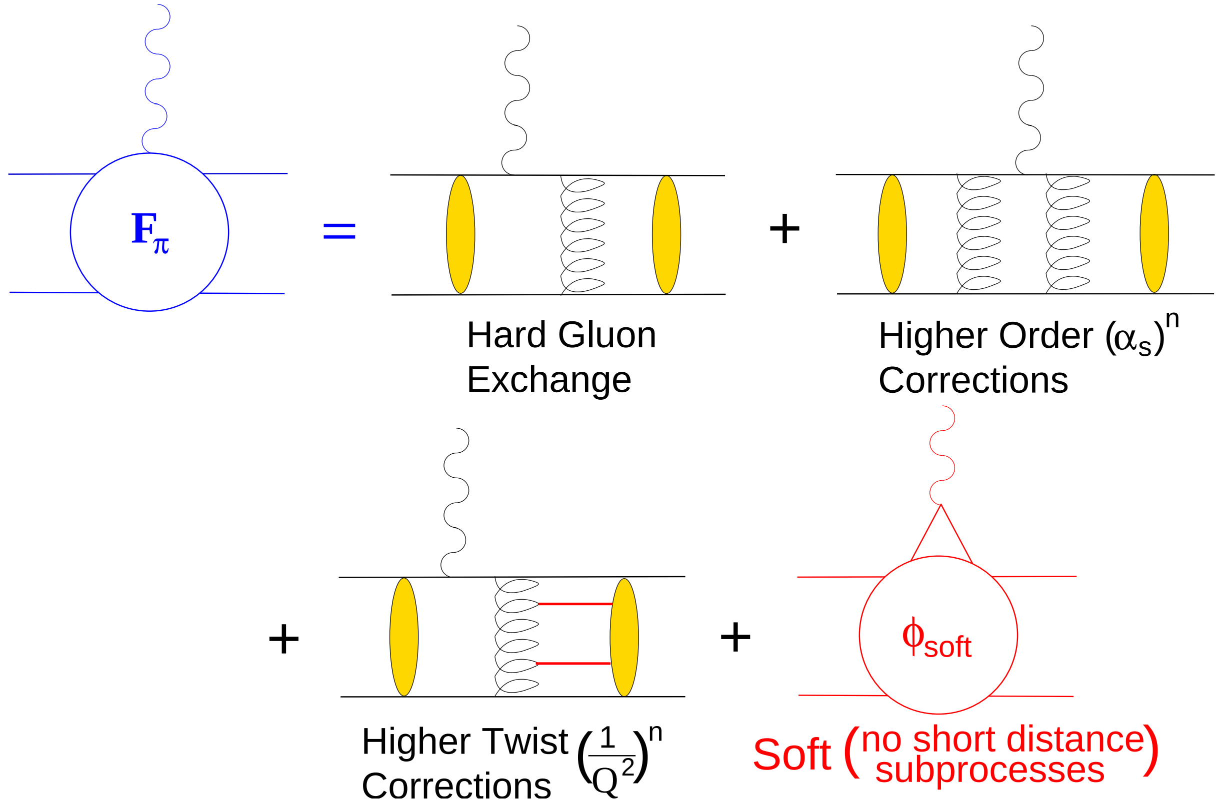
$$F_\pi \to_{Q^2\to\infty} \frac{8\pi\alpha_s f_\pi^2}{Q^2}$$

This asymptotic normalization does **not** exist in the case of the nucleon form factors.

Intermediate Q^2 Scattering Picture:

At experimentally accessible Q^2, the situation is more complicated

$\Rightarrow$ both soft and hard components contribute.



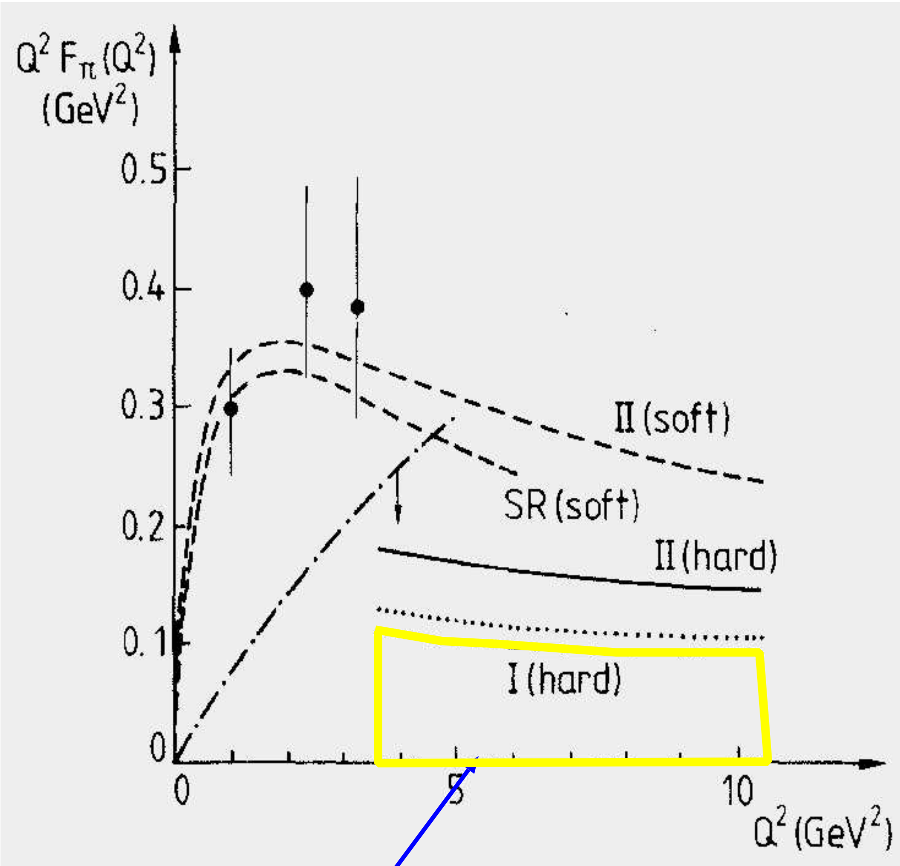
$\Rightarrow$ The interplay between the soft and hard contributions is poorly understood.

