Pion Form Factor Physics

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The Pion Form Factor in QCD

$F_{\pi^+}$ has a unique place in our quest to understand hadronic structure, as the $\bar{q}q$ valence structure of the $\pi^+$ is relatively simple.

In quantum field theory, the pion form factor is given as the overlap integral

$$F_{\pi^+}(Q^2) = \int \phi^*_\pi(p)\phi_\pi(p+q)dp$$

The pion wave function can be separated into a $\phi^{soft}_\pi$ part with only low-momentum contributions ($k < k_0$) and a hard tail $\phi^{hard}_\pi$. While $\phi^{hard}_\pi$ can be treated in pQCD, $\phi^{soft}_\pi$ cannot.

From a theoretical standpoint, the study of the $Q^2$ dependence of $F_\pi$ focusses on finding a description for the soft and hard contributions to the pion wave function.
QCD Hard Scattering Picture:

At very high $Q^2$, perturbative QCD (pQCD) can be used.

\[
F_{\pi^+}(Q^2) = \int_0^1 dx \int_0^1 dy \frac{2g^2}{3xyQ^2} \phi(x) \phi(y)
\]

\[
g^2 = \frac{4}{3} \pi \alpha_s \quad (q - g \text{ coupling const})^2
\]

\[
x yQ^2 \quad \text{virtuality of exchanged gluon.}
\]

As $Q^2 \to \infty$, only the hard portion of the wave function remains

\[
\phi_\pi(x) \to 6f_\pi x(1 - x)
\]

where $f_\pi$ determines the asymptotic normalization of the wave function from $\pi^+ \to \mu^+ \nu_\mu$ decay.

\[
F_\pi \to Q^2 \to \infty \frac{8\pi \alpha_s f_\pi^2}{Q^2}
\]

This asymptotic normalization does not exist in the case of the nucleon form factors.
Intermediate $Q^2$ Scattering Picture:

At experimentally accessible $Q^2$, the situation is more complicated
$\Rightarrow$ both soft and hard components contribute.

$F_\pi = \text{Hard Gluon Exchange} + \text{Higher Order } (\alpha_s)^n$

$+ \text{Higher Twist } \left(\frac{1}{Q^2}\right)^n$

$\Rightarrow \text{Soft (no short distance subprocesses)}$

$\Rightarrow \text{The interplay between the soft and hard contributions is poorly understood.}$

Recent Light Cone Sum Rule calculations up to twist 6:

- soft contributions to $F_\pi(Q^2)$ are large, but there is significant cancellation between hard and soft terms of higher twist near $Q^2 = 5$ GeV$^2$.

- total non-perturbative correction to pQCD result only $\sim 30\%$ at $Q^2 = 1$ GeV$^2$.

[Braun, Khodjamirian, Maul, PRD 61(00)073004.]
An important issue is understanding the transition of the behavior of QCD from the confinement regime to the perturbative regime. "The pion is one of the simplest QCD systems available for study, and the measurement of its elastic form factor is the best hope for seeing this transition experimentally."

Determination of $F_\pi$ via Pion Electroproduction

In the timelike region, $F_\pi$ is determined from the $e^+e^- \rightarrow \pi^+\pi^-$ reaction. Our interest is in the spacelike region.

Up to $Q^2 = 0.3$ GeV$^2$, $F_\pi$ is measured directly from the scattering of 300 GeV pions from atomic electrons.

⇒ $F_\pi$ determined by the charge radius of the pion, $0.657 \pm 0.012$ fm.

To access higher $Q^2$, one must employ the $^1H(e, e'\pi^+)n$ reaction.

- At small $-t < 0.2$ GeV$^2$, the $t$-channel diagram dominates $\sigma_L$.

- In the $t$-pole approximation, $\frac{d\sigma_L}{dt} \propto F_\pi^2$.

⇒ In the actual extraction, a model incorporating the $\pi^+$ production mechanism and the effects of the ‘spectator’ nucleon is used to extract $F_\pi$ from $\sigma_L$.

