Pion Form Factor:
Present and Future

Garth Huber

UNIVERSITY OF REGINA

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Jefferson Lab F_{π} Collaboration

R. Ent, D. Gaskell, M.K. Jones, D. Mack, D. Meekins, J. Roche, G. Smith, W. Vulcan, G. Warren, S.A. Wood

Jefferson Lab, Newport News, VA , USA

E.J. Brash, C. Butuceanu, G.M. Huber, V. Kovaltchouk, G.J. Lolos, S. Vidakovic, C. Xu

University of Regina, Regina, SK, Canada

H.P. Blok, J. Volmer, V. Tvaskis

Vrije Universiteit, Amsterdam, Netherlands

E. Beise, H. Breuer, C.C. Chang, T. Horn, P. King, J. Liu, P.G. Roos

University of Maryland, College Park, MD, USA

W. Boeglin, P. Markowitz, J. Reinhold

Florida International University, FL, USA

J. Arrington, R. Holt, D. Potterveld, P. Reimer, X. Zheng

Argonne National Laboratory, Argonne, IL, USA

H. Mkrtchyan, V. Tadevosyan

Yerevan Physics Institute, Yerevan, Armenia

S. Jin, W. Kim

Kyungook National University, Taegu, Korea

M.E. Christy, C. Keppel, L.G. Tang

Hampton University, Hampton, VA, USA

A. Matsumura, T. Miyoshi, Y. Okayasu

Tohuku University, Sendai, Japan

B. Barrett, A. Sarty

St. Mary's University, Halifax, NS, Canada

K. Aniol, D. Margaziotis

California State University, Los Angeles, CA, USA

L. Pentchev, C. Perdrisat

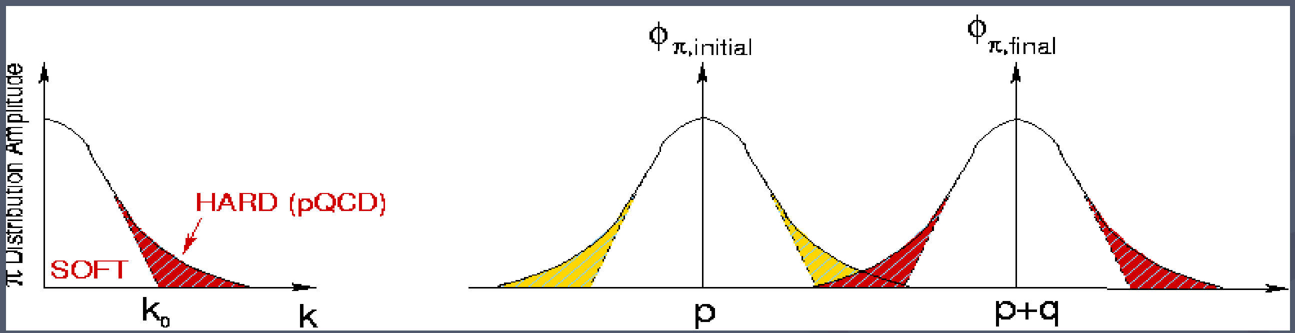
College of William and Mary, Williamsburg, VA, USA

E. Gibson, I. Niculescu, V. Punjabi

The Pion Charge Form Factor

F_π is a topic of fundamental importance to our understanding of hadronic structure, as the $q\bar{q}$ valence structure of the π^+ is relatively simple.

In quantum field theory, F_π is the overlap integral: $F_{\pi^+}(Q^2) = \int \phi_\pi^*(p)\phi_\pi(p+q)dp$



The pion wave function can be separated into φ_π^{soft} with only low momentum contributions ($k<k_0$) and a hard tail φ_π^{hard}.

While φ_π^{hard} can be treated in pQCD, φ_π^{soft} cannot.

