PR12-06-101: Measurement of the Charged Pion Form Factor to High $Q^2$

Spokespersons:
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Scientific Motivation

- The pion form factor is a topic of fundamental importance to our understanding of hadronic structure.
- Pions are the lightest QCD system ($q\bar{q}$).
  - all hadronic structure models use the $\pi^+$ as a test case.
  - “the positronium atom of QCD”.

At large $Q^2$, $F_\pi$ is presumably given by pQCD

$$F_\pi(Q^2) = \frac{4\pi C_F \alpha_s(Q^2)}{Q^2} \sum_{n=0}^{\infty} a_n \left( \log \left( \frac{Q^2}{\Lambda^2} \right) \right)^{-\gamma_n} \left[ 1 + O(\alpha_s(Q^2), m/Q) \right]$$

which in the $Q^2 \to \infty$ limit becomes

$$F_\pi(Q^2) \xrightarrow{Q^2 \to \infty} \frac{16\pi \alpha_s(Q^2) f_\pi^2}{Q^2}$$

where $f_\pi^2 = 93$ MeV is the $\pi^+ \to \mu^+\nu$ decay constant.

$F_\pi$ is the clearest test case for study of transition between non-perturbative and pQCD regions.

- What is the structure of the $\pi^+$ at all $Q^2$?
  - at what value of $Q^2$ will the pQCD contributions dominate?

A difficult question to answer, as both “hard” and “soft” components (such as gluonic effects) must be taken into account.

- non-perturbative hard components of higher twist strongly cancel soft components, even at modest $Q^2$.
  - [Braun et al., PRD 61(2000)073004]

- the situation for nucleon form factors is even more complicated.

Many model calculations exist, but ultimately...

- Reliable $F_\pi(Q^2)$ data are needed to delineate the role of hard versus soft contributions at intermediate $Q^2$.

A program of study unique to Jefferson Lab.
Theory Review Comments:

- The pion form factor is an object of great theoretical interest, especially at larger values of $Q^2$, where one can study nonperturbative dynamics of QCD while searching for the transition to the perturbative regime.

- While the merits of studying this observable are clear, the extraction of the pion form factor from data is a non-trivial exercise.
Determination of $F_{\pi}$ via Pion Electroproduction

Pion charge radius is well known from $e^+e^-\rightarrow\pi^+\pi^-$ experiments.

$r_{\pi} = 0.657 \pm 0.012$ fm

At low $Q^2 < 0.3$ GeV$^2$, the $\pi^+$ form factor can be measured exactly using high energy $\pi^+$ scattering from atomic electrons.

$\Rightarrow$ 300 GeV pions at CERN SPS. [Amendolia et al., NP B277(1986)168]

To access higher $Q^2$, one must employ the $p(e,e'\pi^+)n$ reaction.

- $t$-channel process dominates $\sigma_L$ at small $-t$.

$$\frac{d\sigma_L}{dt} \propto -tQ^2 \frac{g_{\pi NN}^2(t) F_{\pi}^2(Q^2,t)}{(t - m^2_\pi)}$$
Extraction of form factor from $\sigma_L$ data

$p(e,e'\pi^+)n$ data are obtained some distance from the $t=m_\pi^2$ pole.

- No reliable phenomenological extrapolation possible.

A more reliable approach is to use a model incorporating the $\pi^+$ production mechanism and the `spectator’ nucleon to extract $F_\pi$ from $\sigma_L$.

Method check:
- It would be of great value to verify that $F_\pi(Q^2)$ values extracted from electroproduction data are in good agreement with those determined from $e-\pi$ scattering data.
- We propose to take $Q^2=0.30$ GeV$^2$ data at very low $-t=0.005$ GeV$^2$ to rigorously check the electroproduction method.
Previously obtained high $Q^2$ data from Cornell in the 1970’s have many problems

- Problematic L/T separation.
  - High and low $\varepsilon$ from different experiments used, or only low $\varepsilon$ setting taken.
  - In all cases, a model for $\sigma_T$ was used when extracting $\sigma_L$ and $F_\pi^T$.
- Analysis based on assumptions with systematic errors that are difficult to quantify.
  - Data taken far from pole, with $-t_{\text{min}}$ as high as 40 $m_\pi^2$.