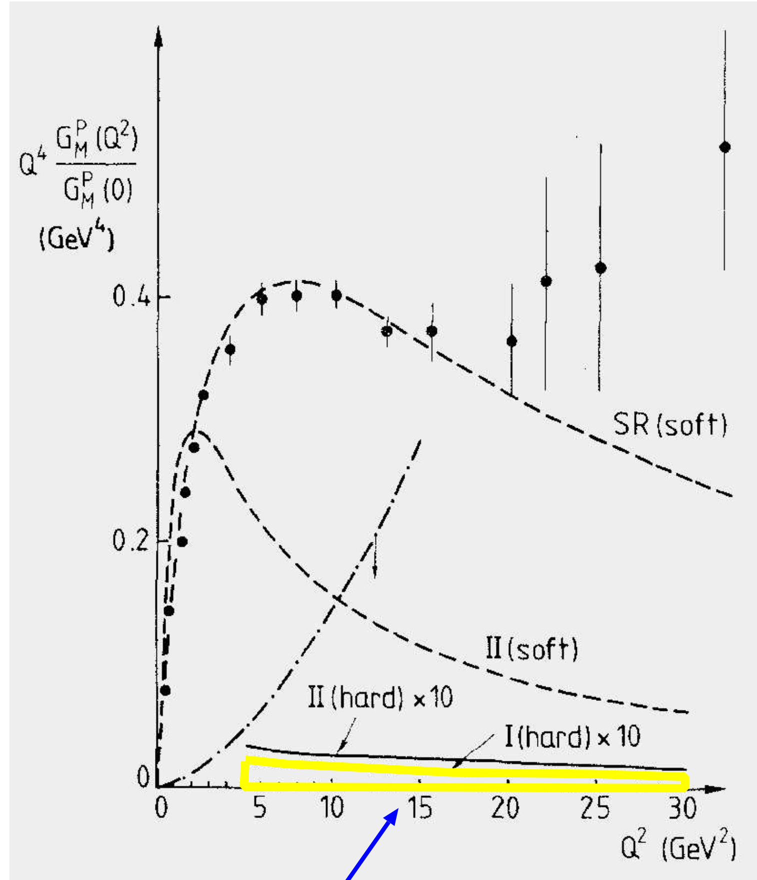
Recent Light Cone Sum Rule calculations up to twist 6:

- soft contributions to $F_\pi(Q^2)$ are large, but there is significant cancellation between hard and soft terms of higher twist near $Q^2 = 5$ GeV^2.
- total non-perturbative correction to pQCD result only $\sim 30\%$ at $Q^2 = 1$ GeV^2.

  [Braun, Khodjamirian, Maul, PRD **61**(00)073004.]

The pion as a QCD laboratory





Isgur & Llewellyn-Smith, PRL **52**(84)1080

Hard contributions
~30-50% at Q^2=5 GeV^2

Hard contributions
≤1% at Q^2=5 GeV^2

An important issue is understanding the transition of the behavior of QCD from the confinement regime to the perturbative regime. *"The pion is one of the simplest QCD systems available for study, and the measurement of its elastic form factor is the best hope for seeing this transition experimentally."*

- 2002 NSAC Report.

Determination of F_π via Pion Electroproduction

In the timelike region, F_π is determined from the $e^+e^- \to \pi^+\pi^-$ reaction. Our interest is in the spacelike region.

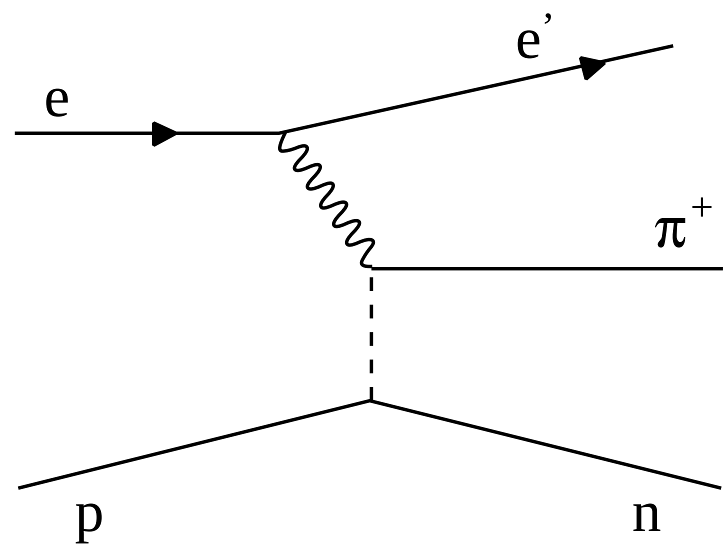
Up to $Q^2 = 0.3$ GeV^2, F_π is measured directly from the scattering of 300 GeV pions from atomic electrons.

$\Rightarrow F_\pi$ determined by the charge radius of the pion, 0.657 ± 0.012 fm.

To access higher Q^2, one must employ the ^{1}H$(e, e'\pi^+)n$ reaction.