⇒ $\pi^+ / \pi^-$ ratios from $^2H(e, e'\pi)$ are measured to test the validity of $t$-pole dominance and the model used.
What type of data do we need?

1. Take data at the smallest available $-t$, so that $\sigma_L$ has maximum contribution from the $\pi^+$ pole.
   
   $\Rightarrow$ For a given $Q^2$, higher $W$ allows smaller $|t|_{min}$.

2. The extraction of $F_\pi$ from $\sigma_L$ requires that the $-t$ dependence of $d\sigma_L/dt$ is known.
   
   $\Rightarrow$ Only three of $W$, $Q^2$, $-t$, and $\theta_\pi$ are independent.

   Vary $\theta_\pi$ to obtain $-t$ dependence of the data.

   $\Rightarrow$ Since non-parallel data are needed, $TT$ and $LT$ must also be determined by the experiment.
Extraction of $F_\pi$ from $\sigma_L$ data

Chew-Low extrapolation using polynomial fit of physical region data does not give reliable answer.

⇒ Better to use model of $p(e, e' \pi^+)n$ reaction and treat $F_\pi$ as free parameter.
⇒ fit of model to $\sigma_L$ data gives $F_\pi$ value at that $Q^2$.

$Q^2 = 0.70$ GeV$^2$ DESY expt [Z.Phys.C 3(79)101] used a Born term (BT) model, with modification to improve the description of the $t$-dependence of the data.

JLab expts use the Vanderhaeghen, Guidal, and Laget Regge model, which provides a better treatment of the $t$-dependence.

$$F_\pi(Q^2) = (1 + \frac{Q^2}{\Lambda^2_\pi})^{-1}$$

“Model uncertainty” of extracted $F_\pi$ result is obtained from fit to $-t$-dependence of data under various assumptions.

DESY data also reanalyzed using Regge model.
⇒ $F_\pi(Q^2 = 0.70)$ increases by 0.05 from result obtained with BT model.
Recent and projected experimental data

E93-021 data is globally consistent with 0.657 fm pion charge radius.

These measurements were recently extended in a new Hall C experiment in the summer of 2003.

- $Q^2 = 2.45 \text{ GeV}^2$ using 6 GeV electron beam.
- Reduce model uncertainties in $F_\pi$ extraction by obtaining data at higher $W = 2.21 \text{ GeV}$.
  ⇒ New data will be closer to $t = m_\pi^2$ pole.
  ⇒ Regge model can be applied with greater authority, so expect smaller model uncertainties.

- $Q^2$ region where $F_\pi$ theoretical calculations begin to diverge.
  ⇒ New data will constrain the treatment of soft contributions in QCD-based models.

Expect preliminary data to be released in second-half 2005.
Selected Theory Developments

Over the next 7 years, new experimental data should be matched by considerable progress in theory. Many $F_\pi$ calculations, as all QCD-based models can be tested in the difficult and poorly understood gap between the “soft” and “hard” regions at intermediate $Q^2$.

Select two areas for discussion:

1) Calculations utilizing Generalized Parton Distributions (GPDs).

- GPDs offer a unified theoretical framework for parton distributions and hadronic form factors.
- GPDs are universal quantities and reflect the structure of the hadron independently of the probing reaction.
- GPD picture applies strictly to the hard-scattering regime, where the interaction can be clearly separated into perturbative (pQCD) and nonperturbative factors.

$\Rightarrow$ the GPD contains the non-perturbative part of the interaction, and represents the interference of quark wavefunctions, differing by momentum fraction $\xi$.

The pion form factor is related to the pion GPD via

$$F_\pi(Q^2) = \int_0^{+1} \sum_q H^q_\pi(\xi, x, Q^2) \, dx$$
Several GPD calculations have been made at intermediate $Q^2$ using Light Front Quark Models.

The construction of GPDs for the pion is in the pioneering stage, but much progress will likely be made over the next 7 years.

Refs:
A. Mukherjee et al., PRD 67(03)073014.
B.C. Tiburzi & G.A. Miller, PRD 67(03)013010.
C. Vogt, PRD 63(01)034013.
2) Lattice QCD (LQCD) calculations.