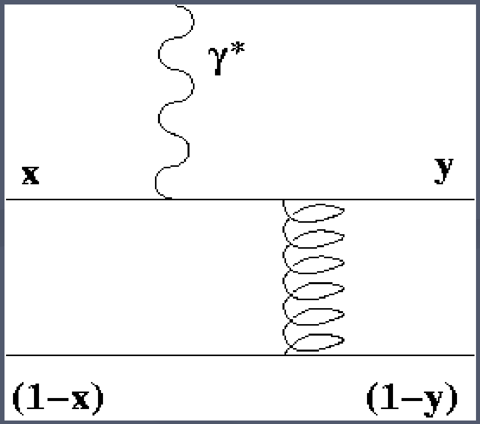
From a theoretical standpoint, the study of the Q^2 dependence of F_π focuses on finding a description for the hard and soft contributions of the pion wave function.

QCD Hard Scattering Picture

At large Q^2, perturbative QCD (pQCD) can be used

$$F_\pi(Q^2) = \frac{4\pi C_F \alpha_S(Q^2)}{Q^2} \left| \sum_{n=0}^{\infty} a_n \left(\log\left(\frac{Q^2}{\Lambda^2}\right)\right)^{-\gamma_n} \right|^2 \left[1 + O\left(\alpha_s(Q^2), \frac{m}{Q}\right)\right]$$

at asymptotically high Q^2, only the hardest portion of the wave function remains



$$\phi_\pi(x) \underset{Q^2\to\infty}{\to} \frac{3 f_\pi}{\sqrt{n_c}} x(1-x)$$

and F_π takes the very simple form

$$F_\pi(Q^2) \underset{Q^2\to\infty}{\to} \frac{16\pi\alpha_s(Q^2) f_\pi^2}{Q^2}$$

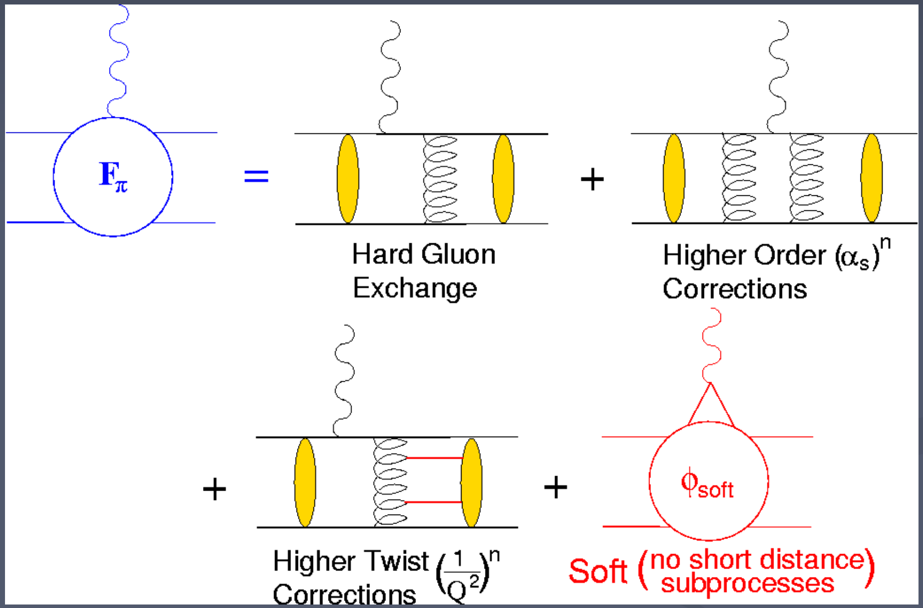
where f^2_π=93 MeV is the $\pi^+ \to \mu^+ \nu$ decay constant.

G.P. Lepage, S.J. Brodsky, Phys.Lett. **87B**(1979)359.

Intermediate Q^2 Scattering Picture

At experimentally-accessible Q^2, the situation is more complicated

- both the "hard" and "soft" components (e.g. transverse momentum effects) contribute.



- The interplay of the hard and soft contributions is poorly understood.

→ Non-perturbative hard components of higher-twist strongly cancel soft components, even at modest Q^2. *[Braun et al., PRD 61(2000)073004]*

→ Different theoretical viewpoints on whether higher-twist mechanisms dominate until very large momentum transfer or not.

F_π is one of the best choices for studying the transition from non-perturbative to perturbative QCD

- **The** π^+ is one of the simplest QCD systems available for study.
  - **What is the structure of the π^+ at all Q^2?**

    → all hadronic structure models use the π^+ as a test case.

