PR12-06-101:
Measurement of the Charged Pion Form Factor to High Q^2

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Scientific Motivation

- The pion form factor is a topic of fundamental importance to our understanding of hadronic structure.
- Pions are the lightest QCD system ($q\bar{q}$).
  - all hadronic structure models use the π^+ as a test case.
  - "the positronium atom of QCD".

At large Q^2 , F_π is presumably given by pQCD

$$F_\pi(Q^2) = \frac{4\pi C_F \alpha_S(Q^2)}{Q^2} \left| \sum_{n=0}^{\infty} a_n \left(\log\left(\frac{Q^2}{\Lambda^2}\right)\right)^{-\gamma_n} \right|^2 \left[1 + O(\alpha_s(Q^2), m/Q)\right]$$

which in the $Q^2 \rightarrow \infty$ limit becomes

$$F_\pi(Q^2) \underset{Q^2 \to \infty}{\longrightarrow} \frac{16\pi\alpha_s(Q^2) f_\pi^2}{Q^2}$$

where f^2_π=93 MeV is the $\pi^+ \rightarrow \mu^+ \nu$ decay constant.

G.P. Lepage, S.J. Brodsky, Phys.Lett. **87B**(1979)359.

F_π is the clearest test case for study of transition between non-perturbative and pQCD regions.

- What is the structure of the π^+ at all Q^2?
  - at what value of Q^2 will the pQCD contributions dominate?
- A difficult question to answer, as both "hard" and "soft" components (such as gluonic effects) must be taken into account.
  - non-perturbative hard components of higher twist strongly cancel soft components, even at modest Q^2.
    *[Braun et al., PRD **61**(2000)073004]*
  - the situation for nucleon form factors is even more complicated.
- Many model calculations exist, but ultimately...
  - Reliable $F_\pi(Q^2)$ data are needed to delineate the role of hard versus soft contributions at intermediate Q^2.
- A program of study unique to Jefferson Lab.

Theory Review Comments:

- The pion form factor is an object of great theoretical interest, especially at larger values of Q^2, where one can study nonperturbative dynamics of QCD while searching for the transition to the perturbative regime.

- While the merits of studying this observable are clear, the extraction of the pion form factor from data is a non-trivial exercise.

Determination of F_π via Pion Electroproduction

Pion charge radius is well known from $e^+e^- \rightarrow \pi^+\pi^-$ experiments.

$$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

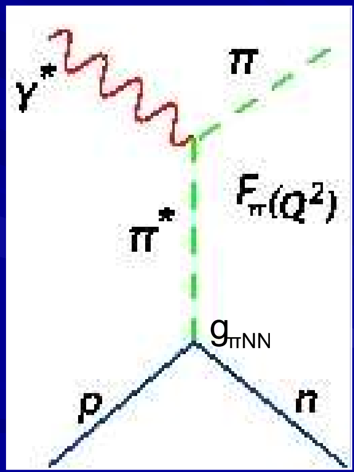
At low Q^2<0.3 GeV², the π^+ form factor can be measured exactly using high energy π^+ scattering from atomic electrons.

$\Rightarrow$ 300 GeV pions at CERN SPS. *[Amendolia et al., NP **B277**(1986)168]*

To access higher Q^2, one must employ the *p(e,e'π⁺)n* reaction.

- *t*-channel process dominates σ_L at small $-t$.

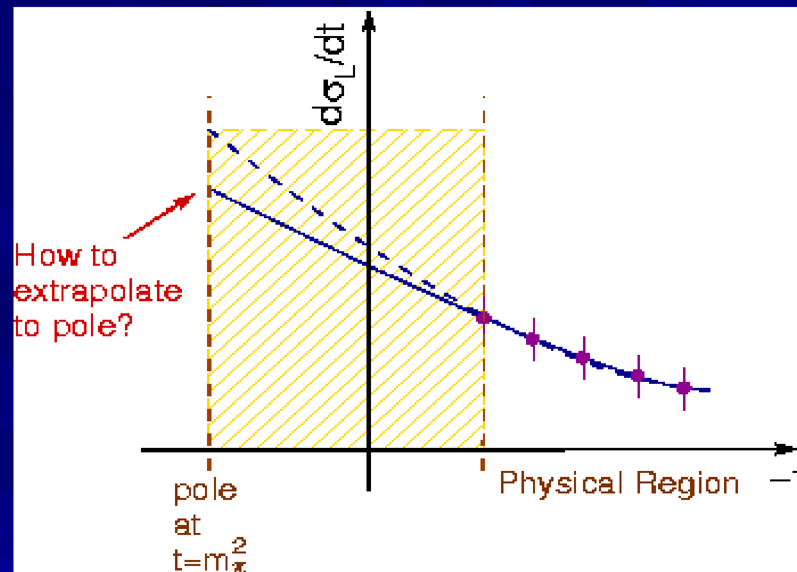
$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t-m_\pi^2)}\; g_{\pi NN}^2(t)\, F_\pi^2(Q^2,t)$$



Extraction of form factor from σ_L data

p(e,e'π⁺)n data are obtained some distance from the $t=m_\pi^2$ pole.

– No reliable phenomenological extrapolation possible.