One feature shared by all QCD-based models is that confinement must be put in by hand. Lattice QCD allows the calculation to proceed from first principles.

Although based on the QCD Lagrangian, LQCD involves approximations:

- Lattice discretization errors. → Use improved lattice QCD actions.
- Chiral extrapolation of lattice results in the pion mass.
- Quenching errors. → Need to include disconnected quark loops.

The first LQCD calculations of $F_\pi$ (1980’s) used $m_\pi \sim 1$ GeV.

⇒ Calculation up to $Q^2 = 1$ GeV$^2$ consistent with monopole charge radius, within error.

Today, three different Lattice groups are pursuing $F_\pi$ calculations.

⇒ Goal is to perform calculation with significantly smaller quark masses than before, and eventually to attain larger values of $Q^2$.

- Lower pion mass → Larger $N_s \times N_s \times N_s \times N_t$ lattice → more rapidly converging action and faster CPU.
- Higher $Q^2$ → finer lattice spacing → improved pion operators.
Recent LQCD calculations:

All calculations - use similar quark masses. - are without Chiral extrapolation.

The difference between the solid and dashed lines indicates the expected effect of the Chiral extrapolation.

Quenching errors are not shown.

Wilson action has $O(a)$ errors; other actions incorporate techniques for lattice spacing error suppression.
[F.D.R. Bonnet, et al., hep-lat/0310053]

Clover action requires tuning of an additional lattice spacing cutoff parameter (outside QCD).
[J. van der Heide, et al., hep-lat/0312023]

Twisted mass action is CPU efficient, but does not preserve Chiral symmetry as $m_q \to 0$.
[A.M. Abdel-Rehim, R. Lewis, in preparation]

Domain wall action has exact Chiral symmetry, but is CPU expensive.
[Y. Nemoto, et al., hep-lat/0309173]

Unquenched $F_\pi$ calculations using domain wall action.
[T. Fleming et al., in progress]

Now: LQCD calculations are consistent with experimental data, within large statistical and systematic (chiral and quenching) errors.
⇒ Primary aim is to test the proof-of-principle of various calculational techniques.

In 7 years: hope to see dynamical (unquenched) calculations of $F_\pi$ with pion mass sufficiently low to yield small chiral extrapolation uncertainties.
⇒ The comparison between experiment and LQCD data will become more challenging.
The SHMS+HMS in Hall C will allow $F_\pi$ to be measured to $Q^2 = 6 \text{ GeV}^2$, and possibly higher, depending on the favorability of the $\sigma_L/\sigma_T$ ratio at large $Q^2$.

The 5.5° forward angle capability of the SHMS is specifically driven by $F_\pi$ requirement to access low -$t$.

These higher $Q^2$ data will have an unprecedented ability to test the state-of-the-art QCD calculations anticipated by that time.
Next 7 years:
Longitudinal Photon, Transverse Nucleon,
Single Spin Asymmetry in Exclusive $p(e,e'\pi)n$

L.L. Frankfurt, M.V. Polyakov, M. Strikman, M. Vanderhaeghen, PRL 84(00)2589.

$A_{\pi N}$ is especially sensitive to the spin-flip GPD $E$-tilde, which can only be probed via hard exclusive pseudoscalar meson production.

⇒ Precocious scaling is expected to set in as early as $Q^2 \sim 2$-4 GeV$^2$.

Measure $A_{\pi N}$ to constrain non-pole contributions to $\sigma_L$.

⇒ improve future extractions of $F_\pi$ from $p(e,e'\pi^+)n$ data, by significantly reducing the model uncertainty.

Since $F_\pi$ has been identified as a key 12 GeV experiment, this measurement should be pursued in support of that program.

A measurement over $Q^2 \sim 2.5$-4 GeV$^2$ is feasible with 6 GeV beam, although time consuming (~65 days).

⇒ Use transversely polarized target and the HMS in coincidence with the BigCal calorimeter.

Calculations courtesy of A. Belitsky.