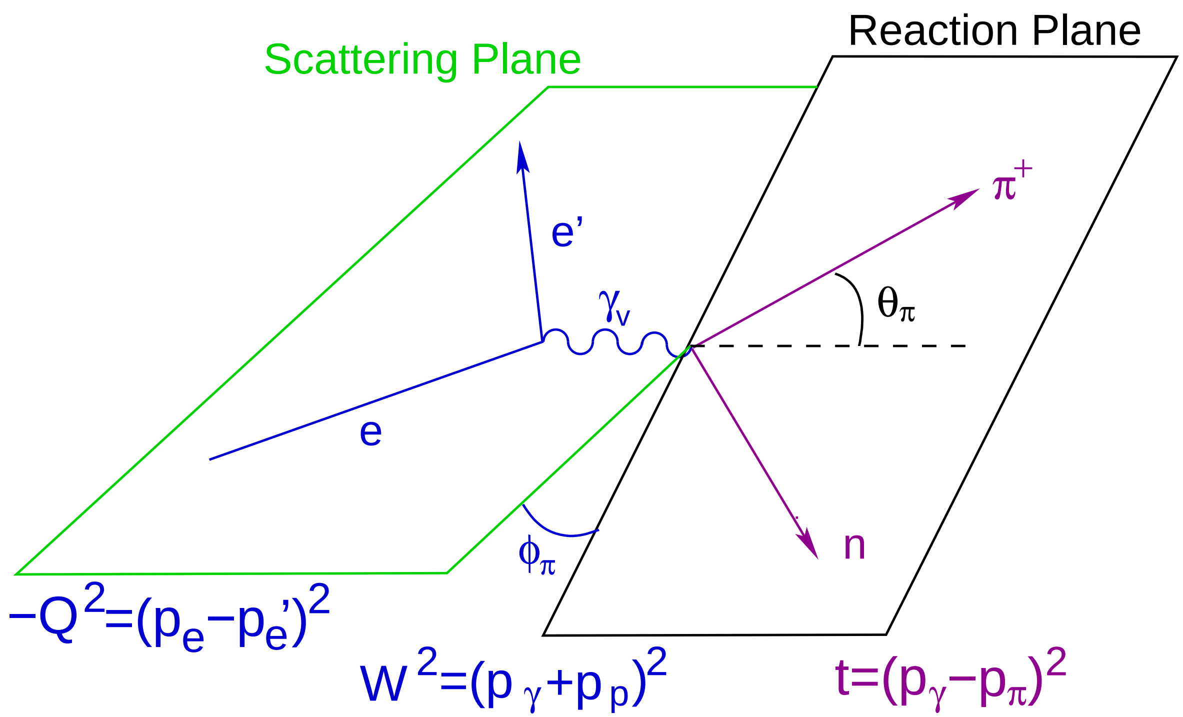
- At small $-t < 0.2$ GeV^2, the t-channel diagram dominates σ_L.
- In the t-pole approximation, $\frac{\mathbf{d\sigma_L}}{\mathbf{dt}} \propto \mathbf{F_\pi^2}$.

$\Rightarrow$ In the actual extraction, a model incorporating the π^+ production mechanism and the effects of the 'spectator' nucleon is used to extract F_π from σ_L.



$\Rightarrow \pi^+/\pi^-$ ratios from $^2H(e, e'\pi)$ are measured to test the validity of t-pole dominance and the model used.

What type of data do we need?



$$2\pi\frac{d^2\sigma}{dt\,d\phi}=\epsilon\frac{d\sigma_L}{dt}+\frac{d\sigma_T}{dt}+\sqrt{2\epsilon(\epsilon+1)}\frac{d\sigma_{LT}}{dt}cos\phi+\epsilon\frac{d\sigma_{TT}}{dt}cos2\phi$$

1. Take data at the smallest available $-t$, so that σ_L has maximum contribution from the π^+ pole.

   $\Rightarrow$ For a given Q^2, higher W allows smaller $|t|_{min}$.

2. The extraction of F_π from σ_L requires that the $-t$ dependence of $d\sigma_L/dt$ is known.

   $\Rightarrow$ Only three of W, Q^2, $-t$, and θ_π are independent.
   Vary θ_π to obtain $-t$ dependence of the data.

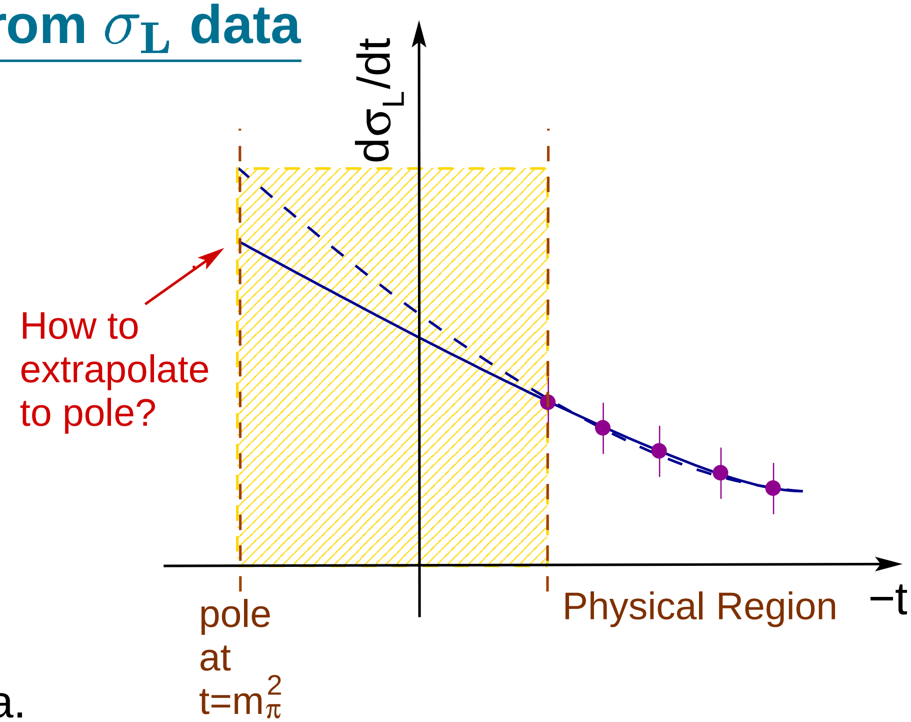
   $\Rightarrow$ Since non-parallel data are needed, TT and LT must also be determined by the experiment.