- At what value of Q^2 will the hard pQCD contributions dominate?

    → The situation for nucleon form factors is even more complicated.

- **Reliable $F_\pi(Q^2)$ data are required to delineate the role of hard versus soft contributions at intermediate Q^2.**
  - Jefferson Lab is the only experimental facility capable of the necessary measurements.

Determination of F_π via Pion Electroproduction

At low Q^2<0.3 GeV^2, the π^+ form factor can be measured **exactly** using high energy π^+ scattering from atomic electrons.

⇒ 300 GeV pions at CERN SPS. *[Amendolia et al., NP **B277**(1986)168]*

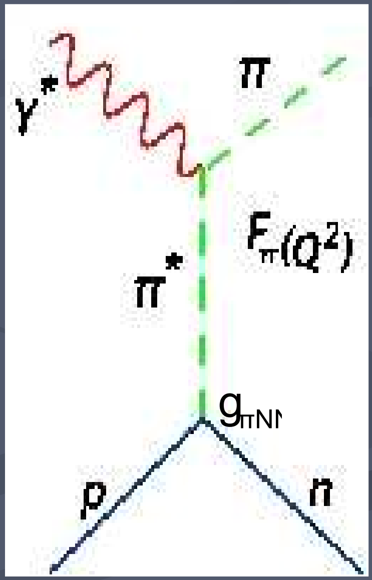
⇒ Provides an accurate measure of the π^+ charge radius.

$$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

To access higher Q^2, one must employ the *p(e,e'π$^+$)n* reaction.

- t-channel process dominates σ_L at small $-t$.
- In the Born term model:

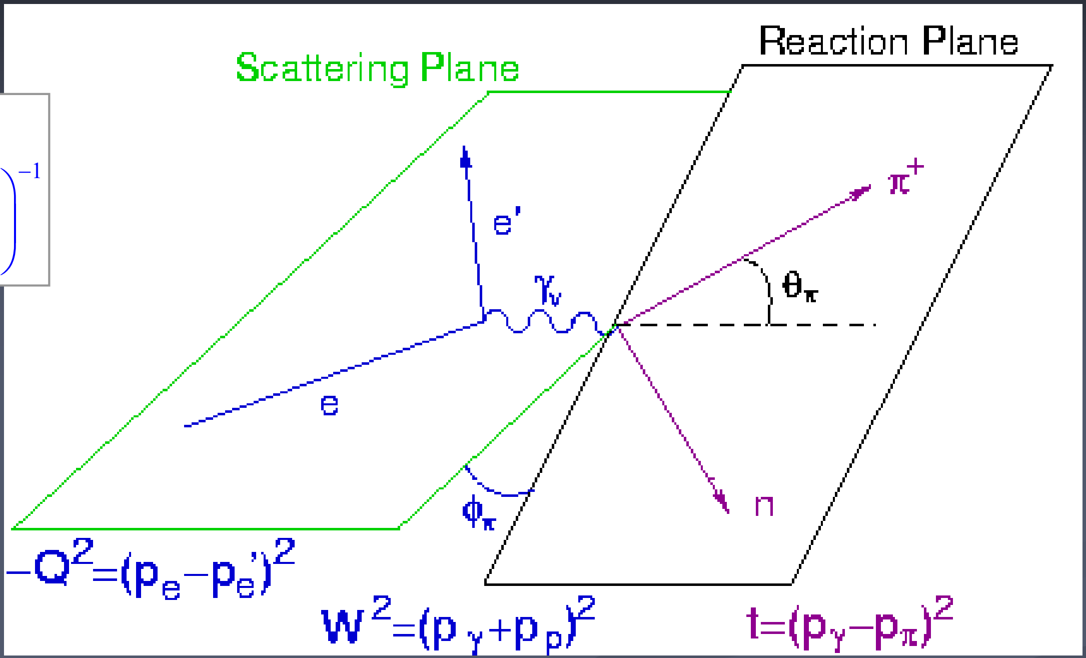
$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t-m_\pi^2)} \, g_{\pi NN}^2(t)\, F_\pi^2(Q^2,t)$$



$$2\pi \frac{d^2\sigma}{dt\,d\phi} = \varepsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon(\varepsilon+1)}\, \frac{d\sigma_{LT}}{dt}\cos\phi + \varepsilon \frac{d\sigma_{TT}}{dt}\cos 2\phi$$