“[we] question whether $F_\pi$ has been truly determined for large $Q^2$.”

The importance of appropriately-chosen kinematics

- Experiment must access small $-t$ to ensure $t$-channel dominance.

- Carlson and Milana [PRL 65(1990)1717] looked at competing non-pole QCD processes complicating the extraction of $F_\pi$ at large $Q^2$.
  - background ratio $M_{pQCD}/M_{pole}$ rises dramatically once $-t_{min}>0.20$.
  - “more reliable measurements of $F_\pi$ at high $Q^2$ require smaller $|t|$ and thus higher electron energy loss $\nu$.”

- 11 GeV upgrade and SHMS small angle capability are crucial for this task.

  - $\Rightarrow$ large $\nu \Rightarrow$ large $W \Rightarrow$ smaller $|t_{min}|$.
  - $\Rightarrow$ reduced model uncertainty in $F_\pi$ extraction.
  - $\Rightarrow$ expected smaller background to $\pi$ pole diagram.
\[
2\pi \frac{d\sigma}{dtd\phi} = \epsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\epsilon} \epsilon + 1 \frac{d\sigma_{LT}}{dt} \cos \phi + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi
\]

- **Extraction of** \(F_\pi\) **requires** \(t\) **dependence of** \(\sigma_L\) **to be known.**
  - Only three of \(Q^2, W, t, \theta_\pi\) are independent.
  - Vary \(\theta_\pi\) to measure \(t\) dependence.
  - Since non-parallel data needed, LT and TT must also be determined.
Simulated SHMS+HMS: $-t$ vs. $\phi$ coverage

$Q^2=6.0\ GeV^2, \ \varepsilon=0.435$

$Q^2=6.0\ GeV^2, \ \varepsilon=0.177$

- Multiple SHMS settings $\pm 2^\circ$ left and right of the $q$-vector are used to obtain good $\phi$-coverage over a range of $-t$.
  - Measurements over $0< \phi < 2\pi$ are required to determine LT, TT contributions versus $-t$.

Radial coordinate ($-t$). Azimuthal coordinate ($\phi$).
The different pion arm settings are combined to yield \(\phi\)-distributions for each \(t\)-bin

\[
\frac{d^2\sigma}{dt\,d\phi} = \varepsilon \frac{d\sigma_L}{dt\,d\phi} + \frac{d\sigma_T}{dt\,d\phi} + \sqrt{2\varepsilon(\varepsilon+1)} \frac{d\sigma_{LT}}{dt\,d\phi} \cos\phi + \varepsilon \frac{d\sigma_{TT}}{dt\,d\phi} \cos 2\phi
\]

- Extract all four response functions via a simultaneous fit using measured azimuthal angle \(\phi_\pi\) and knowledge of photon polarization \(\varepsilon\).

- This technique demands the good knowledge of the magnetic spectrometer acceptances.

F_{\pi-2} data: nucl-ex/0607005
Magnetic Spectrometer Calibrations

- Similarly to Fπ-2, we propose to use the over-constrained $p(e,e'p)$ reaction and inelastic $e^{+12}C$ in the DIS region to calibrate spectrometer acceptances, momenta, offsets, etc.
  - Fπ-2 beam energy and spectrometer momenta determined to <0.1%.
  - Spectrometer angles <0.5 mr.
  - Fπ-2 agreement with published $p+e$ elastics cross sections <2%.

<table>
<thead>
<tr>
<th>Projected Systematic Uncertainty Source</th>
<th>Pt-Pt $\epsilon$-random t-random</th>
<th>$\epsilon$-uncorrelated common to all t-bins</th>
<th>Scale $\epsilon$-global t-global</th>
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</thead>
<tbody>
<tr>
<td>Spectrometer Acceptance</td>
<td>0.4%</td>
<td>0.4%</td>
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<td>Target Thickness</td>
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<td>Kinematic Offsets</td>
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<td></td>
</tr>
</tbody>
</table>

Uncorrelated uncertainties in $\sigma_{UNS}$ are amplified by $1/\Delta\epsilon$ in L-T separation.
Scale uncertainty propagates directly into separated cross section.
**F\(\pi\)-1 and F\(\pi\)-2 used the VGL Regge Model to extract \(F_{\pi}(Q^2)\) from the \(\sigma_L\) data**

- **Feynman propagator** \(\frac{1}{t - m_{\pi}^2}\)
  - Represents the exchange of a **series** of particles, compared to a **single** particle.