Extraction of F_π from σ_L data

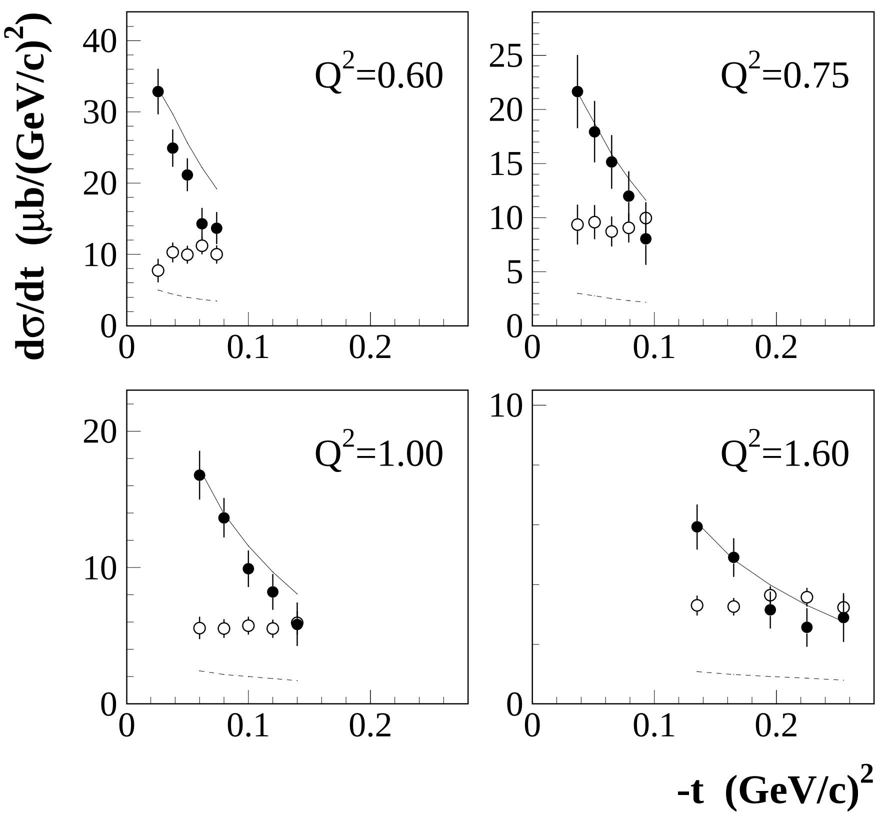
Chew-Low extrapolation using polynomial fit of physical region data does not give reliable answer.

$\Rightarrow$ Better to use model of $p(e,e'\pi^+)n$ reaction and treat F_π as free parameter.

$\Rightarrow$ fit of model to σ_L data gives F_π value at that Q^2.



$Q^2 = 0.70$ GeV^2 DESY expt [Z.Phys.C **3**(79)101] used a Born term (BT) model, with modification to improve the description of the t-dependence of the data.



JLab expts use the Vanderhaeghen, Guidal, and Laget Regge model, which provides a better treatment of the t-dependence.

$$F_\pi(Q^2) = (1 + \frac{Q^2}{\Lambda_\pi^2})^{-1}$$

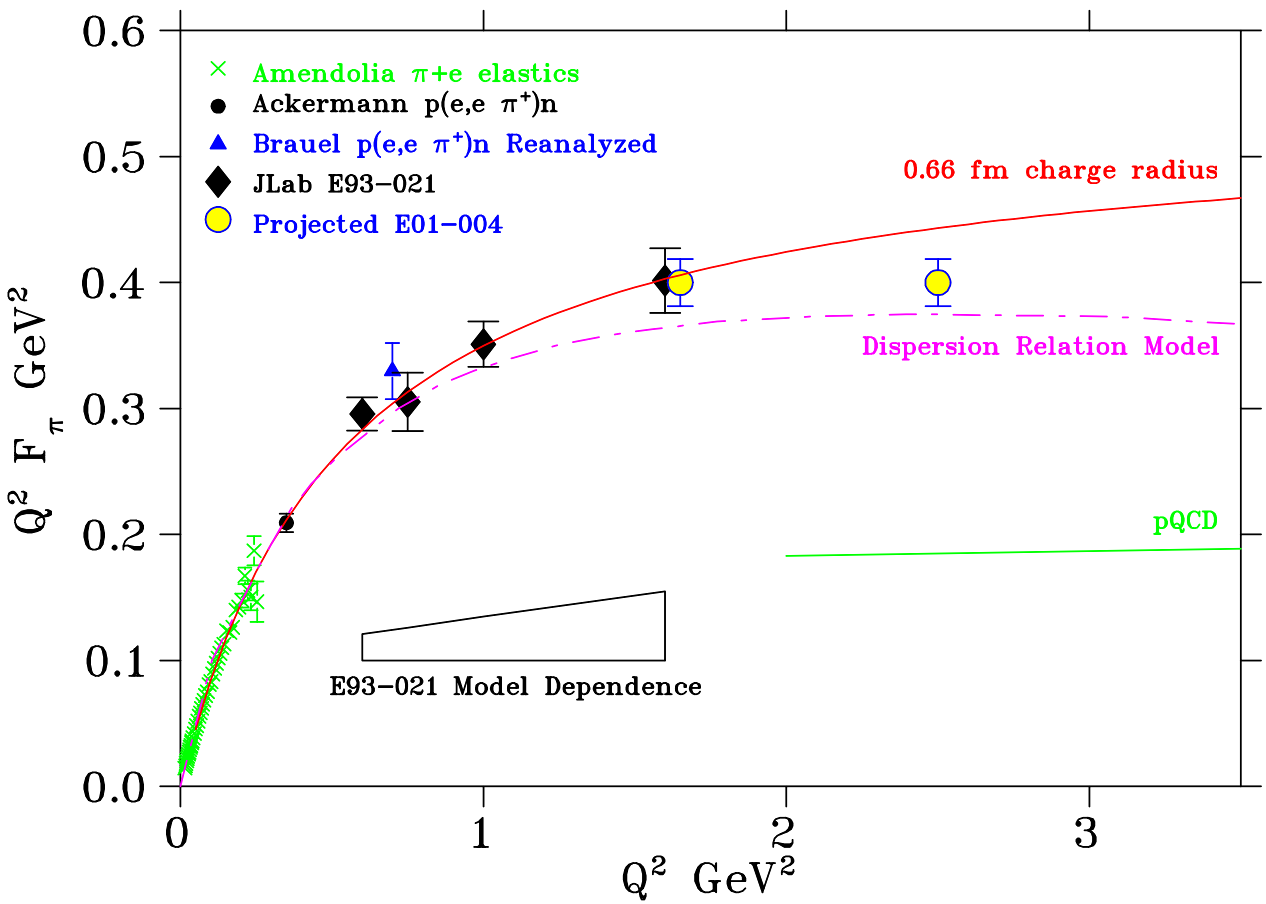
"Model uncertainty" of extracted F_π result is obtained from fit to $-t$-dependence of data under various assumptions.

DESY data also reanalyzed using Regge model.
$\Rightarrow F_\pi(Q^2 = 0.70)$ increases by 0.05 from result obtained with BT model.