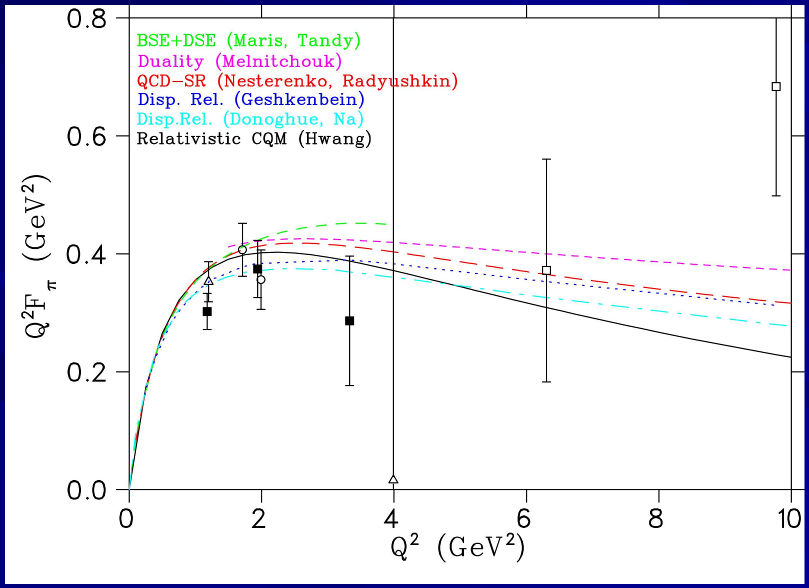
A more reliable approach is to use a model incorporating the π^+ production mechanism and the `spectator' nucleon to extract F_π from σ_L.

Method check:

- It would be of great value to verify that $F_\pi(Q^2)$ values extracted from electroproduction data are in good agreement with those determined from *e*-π scattering data.
- We propose to take Q^2=0.30 GeV^2 data at very low $-t$=0.005 GeV^2 to rigorously check the electroproduction method.

Previously obtained high Q^2 data from Cornell in the 1970's have many problems

- Problematic L/T separation.
  - High and low ε from different experiments used, or only low ε setting taken.
  - In all cases, a model for σ_T was used when extracting σ_L and F_π.
- Analysis based on assumptions with systematic errors that are difficult to quantify.
  - Data taken far from pole, with $-t_{min}$ *as high as 40* m_π^2.



"[we] question whether F_π has been truly determined for large Q^2."

C.E. Carlson, J. Milana, PRL **65**(1990)1717.

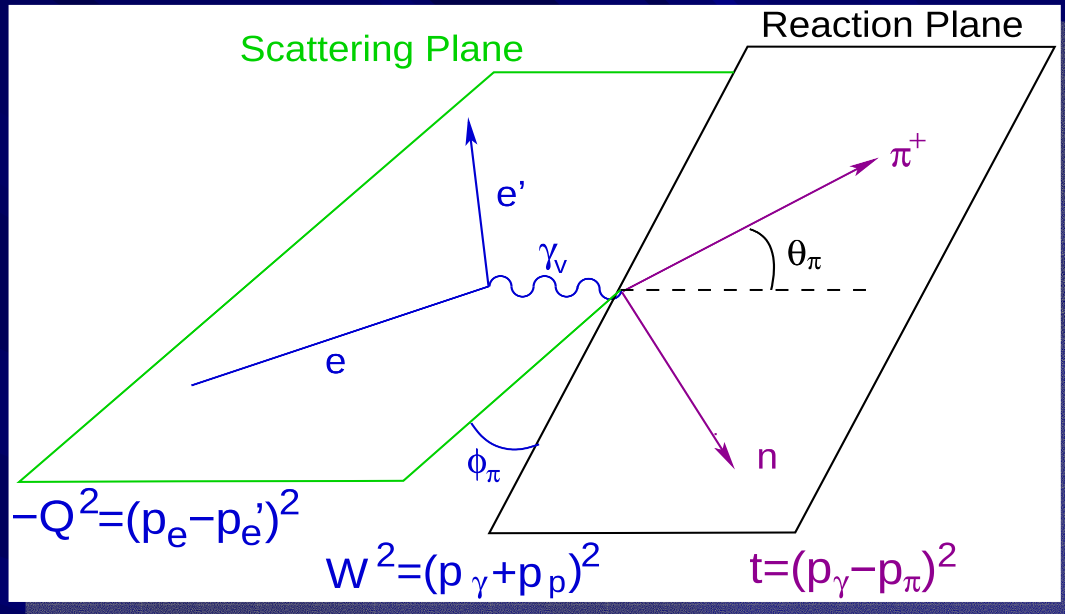
The importance of appropriately-chosen kinematics

- Experiment must access small $-t$ to ensure t-channel dominance.
- Carlson and Milana [PRL **65**(1990)1717] looked at competing non-pole QCD processes complicating the extraction of F_π at large Q^2.
  – background ratio M_{pQCD}/M_{pole} rises dramatically once $-t_{min}>0.20$.
  – "more reliable measurements of F_π at high Q^2 require smaller $|t|$ and thus higher electron energy loss ν."
- 11 GeV upgrade and SHMS small angle capability are crucial for this task.

  $\Rightarrow$ large ν $\Rightarrow$ large W $\Rightarrow$ smaller $|t_{min}|$.

  $\Rightarrow$ reduced model uncertainty in F_π extraction.

  $\Rightarrow$ expected smaller background to π pole diagram.