- **Model parameters** fixed from pion photoproduction.

- **Free parameters**: \(\Lambda_\pi\), \(\Lambda_\rho\)
  (trajectory cutoff).

  [Vanderhaeghen, Guidal, Laget, PRC 57(1998)1454]

\[ F_{\pi} = \frac{1}{1 + Q^2 / \Lambda_\pi^2} \]

Fit to \(\sigma_L\) to model gives \(F_{\pi}\) at each \(Q^2\).

Error bars indicate statistical and random (pt-pt) systematic uncertainties in quadrature.
Yellow band indicates the correlated (scale) and partly correlated (t-corr) systematic uncertainties.

\[ \Lambda_\pi^2 = 0.513, \ 0.491 \text{ GeV}^2, \ \Lambda_\rho^2 = 1.7 \text{ GeV}^2. \]
The experimental result is not permanently “locked in” to a specific model.

In principle, the experimentalist would like to use a variety of models to extract $F_\pi(Q^2)$ from the electroproduction data, so that the model dependence can be better understood.

- Unfortunately, the VGL model is the only reliable model available for our use at present.

We intend to publish our experimentally measured $d\sigma_L/dt$, so that updated values of $F_\pi(Q^2)$ could be extracted in the future.

Theory review:

“The fact that the collaboration plans to publish the cross section data is to be applauded, as this would enable any future theoretical advances to subsequently improve on the extracted number.”
Data point at $Q^2=1.60$ GeV$^2$

to check model dependence of form factor extraction.

- $F_{\pi^-1} (W=1.95$ GeV$)$ and $F_{\pi^-2} (W=2.22$ GeV$)$ agree to $\sim 4\%$.

- New point is 30\% closer to pion pole, with significantly reduced uncertainties.

Proposed Kinematics (1)

\[ Q^2 = 0.30 \text{ GeV}^2 \]
- Precision low \(-t\) data to test the electroproduction method of \( F_{\pi^*} \).
- Perform a direct comparison with exact values from \( \pi^-e \) elastics.
- \(-t_{\text{min}} = 0.005\) is 50\% smaller than any previous electroproduction data.
- Measurement requires 5.5° \( \pi^+ \) arm (SHMS), 2.8-4.2 GeV beam.
- Use 30 \( \mu \)A on 4cm \( \text{LH}_2 \) target to avoid potentially high accidental coincidence rates.

\[ Q^2 = 1.60, 2.45 \text{ GeV}^2 \]
- Repeat measurements taken in \( F_{\pi^-1} \) and \( F_{\pi^-2} \) but at widely different \( W \) and \( t_{\text{min}} \).
- Needed to better understand model-dependence of \( F_{\pi} \) results.
Proposed Kinematics (2)

$Q^2=5.25, 6.00$ GeV$^2$

- Constraints: $|t_{\text{min}}| \approx 0.2$, $\Delta \epsilon \approx 0.3$, 10.9 GeV beam, 5.5° π$^+$ arm.
  - $\Rightarrow$ maximum $Q^2$ near 6.0 GeV$^2$.
  - Take $Q^2=5.25$ GeV$^2$ “nearby” where expected precision is better.

| $Q^2$ (GeV) | $W$ (GeV) | $\Delta \epsilon$ | $|t|$ (GeV$^2$) |
|-------------|-----------|---------------------|----------------|
| 5.25        | 3.20      | 0.31                | 0.17           |
| 6.00        | 3.20      | 0.26                | 0.21           |

$Q^2=3.50, 4.50$ GeV$^2$

- These points are crucial if highest $Q^2$ points suggest a “turnover” in $Q^2 F_\pi$ and pQCD limit being reached.

| $Q^2$ (GeV) | $W$ (GeV) | $\Delta \epsilon$ | $|t|$ (GeV$^2$) |
|-------------|-----------|---------------------|----------------|
| 3.50        | 3.10      | 0.37                | 0.10           |
| 4.25        | 3.28      | 0.30                | 0.12           |
How to verify that $\sigma_L$ is dominated by the $t$-channel process