Virtual-photon polarization:

$$\varepsilon = \left(1 + 2\frac{(E_e - E_{e'})^2 + Q^2}{Q^2}\tan^2\frac{\theta_{e'}}{2}\right)^{-1}$$

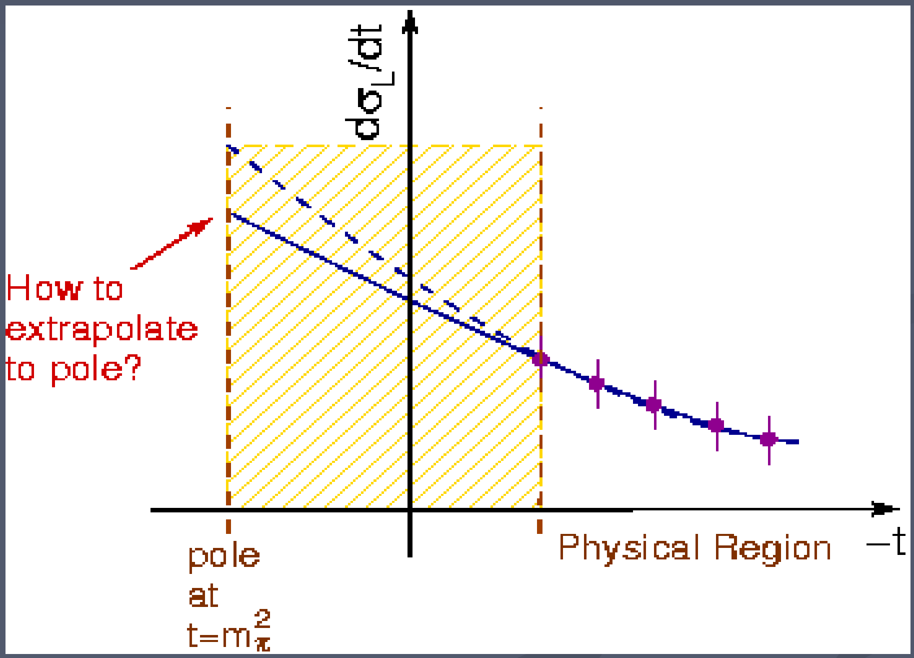


1. Need to take data at smallest available $-t$, so σ_L has maximum contribution from the π^+ pole.
   - Reduced model uncertainty in F_π extraction.
   - For given Q^2, higher W allows smaller $|t_{min}|$.
2. Extraction of F_π requires t dependence of σ_L to be known.
   - Only three of Q^2, W, t, θ_π are independent.
   - Vary θ_π to measure t dependence.
   - Since non-parallel data needed, LT and TT must also be determined.

Extraction of pion form factor from σ_L data

$p(e,e'\pi^+)n$ data are obtained some distance from the $t=m_\pi^2$ pole.

- "Chew Low" extrapolation method requires knowing the analytic dependence of $d\sigma_L/dt$ through the unphysical region.
  - Very large systematic uncertainties.
  - Reliable phenomenological extrapolation is not possible.
  - Extrapolation method not used since the early 1970's.



A more reliable approach is to use a model incorporating the π^+ production mechanism and the `spectator' nucleon to **extract** F_π from σ_L.

Electroproduction Method Check

"What is at best measured in electroproduction is the transition amplitude between a mesonic state with an effective space-like mass $m^2=t<0$ and the physical pion.