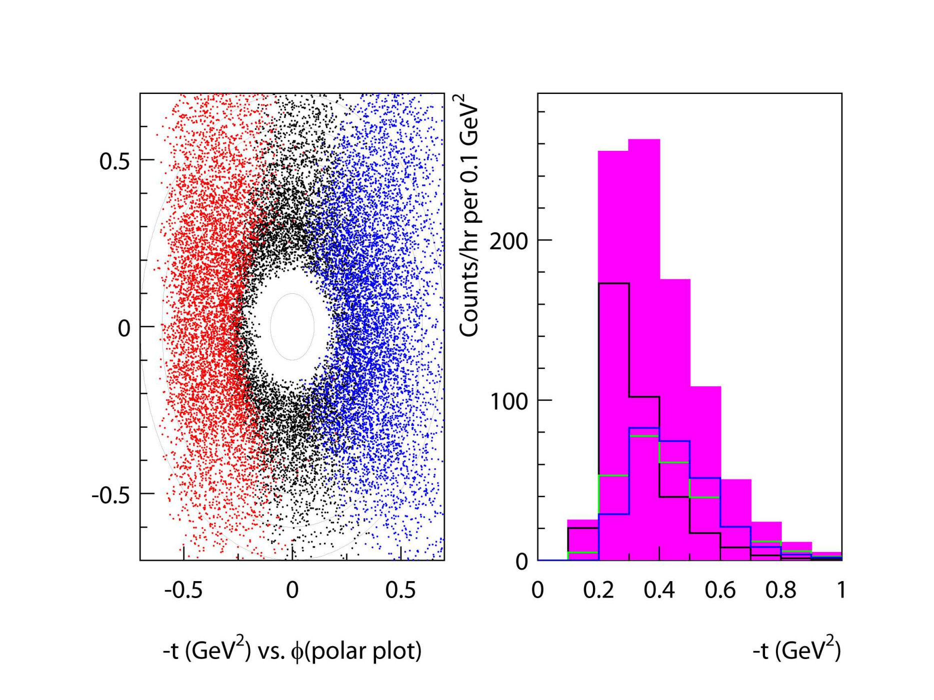


$$2\pi \frac{d\sigma}{dt\,d\phi} = \varepsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon(\varepsilon+1)}\,\frac{d\sigma_{LT}}{dt}\cos\phi + \varepsilon \frac{d\sigma_{TT}}{dt}\cos 2\phi$$

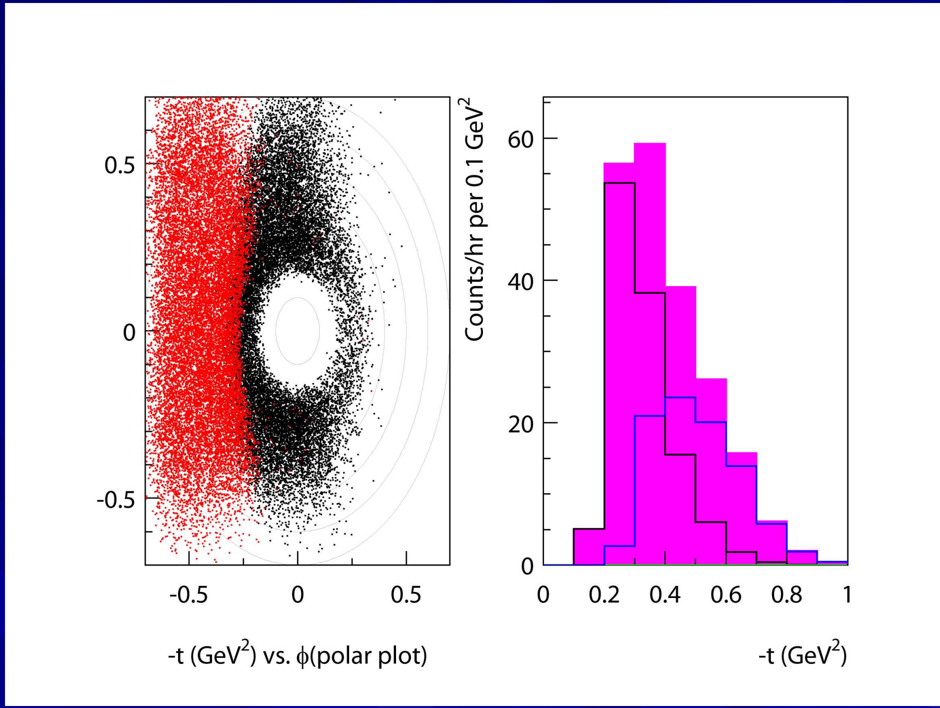
- Extraction of F_π requires t dependence of σ_L to be known.
  - Only three of Q^2, W, t, θ_π are independent.
  - Vary θ_π to measure t dependence.
  - Since non-parallel data needed, LT and TT must also be determined.