- $\pi^+$ $t$-channel diagram is purely isovector.
- Measure

$$R_L = \frac{\sigma_L[n(e, e'\pi^-)p]}{\sigma_L[p(e, e'\pi^+)n]} = \frac{|A_Y - A_S|^2}{|A_Y + A_S|^2}$$

using a deuterium target.
- Isoscalar backgrounds (such as $b_1(1235)$ contributions to the $t$-channel) will dilute the ratio.
- We propose the same tests at $Q^2=1.60$ and $3.50$ GeV$^2$.

Because one of the many problems encountered by the historical data was isoscalar contamination, allocation of beamtime to this test will increase the confidence in the extraction of $F_{\pi}(Q^2)$ from our $\sigma_L$ data.
Question: Two $\gamma$ exchange?

In the Rosenbluth separation of the proton electric form factor, $2\gamma$ contributions may be important because one is trying to separate a small cross section (electric) from a much larger (magnetic) one.

$2\gamma$ exchange is not expected to be a significant issue in the extraction of $\sigma_L$ in pion electroproduction.

$Q^2=6$ GeV$^2$ calculation performed by Tjon and Melnitchouk.

Correction=$\delta_{\text{FULL}} - \delta_{\text{Mo&Tsai}}$
Question: Target Cell Length

Technical Review:
“The experiment assumes beyond the standard 4 cm targets also non-standard 8 cm long targets. Although certainly possible, it would be useful to have an allowable range for ease of scheduling and cost reasons.”

- The SHMS has a very large $y_{tar}$ acceptance and sits at forward angles, so is not an issue.
- The HMS is at angle up to $47^\circ$ and poses the main limitation, although not as extreme as SOS.
- Simulations indicate a cell length up to 10 cm is probably okay.
Projected Error Bars

- Rates and uncertainties are based on an empirical parameterization of existing electroproduction data.
  - conservative assumptions used when extrapolating to poorly measured $Q^2$.
- Error is amplified by $\Delta \varepsilon$ and potentially large $r$.

<table>
<thead>
<tr>
<th>$Q^2$ (GeV)</th>
<th>$W$ (GeV)</th>
<th>Projected $r_{\pi^+}/r_{L}$</th>
<th>$\Delta \varepsilon$</th>
<th>$\Delta F_{\pi^+}/F_{\pi}$ (%)</th>
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<td>0.73</td>
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F$_\pi$-2 Final Errors

- 1.60 2.22 0.48 0.27 4.9
- 2.45 2.22 0.80 0.28 6.0
## Beam Time Estimate

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<th>$\varepsilon$ settings</th>
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<th>LD+ Hours</th>
<th>LD- Hours</th>
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</table>

**Grand Total: 1339 hrs (56 days)**

- Calibration measurements detailed in the proposal indicate that a useful set of $^1$H(e,e'p) coincidence data can be compiled with a reasonable investment of beam time.
- The LD-, $Q^2=0.30$ and some of the optics runs are unaffected if the maximum beam current is <90 $\mu$A.
Summary

To reliably extract $F_\pi$ at higher $Q^2$, measure $p(e,e'\pi^+)n$ close to the pole.
- Measure $d\sigma_L$ at $-t<0.2$ GeV$^2$, $W>3$ GeV.
- 11 GeV beam and forward angle capability of SHMS are essential.

Use best available model(s) for $\sigma_L$ to extract $F_\pi$.
- $d\sigma_L/dt$ vs $-t$ to test reliability of model.
- Non parallel kinematics used. $\Rightarrow LT, TT$.

Take $\pi^\pm$ data to verify $t$-channel dominance.
- Deuterium target.

Extraction of $F_\pi$ to $Q^2=6$ GeV$^2$ would challenge QCD-based calculations in the most rigorous manner.

Test electroproduction method by taking low $Q^2$ data very close to the pole.

A unique opportunity for JLab to dramatically improve the $F_\pi(Q^2)$ database.

→ An essential part of the Hall C program 18-24 months after the start of SHMS commissioning.