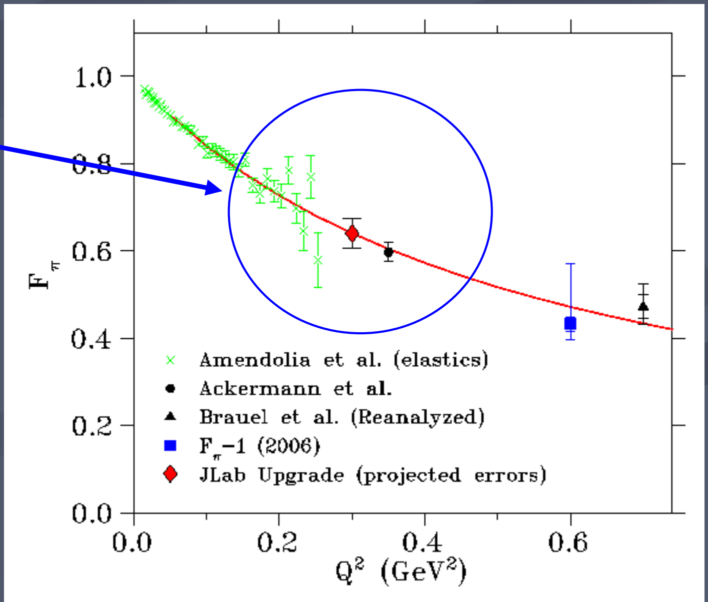
It is theoretically possible that the off-shell form factor $F_\pi(Q^2,t)$ is significantly larger than the physical form factor because of its bias towards more point-like $q\bar{q}$ valence configurations within its Fock state structure."

--S.J. Brodsky, Handbook of QCD, 2001.

Can test the electroproduction method by directly comparing $F_\pi(Q^2)$ values extracted from very low $-t$ electroproduction with the exact values measured in elastic e-π scattering.

METHOD PASSES CHECKS:

- Existing Q^2=0.35 GeV^2 data from DESY are consistent with the limit of the elastic scattering data within uncertainties.
  [H. Ackermann, et al., NP ***B137****(1978)294]*
- We intend to do a much better check by taking Q^2=0.30 GeV^2 data at 50% lower $-t$ (0.005 GeV^2) as part of our "JLab 12 GeV" experiment.

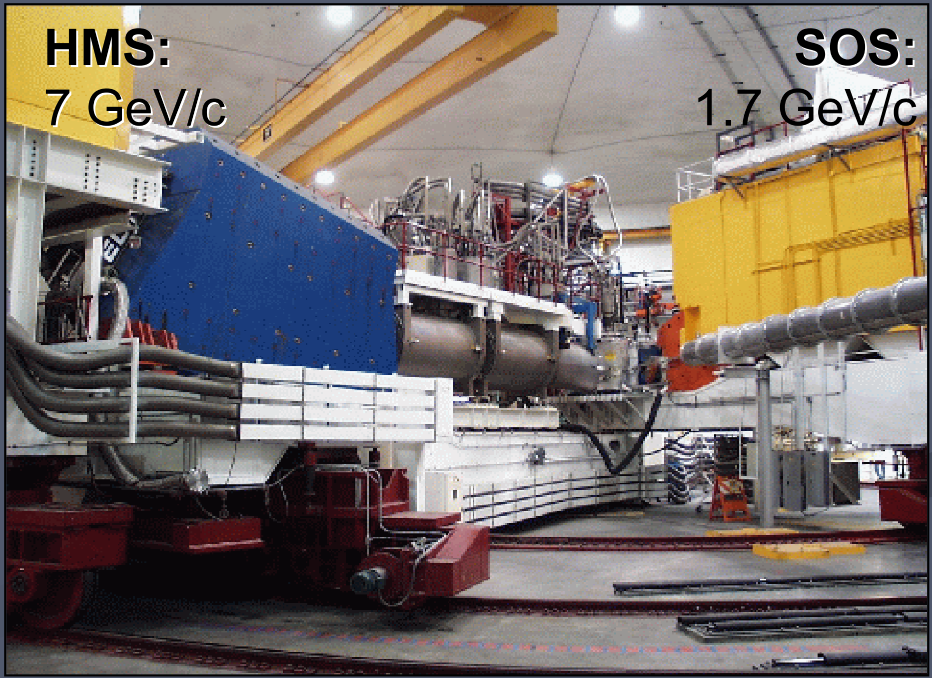


F_π-1 and F_π-2 Experiments at Jefferson Lab

F_π-2 Goals:

- Extension of our earlier F_π-1 to the highest Q^2 possible with JLab 6 GeV electron beam.
- Higher W above resonance region.
- Repeat Q^2=1.60 GeV^2 closer to $t=m_\pi^2$ pole.
  - reduced model uncertainties.