Simulated SHMS+HMS $-t$ vs. φ coverage

Q^2=6.0 GeV^2, ε=0.435



Q^2=6.0 GeV^2, ε=0.177



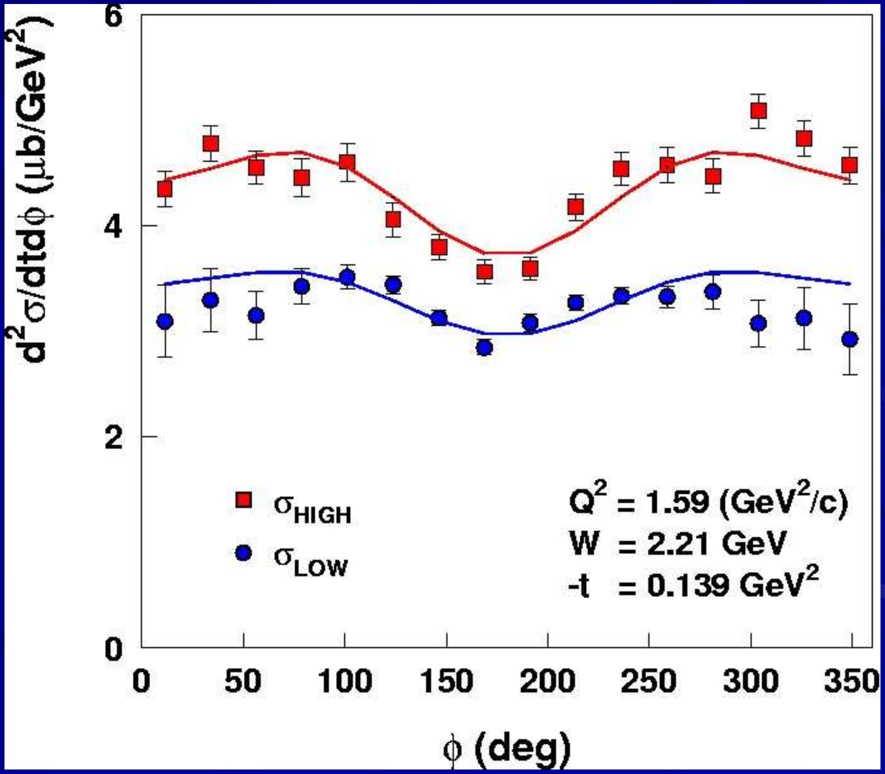
- Multiple SHMS settings ±2° left and right of the q-vector are used to obtain good φ-coverage over a range of $-t$.
  - Measurements over $0<\varphi<2\pi$ are required to determine LT, TT contributions versus $-t$.

Radial coordinate ($-t$). Azimuthal coordinate (φ).

The different pion arm settings are combined to yield φ-distributions for each *t*-bin

$$\frac{d^2\sigma}{dt\,d\phi} = \varepsilon\frac{d\sigma_L}{dtd\phi} + \frac{d\sigma_T}{dt\,d\phi} + \sqrt{2\varepsilon(\varepsilon+1)}\frac{d\sigma_{LT}}{dt\,d\phi}\cos\phi_\pi + \varepsilon\frac{d\sigma_{TT}}{dt\,d\phi}\cos 2\phi_\pi$$

- Extract all four response functions via a simultaneous fit using measured azimuthal angle (φ_π) and knowledge of photon polarization (ε).
- This technique demands the good knowledge of the magnetic spectrometer acceptances.



Fπ-2 data: nucl-ex/0607005

Magnetic Spectrometer Calibrations

- Similarly to Fπ-2, we propose to use the over-constrained *p(e,e'p)* reaction and inelastic $e+{}^{12}\mathrm{C}$ in the DIS region to calibrate spectrometer acceptances, momenta, offsets, etc.
  - Fπ-2 beam energy and spectrometer momenta determined to <0.1%.
  - Spectrometer angles <0.5 mr.
  - Fπ-2 agreement with published *p+e* elastics cross sections <2%.

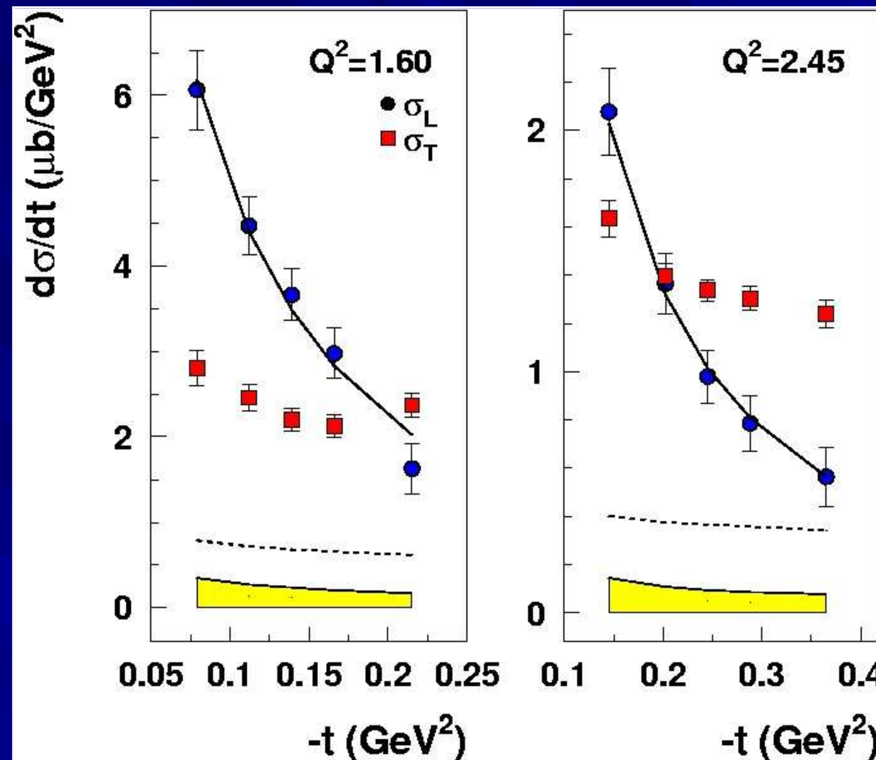
Projected Systematic Uncertainty Source	Pt-Pt ε-random t-random	ε-uncorrelated common to all t-bins	Scale ε-global t-global
Spectrometer Acceptance	0.4%	0.4%	1.0%
Target Thickness		0.2%	0.8%
Beam Charge	-	0.2%	0.5%
HMS+SHMS Tracking	0.1%	0.4%	1.5%
Coincidence Blocking		0.2%	
PID		0.4%	
Pion Decay Correction	0.03%	-	0.5%
Pion Absorption Correction	-	0.1%	1.5%
MC Model Dependence	0.2%	1.0%	0.5%
Radiative Corrections	0.1%	0.4%	2.0%
Kinematic Offsets	0.4%	1.0%	-