Recent and projected experimental data

E93-021 data is globally consistent with 0.657 fm pion charge radius.

These measurements were recently extended in a new Hall C experiment in the summer of 2003.



- $Q^2 = 2.45$ GeV^2 using 6 GeV electron beam.
- Reduce model uncertainties in F_π extraction by obtaining data at higher $W = 2.21$ GeV.

  $\Rightarrow$ New data will be closer to $t = m_\pi^2$ pole.

  $\Rightarrow$ Regge model can be applied with greater authority, so expect smaller model uncertainties.
- Q^2 region where F_π theoretical calculations begin to diverge.

  $\Rightarrow$ New data will constrain the treatment of soft contributions in QCD-based models.

Expect preliminary data to be released in second-half 2005.

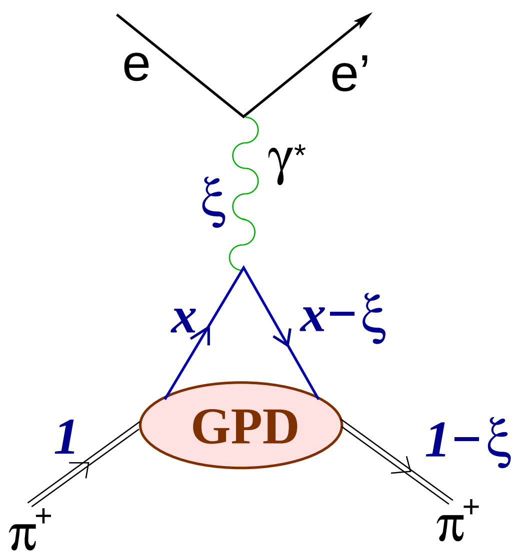
Selected Theory Developments

Over the next 7 years, new experimental data should be matched by considerable progress in theory.

Many F_π calculations, as all QCD-based models can be tested in the difficult and poorly understood gap between the “soft” and “hard” regions at intermediate Q^2.

Select two areas for discussion:

1) Calculations utilizing Generalized Parton Distributions (GPDs).



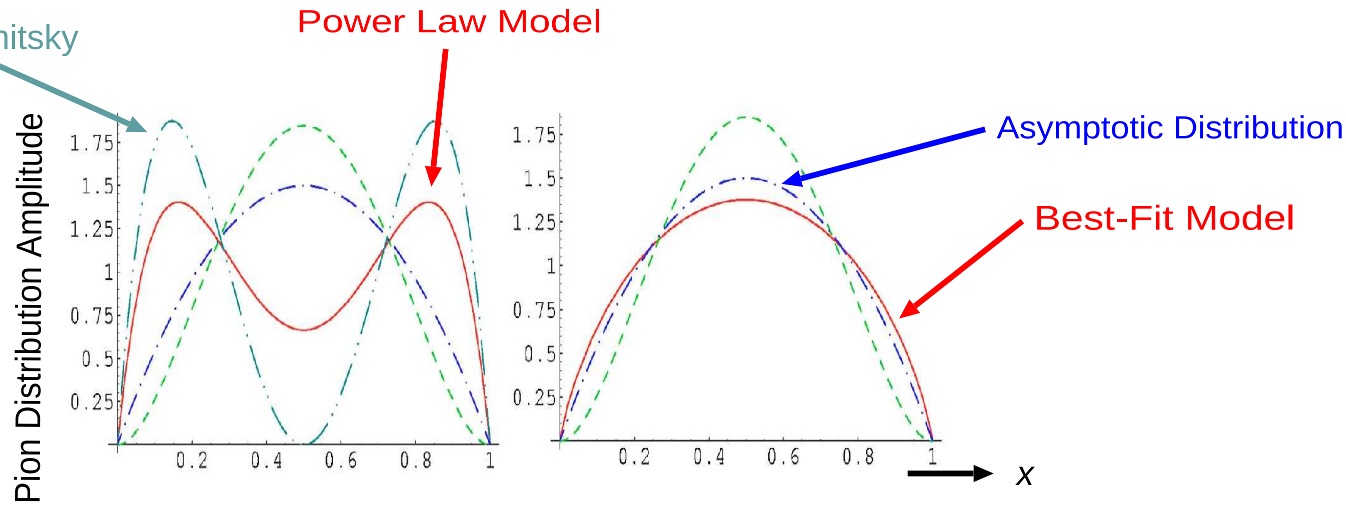
- GPDs offer a unified theoretical framework for parton distributions and hadronic form factors.
- GPDs are universal quantities and reflect the structure of the hadron independently of the probing reaction.
- GPD picture applies strictly to the hard-scattering regime, where the interaction can be clearly separated into perturbative (pQCD) and nonperturbative factors.

$\Rightarrow$ the GPD contains the non-perturbative part of the interaction, and represents the interference of quark wavefunctions, differing by momentum fraction ξ.