Expt	Q^2 (GeV^2)	W (GeV)	$\|t_{min}\|$ (Gev^2)	E_e (GeV)
F_π-1	0.6-1.6	1.95	0.03-0.150	2.445-4.045
F_π-2	1.6,2.45	2.22	0.093,0.189	3.779-5.246

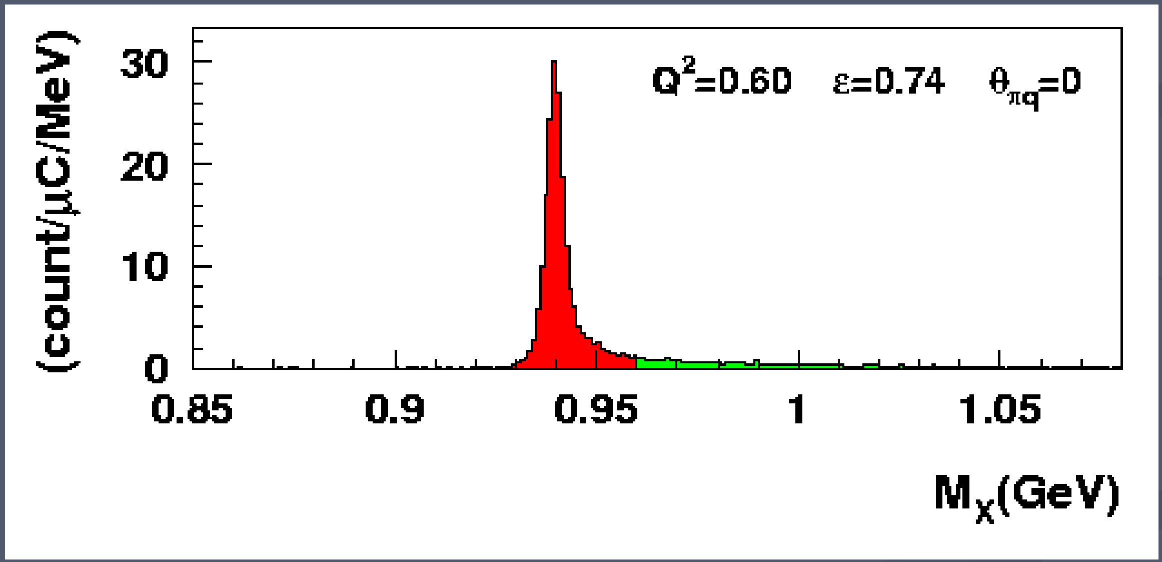


Experiment:

- Extract F_π via L/T/LT/TT Rosenbluth separation in $p(e,e'\pi^+)n$.
- Coincidence measurement between charged pions in HMS and electrons in SOS.
- Data acquired: F_π-1: 1997, F_π-2: 2003.