- Uncorrelated uncertainties in σ_{UNS} are amplified by $1/\Delta\varepsilon$ in L-T separation.
- Scale uncertainty propagates directly into separated cross section.

Fπ-1 and Fπ-2 used the VGL Regge Model to extract $F_\pi(Q^2)$ from the σ_L data

- Feynman propagator $\left(\frac{1}{t-m_\pi^2}\right)$ replaced by π and ρ Regge propagators.
  - Represents the exchange of a series of particles, compared to a single particle.
- Model parameters fixed from pion photoproduction.
- Free parameters: Λ_π, Λ_ρ (trajectory cutoff).

[Vanderhaeghen, Guidal, Laget, PRC **57**(1998)1454]



Fπ-2 data: nucl-ex/0607005

$$F_\pi = \frac{1}{1+Q^2/\Lambda_\pi^{\;2}}$$

Error bars indicate statistical and random (pt-pt) systematic uncertainties in quadrature.
Yellow band indicates the correlated (scale) and partly correlated (t-corr) systematic uncertainties.

Fit to σ_L to model gives F_π at each Q^2.

Λ_π^2=0.513, 0.491 GeV^2, Λ_ρ^2=1.7 GeV^2.

The experimental result is not permanently "locked in" to a specific model.

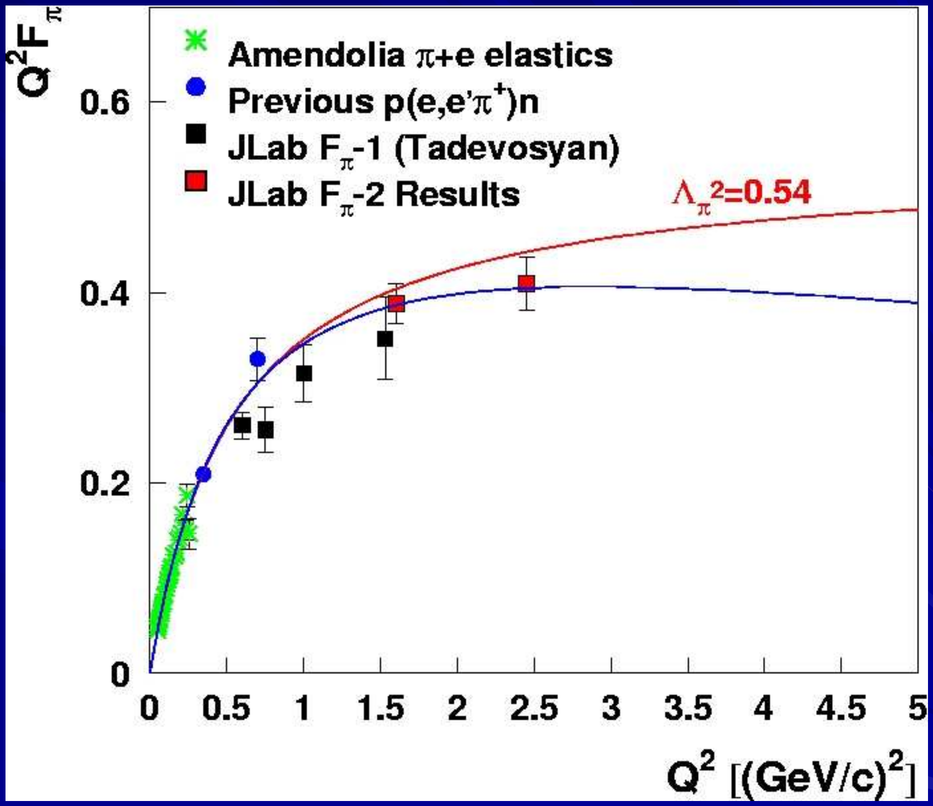
- In principle, the experimentalist would like to use a variety of models to extract $F_\pi(Q^2)$ from the electroproduction data, so that the model dependence can be better understood.
  - Unfortunately, the VGL model is the only reliable model available for our use at present.
- We intend to publish our experimentally measured $d\sigma_L/dt$, so that updated values of $F_\pi(Q^2)$ could be extracted in the future.
- Theory review:

  "The fact that the collaboration plans to publish the cross section data is to be applauded, as this would enable any future theoretical advances to subsequently improve on the extracted number."