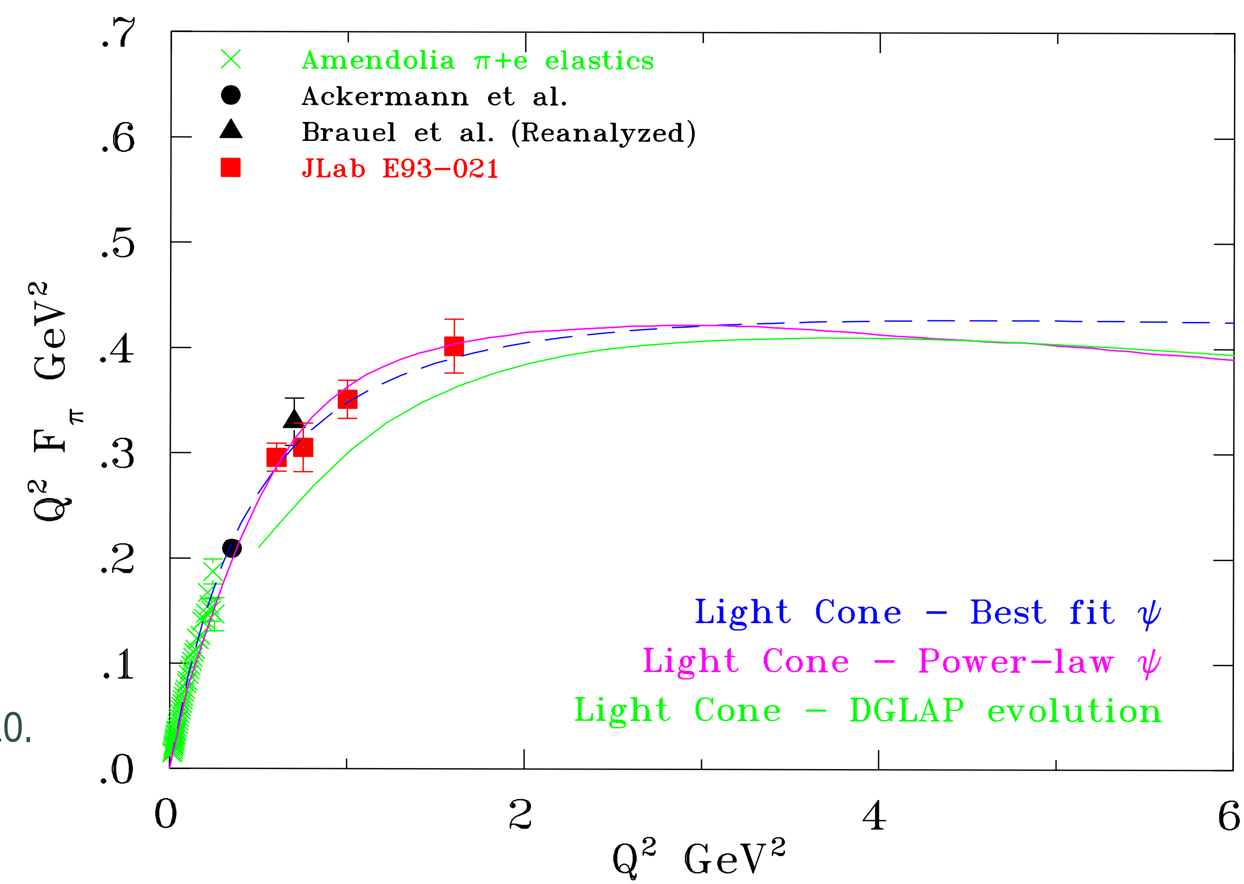
The pion form factor is related to the pion GPD via

$$F_\pi(Q^2) = \int_0^{+1} \sum_q H_\pi^q(\xi, x, Q^2)dx$$

Several GPD calculations have been made at intermediate Q^2 using Light Front Quark Models.



The construction of GPDs for the pion is in the pioneering stage, but much progress will likely be made over the next 7 years.



Refs:

A. Mukherjee et al., PRD **67**(03)073014.

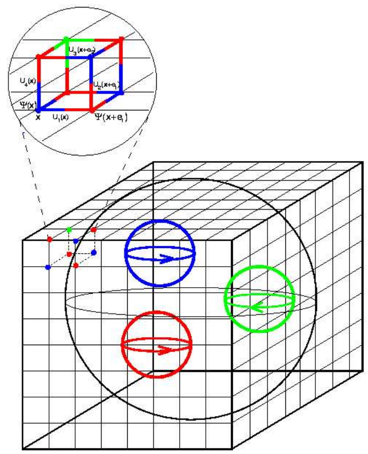
B.C. Tiburzi & G.A. Miller, PRD **67**(03)013010.

C. Vogt, PRD **63**(01)034013.

2) Lattice QCD (LQCD) calculations.

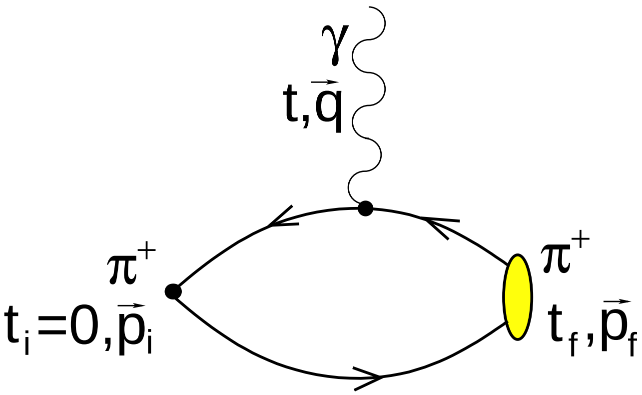
One feature shared by all QCD-based models is that confinement must be put in by hand.

Lattice QCD allows the calculation to proceed from first principles.



Although based on the QCD Lagrangian, LQCD involves approximations:

- Lattice discretization errors. $\rightarrow$ Use improved lattice QCD actions.
- Chiral extrapolation of lattice results in the pion mass.
- Quenching errors. $\rightarrow$ Need to include disconnected quark loops.



The first LQCD calculations of F_π (1980's) used $m_\pi \sim 1$ GeV.

$\Rightarrow$ Calculation up to $Q^2 = 1$ GeV^2 consistent with monopole charge radius, within error.

Today, three different Lattice groups are pursuing F_π calculations.

$\Rightarrow$ Goal is to perform calculation with significantly smaller quark masses than before, and eventually to attain larger values of Q^2.

- Lower pion mass $\rightarrow$ Larger $N_s \times N_s \times N_s \times N_t$ lattice $\rightarrow$ more rapidly converging action and faster CPU.
- Higher Q^2 $\rightarrow$ finer lattice spacing $\rightarrow$ improved pion operators.

Recent LQCD calculations:

All calculations - use similar quark masses. - are without Chiral extrapolation.

The difference between the solid and dashed lines indicates the expected effect of the Chiral extrapolation.

Quenching errors are not shown.