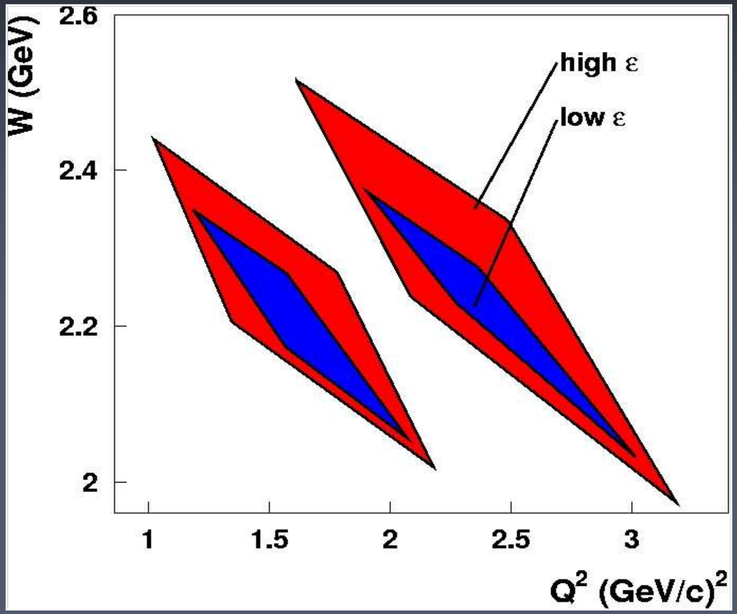
$p(e,e'\pi^+)n$ Event Selection

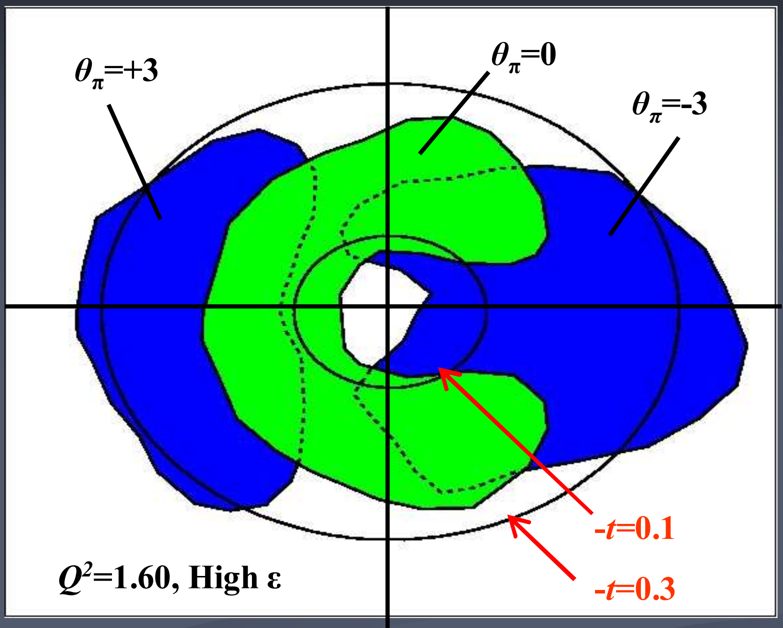
- **Coincidence measurement between charged pions in HMS and electrons in SOS.**
- π^+ detected in HMS – Aerogel Cerenkov and Coincidence time for PID.
- Electrons in SOS – identified by Cerenkov /Calorimeter.
- **After PID cuts, almost no random coincidences remain.**



Missing mass cut assures exclusivity.

F_π-2 Kinematic Coverage





Radial coordinate (-*t*) Azimuthal coordinate (φ).

- Overlapping data at high and low ε are required for L/T separation.
- Diamond cuts define common *(W,Q^2)* coverage at both ε.
- Measurements over $0<\varphi<2\pi$ are required to determine LT, TT contributions versus -*t*.
- HMS settings $\pm 3^o$ left and right of the *q*-vector are used to obtain good φ-coverage over a range of –*t*.
- Technique demands good knowledge of spectrometer acceptances.

Magnetic Spectrometer Calibrations

- **Over-constrained *p(e,e'p)* reaction and *e*+^{12}C reactions used to calibrate spectrometer acceptances, momenta, offsets, etc.**
  - **Beam energy and spectrometer momenta determined to <0.1%.**
  - **Spectrometer angles to ~0.5 mr.**
  - **Agreement with published *p+e* elastics cross sections <2%.**

- Per data *t*-bin (F_π-2):
  - Typical statistical error: **1-2%.**
  - Uncorrelated syst. unc. in σ_{UNS} common to all *t* bins: **1.8(1.9)%.**
  - Additional uncorrelated unc. also uncorrelated in *t*: **1.1(0.9)%.**
  - Total correlated uncertainty: **3.5%.**