F_π-1 and F_π-2 Results

Fπ-1: nucl-ex/0607007.
Fπ-2: nucl-ex/0605005.

- Data point at Q^2=1.60 GeV^2 to check model dependence of form factor extraction.
  - –F_π-1 (W=1.95 GeV) and F_π-2 (W=2.22 GeV) agree to ~4%.
  - –New point is 30% closer to pion pole, with significantly reduced uncertainties.



S.R. Amendolia, et al., Nucl. Phys. **B277** (1986) 168.
H. Ackermann, et al, Nucl. Phys. **B137** (1978) 294.
P. Brauel, et al., Z. Phys. **C3** (1979) 101.

Proposed Kinematics (1)

- Q^2=0.30 GeV^2
  - Precision low $-t$ data to test the electroproduction method of F_π.
  - Perform a direct comparison with exact values from π-e elastics.
  - $-t_{min}$=0.005 is 50% smaller than any previous electroproduction data.
  - Measurement requires 5.5° π^+ arm (SHMS), 2.8-4.2 GeV beam.
  - Use 30 μA on 4cm LH_2 target to avoid potentially high accidental coincidence rates.
- Q^2=1.60, 2.45 GeV^2
  - Repeat measurements taken in Fπ-1 and Fπ-2 but at widely different W and t_{min}.
  - Needed to better understand model-dependence of F_π results.

Q^2 (GeV)	W (GeV)	|t| (GeV^2)
1.60	3.00	0.029
	2.22	0.095
	1.95	0.150
2.45	3.20	0.048
	2.22	0.186

Proposed Kinematics (2)

- Q^2=5.25, 6.00 GeV^2
  - Constraints: $|t_{min}|\approx 0.2$, $\Delta\varepsilon\approx 0.3$, 10.9 GeV beam, 5.5^o π^+ arm.

    $\Rightarrow$ maximum Q^2 near 6.0 GeV^2.
  - Take Q^2=5.25 GeV^2 "nearby" where expected precision is better.

Q^2 (GeV)	W (GeV)	Δε	|t| (GeV^2)
5.25	3.20	0.31	0.17
6.00	3.20	0.26	0.21

- Q^2=3.50, 4.50 GeV^2
  - These points are crucial if highest Q^2 points suggest a "turnover" in Q^2F_π and pQCD limit being reached.

Q^2 (GeV)	W (GeV)	Δε	|t| (GeV^2)
3.50	3.10	0.37	0.10
4.25	3.28	0.30	0.12

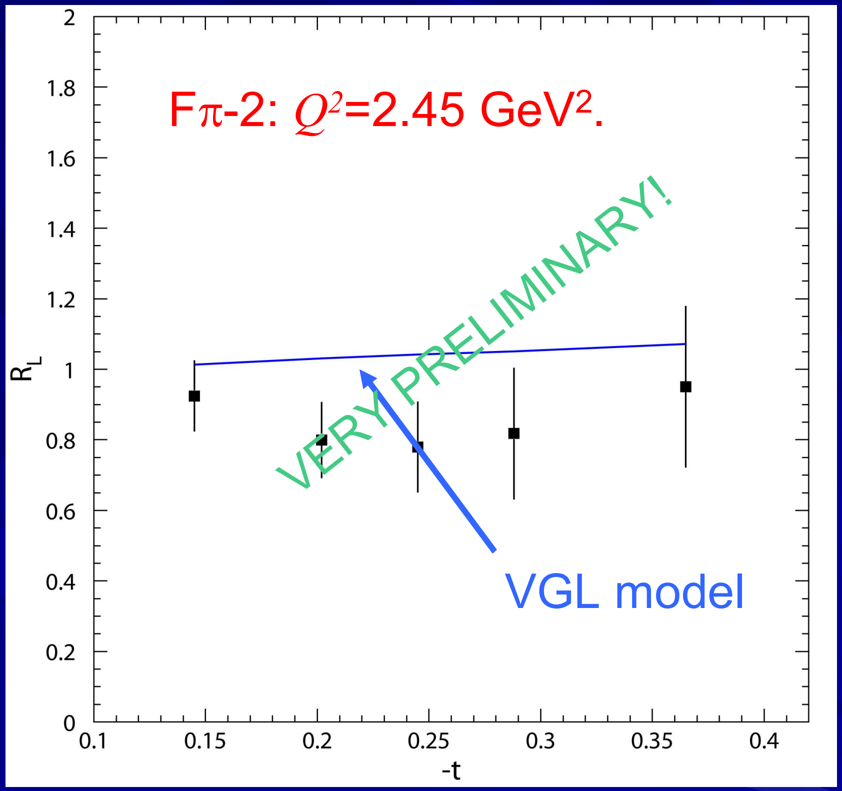
How to verify that σ_L is dominated by the t-channel process

- π^+ t-channel diagram is purely isovector.
- measure

$$R_L = \frac{\sigma_L[n(e,e'\pi^-)p]}{\sigma_L[p(e,e'\pi^+)n]} = \frac{|A_V - A_S|^2}{|A_V + A_S|^2}$$

using a deuterium target.