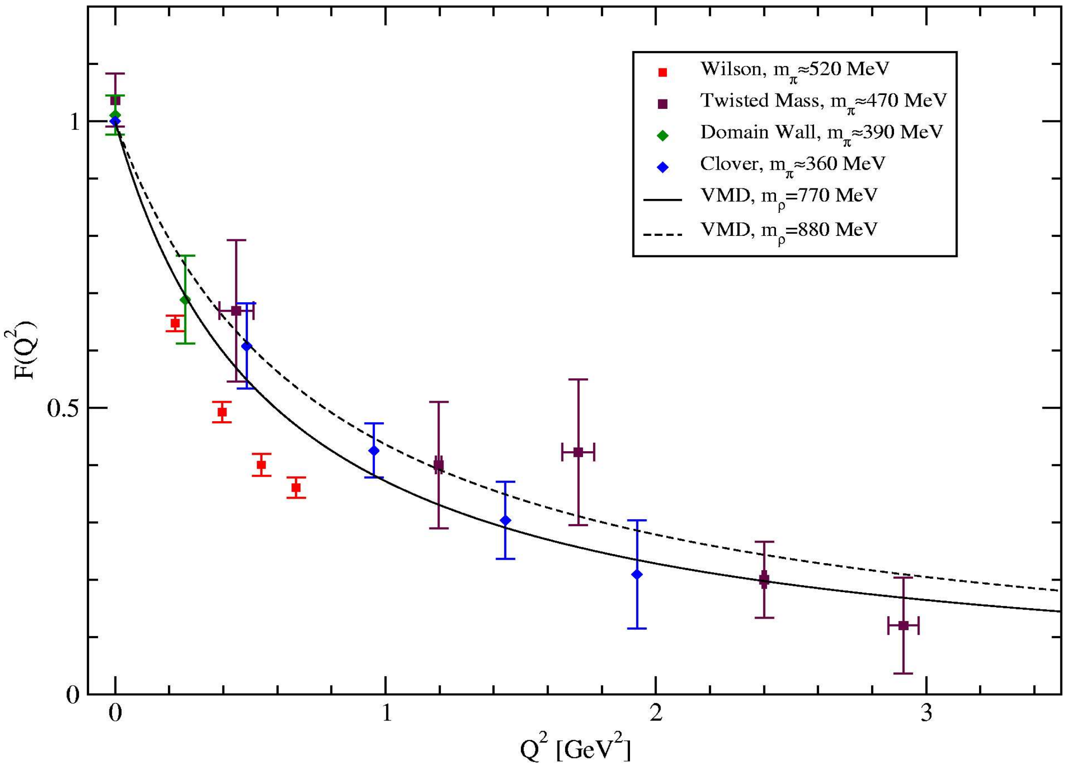
Wilson action has O(a) errors; other actions incorporate techniques for lattice spacing error suppression.
[F.D.R. Bonnet, et al., hep-lat/0310053]

Clover action requires tuning of an additional lattice spacing cutoff parameter (outside QCD).
[J. van der Heide, et al., hep-lat/0312023]

Twisted mass action is CPU efficient, but does not preserve Chiral symmetry as $m_q \rightarrow 0$.
[A.M. Abdel-Rehim, R. Lewis, in preparation]

Domain wall action has exact Chiral symmetry, but is CPU expensive.
[Y. Nemoto, et al., hep-lat/0309173]

Unquenched F_π calculations using domain wall action.
[T. Fleming et al., in progress]



Now: LQCD calculations are consistent with experimental data, within large statistical and systematic (chiral and quenching) errors.

⇒ Primary aim is to test the proof-of-principle of various calculational techniques.

In 7 years: hope to see dynamical (unquenched) calculations of F_π with pion mass sufficiently low to yield small chiral extrapolation uncertainties.

⇒ The comparison between experiment and LQCD data will become more challenging.

F_π Long Term - 12 GeV Upgrade

The SHMS+HMS in Hall C will allow F_π to be measured to Q^2 =6 GeV^2, and possibly higher, depending on the favorability of the σ_L/σ_T ratio at large Q^2.

The 5.5^o forward angle capability of the SHMS is specifically driven by F_π requirement to access low *-t*.

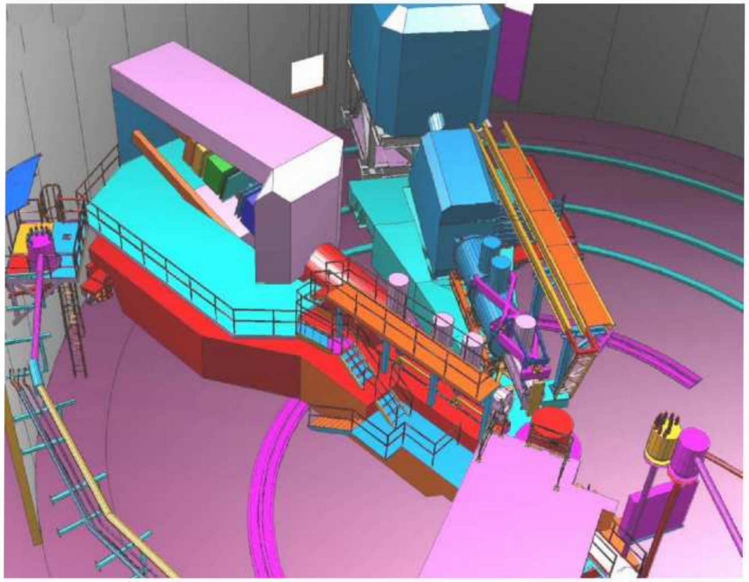
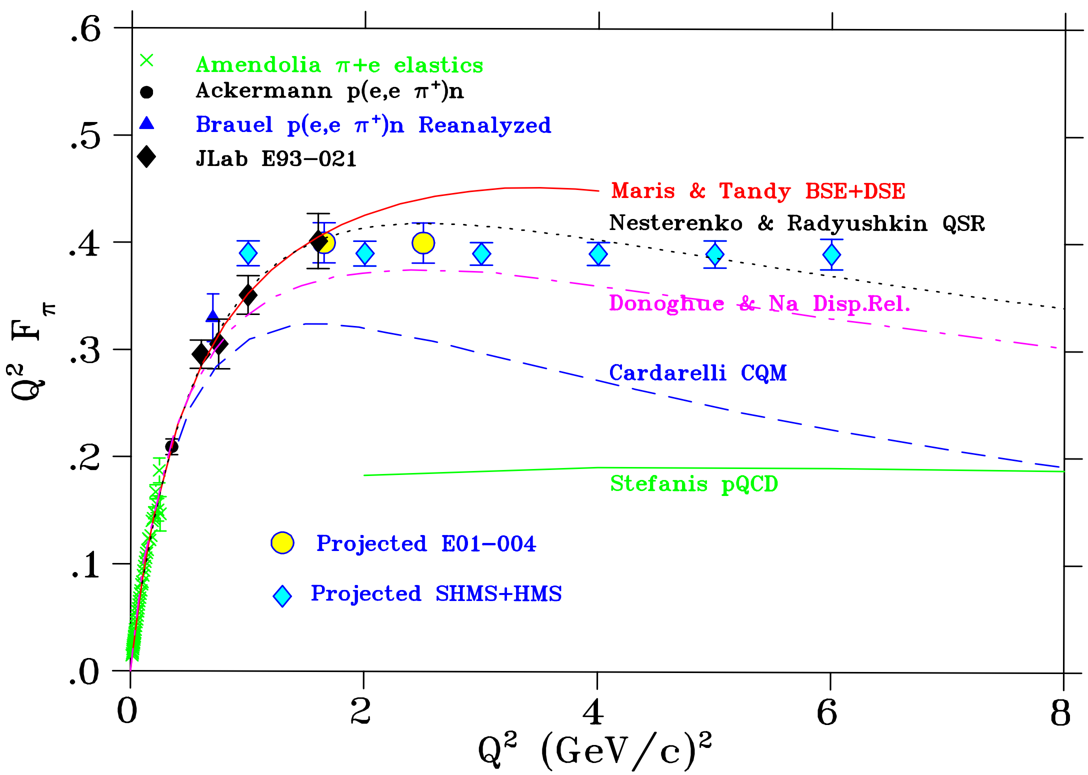


Figure 4.3: Artist's Rendering of SHMS and HMS Spectrometers in Hall C. The top of the SOS Spectrometer is in the foreground.



These higher Q^2 data will have an unprecendented ability to test the state-of-the-art QCD calculations anticipated by that time.

Next 7 years:

Longitudinal Photon, Transverse Nucleon, Single Spin Asymmetry in Exclusive *p(e,e'π)n*

L.L. Frankfurt, M.V. Polyakov, M. Strikman, M. Vanderhaeghen, PRL **84**(00)2589.

$A_{\pi N}$ is especially sensitive to the spin-flip GPD *E*-tilde, which can only be probed via hard exclusive pseudoscalar meson production.

⇒ Precocious scaling is expected to set in as early as Q^2~2-4 GeV^2.

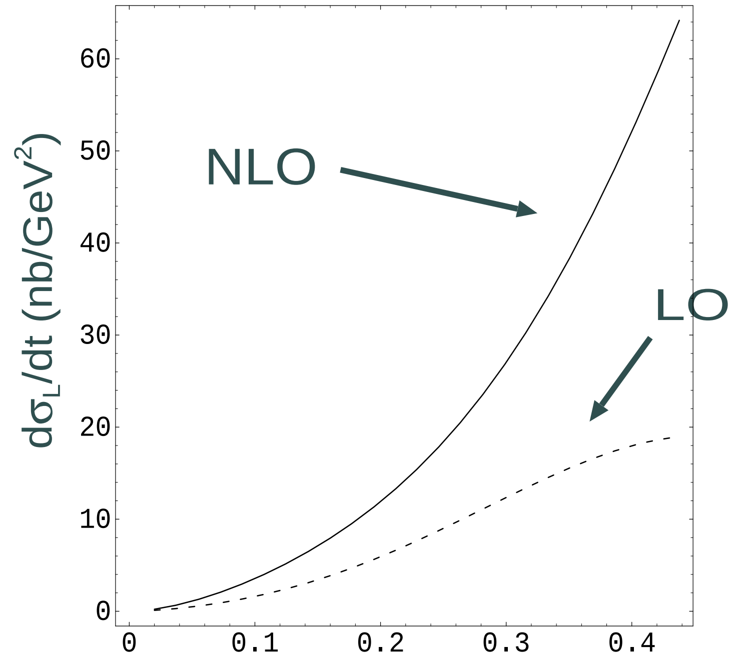
Measure $A_{\pi N}$ to constrain non-pole contributions to σ_L.

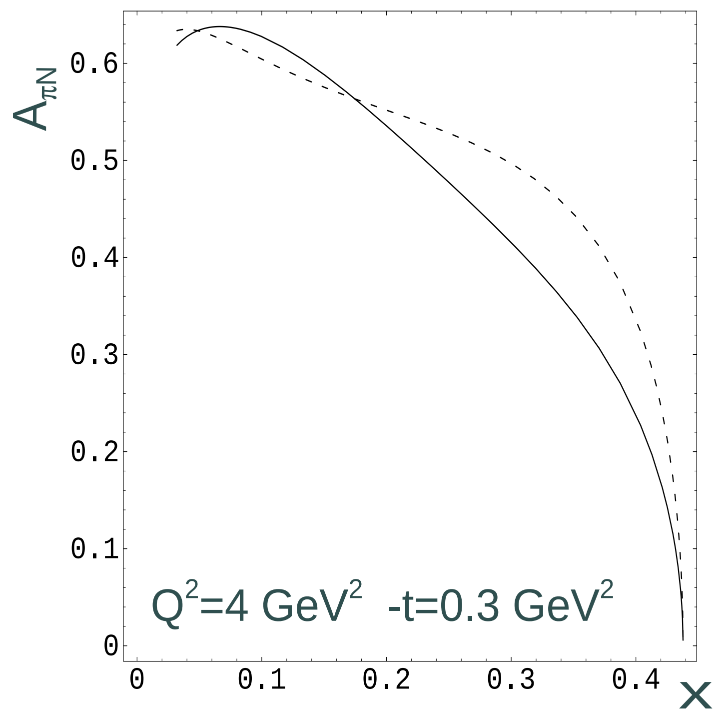
⇒ improve future extractions of F_π from *p(e,e'π⁺)n* data, by significantly reducing the model uncertainty.

Since F_π has been identified as a key 12 GeV experiment, this measurement should be pursued in support of that program.

A measurement over Q^2~2.5-4 GeV^2 is feasible with 6 GeV beam, although time consuming (~65 days).

⇒ Use transversely polarized target and the HMS in coincidence with the BigCal calorimeter.





Calculations courtesy of A. Belitsky.