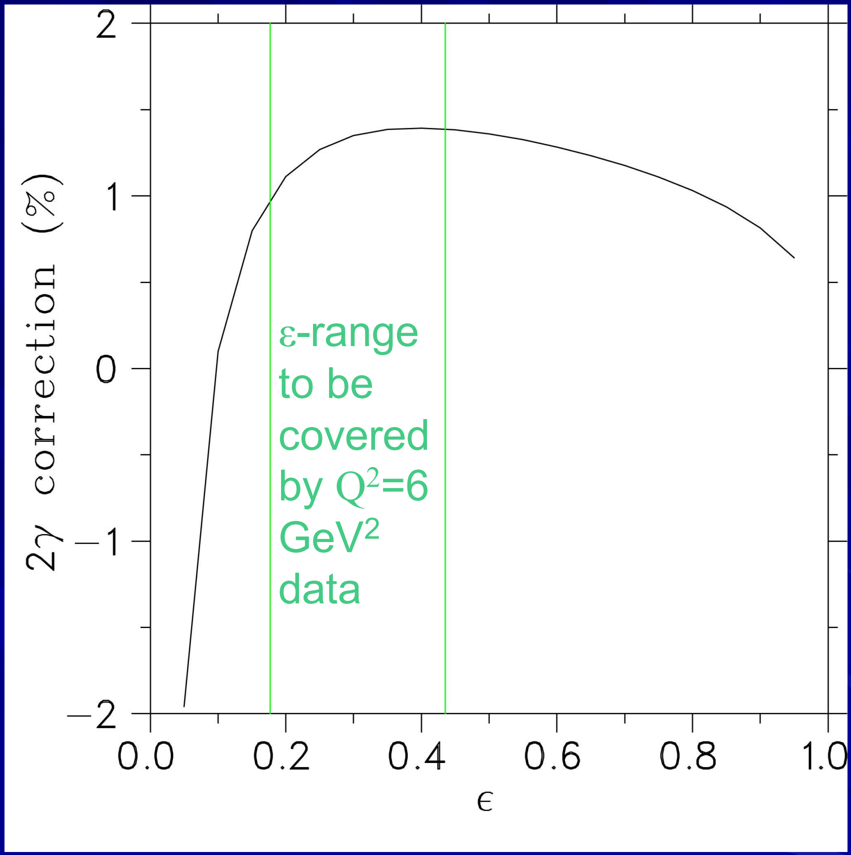
- isoscalar backgrounds (such as $b_1(1235)$ contributions to the t-channel) will dilute the ratio.
- We propose the same tests at Q^2=1.60 and 3.50 GeV².



Because one of the many problems encountered by the historical data was isoscalar contamination, allocation of beamtime to this test will increase the confidence in the extraction of $F_\pi(Q^2)$ from our σ_L data.

Question: Two γ exchange?

- In the Rosenbluth separation of the proton electric form factor, 2γ contributions may be important because one is trying to separate a small cross section (electric) from a much larger (magnetic) one.
- 2γ exchange is not expected to be a significant issue in the extraction of σ_L in pion electroproduction.



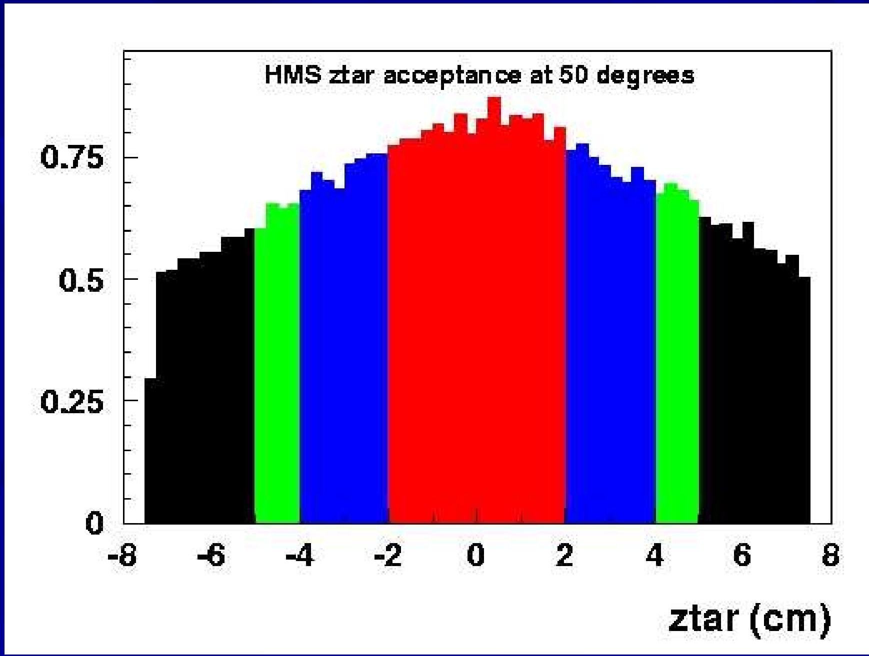
Q^2=6 GeV² calculation performed by Tjon and Melnitchouk.
Correction=δ_{FULL}-$\delta_{Mo\&Tsai}$

Question: Target Cell Length

Technical Review:

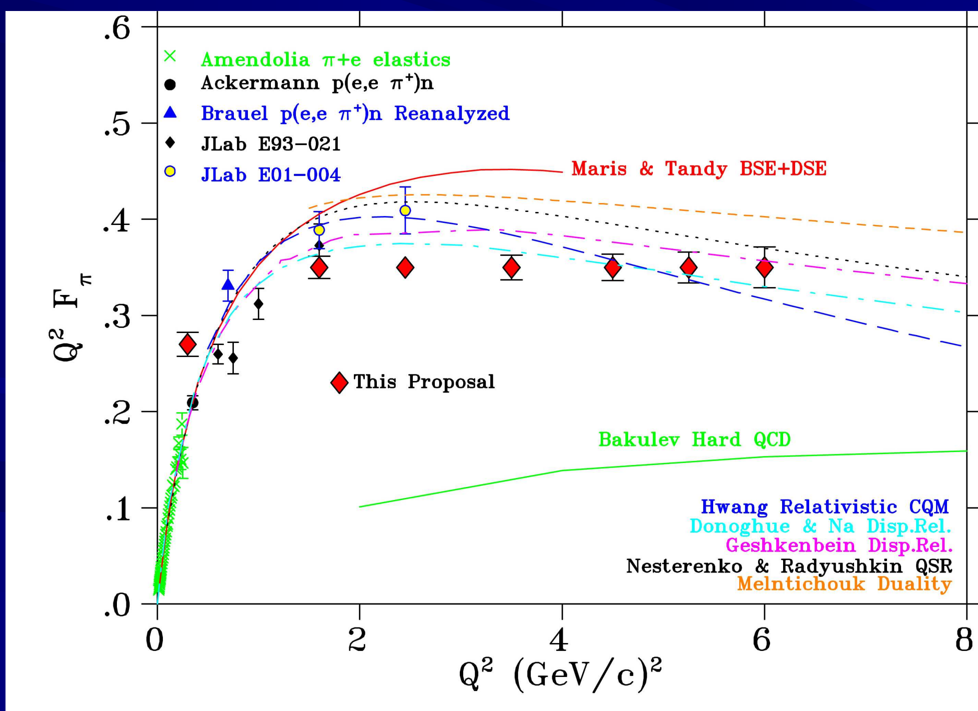
"The experiment assumes beyond the standard 4 cm targets also non-standard 8 cm long targets. Although certainly possible, it would be useful to have an allowable range for ease of scheduling and cost reasons."

- The SHMS has a very large y_{tar} acceptance and sits at forward angles, so is not an issue.
- The HMS is at angle up to 47° and poses the main limitation, although not as extreme as SOS.
- Simulations indicate a cell length up to 10 cm is probably okay.



Projected Error Bars

- Rates and uncertainties are based on an empirical parameterization of existing electroproduction data.
  - conservative assumptions used when extrapolating to poorly measured Q^2.
- Error is amplified by $\Delta\varepsilon$ and potentially large *r*.



Q^2 (GeV)	*W* (GeV)	Projected $r=\sigma_T/\sigma_L$	$\Delta\varepsilon$	$\Delta F_\pi/F_\pi$ (%)
0.30	2.20	0.63	0.41	5.2
1.60	3.00	0.18	0.38	3.6
2.45	3.20	0.19	0.44	3.0
3.50	3.20	0.32	0.37	4.0
4.50	3.28	0.38	0.30	4.3
5.25	3.20	0.56	0.31	5.0
6.00	3.20	0.73	0.26	6.6
Fπ-2 Final Errors				
1.60	2.22	0.48	0.27	4.9
2.45	2.22	0.80	0.28	6.0

Beam Time Estimate

Q^2 (GeV^2)	ε settings	LH+ Hours	LD+ Hours	LD- Hours	Over-head	Total Hours
6.00	3	376	--	--	12	388
5.25	3	231	--	--	12	243
4.50	3	125	--	--	12	137
3.50	3	39	31	153	20	243
2.45	4	43	--	--	16	59
1.60	3	24	16	21	20	81
0.30	3	24	--	--	12	36
Subtotals		862	47	174	104	
^{1}H(*e,e'p*) + Optics						80
9 Beam Changes						72
Grand Total: 1339 hrs (56 days)						

- Calibration measurements detailed in the proposal indicate that a useful set of ^{1}H(*e,e'p*) coincidence data can be compiled with a reasonable investment of beam time.
- The LD-, Q^2=0.30 and some of the optics runs are unaffected if the maximum beam current is <90 μA.

Summary

- To reliably extract F_π at higher Q^2, measure $p(e,e'\pi^+)n$ close to the pole.
  - Measure $d\sigma_L$ at $-t$<0.2 GeV^2, W>3 GeV.
  - 11 GeV beam and forward angle capability of SHMS are essential.
- Use best available model(s) for σ_L to extract F_π.
  - $d\sigma_L/dt$ vs $-t$ to test reliability of model.
  - Non parallel kinematics used. $\Rightarrow$ *LT, TT*.
- Take $\pi^\pm$ data to verify t-channel dominance.
  - Deuterium target.
- Extraction of F_π to Q^2=6 GeV^2 would challenge QCD-based calculations in the most rigorous manner.
- Test electroproduction method by taking low Q^2 data very close to the pole.

A unique opportunity for JLab to dramatically improve the $F_\pi(Q^2)$ database.

→ An essential part of the Hall C program 18-24 months after the start of SHMS commissioning.