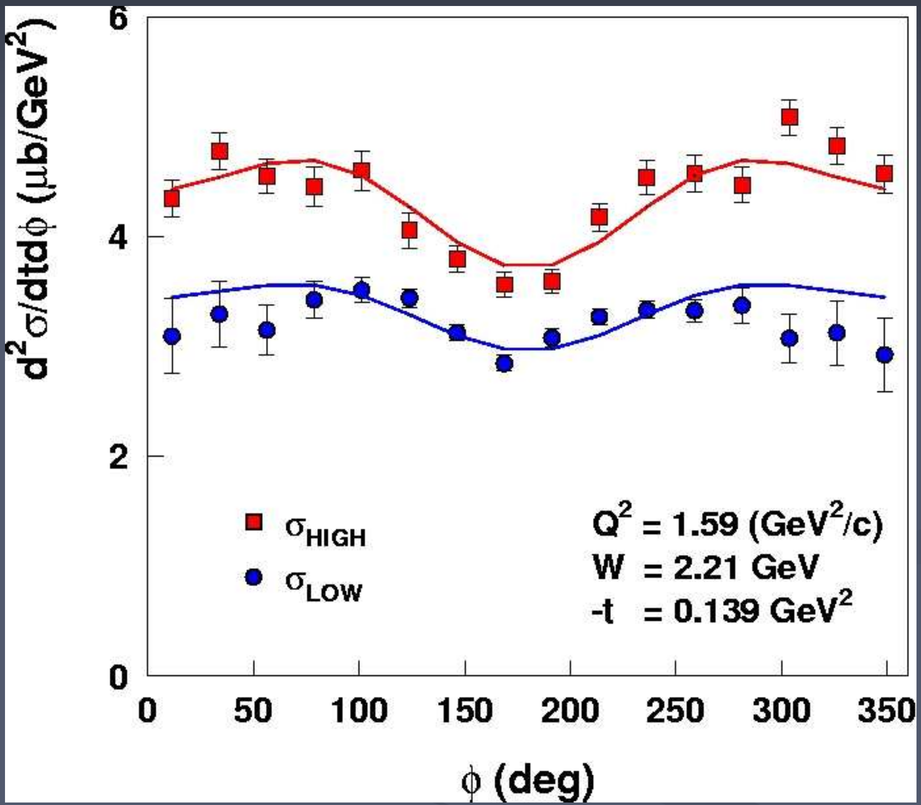
- Uncorrelated uncertainties in σ_{UNS} are amplified by $1/\Delta\varepsilon$ in L-T separation.
- Scale uncertainty propagates directly into separated cross section.

Experimental Cross Section Determination

- Use of Monte Carlo to replicate the physical acceptance for the channel studied:
  - *p(e,e'π⁺)n* model based on pion electroproduction data.
  - Spectrometer optics (COSY model).
  - Radiative effects, pion decay, energy loss, multiple scattering.

$$\left(\frac{d\sigma(\overline{W},\overline{Q}^2,t,\phi)}{dt}\right)_{\exp} = \frac{\langle Y_{\exp}\rangle}{\langle Y_{MC}\rangle}\left(\frac{d\sigma(\overline{W},\overline{Q}^2,t,\varphi)}{dt}\right)_{MC}$$

F_π-2 data: T. Horn *et al.*, PRL **97**(2006)192001.



Only Statistical Uncertainties Shown.

- Extract σ_L by simultaneous fit to the measured yields versus azimuthal angle (φ_π) and virtual photon polarization parameter (ε).

$$2\pi\frac{d^2\sigma}{dtd\phi} = \varepsilon\frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon(\varepsilon+1)}\frac{d\sigma_{LT}}{dt}\cos\phi + \varepsilon\frac{d\sigma_{TT}}{dt}\cos 2\phi$$

After σ_L is determined, a model is required to extract $F_\pi(Q^2)$

Model incorporates π^+ production mechanism and spectator neutron effects:

- **The experimentalist would like to use a variety of models to extract $F_\pi(Q^2)$ from the electroproduction data, so that the model dependence can be better understood.**
  - The Vanderhaeghen-Guidal-Laget (VGL) Regge model is the only reliable model available for our use at present.
  - It would be useful to have additional models for the pion form factor extraction.
- **Our philosophy remains to publish our experimentally measured $d\sigma_L/dt$, so that updated values of $F_\pi(Q^2)$ could be extracted in the future.**

The experimental $F_\pi(Q^2)$ result is not permanently "locked in" to a specific model.

Extraction of $F_\pi(Q^2)$ from the F_π-2 σ_L data

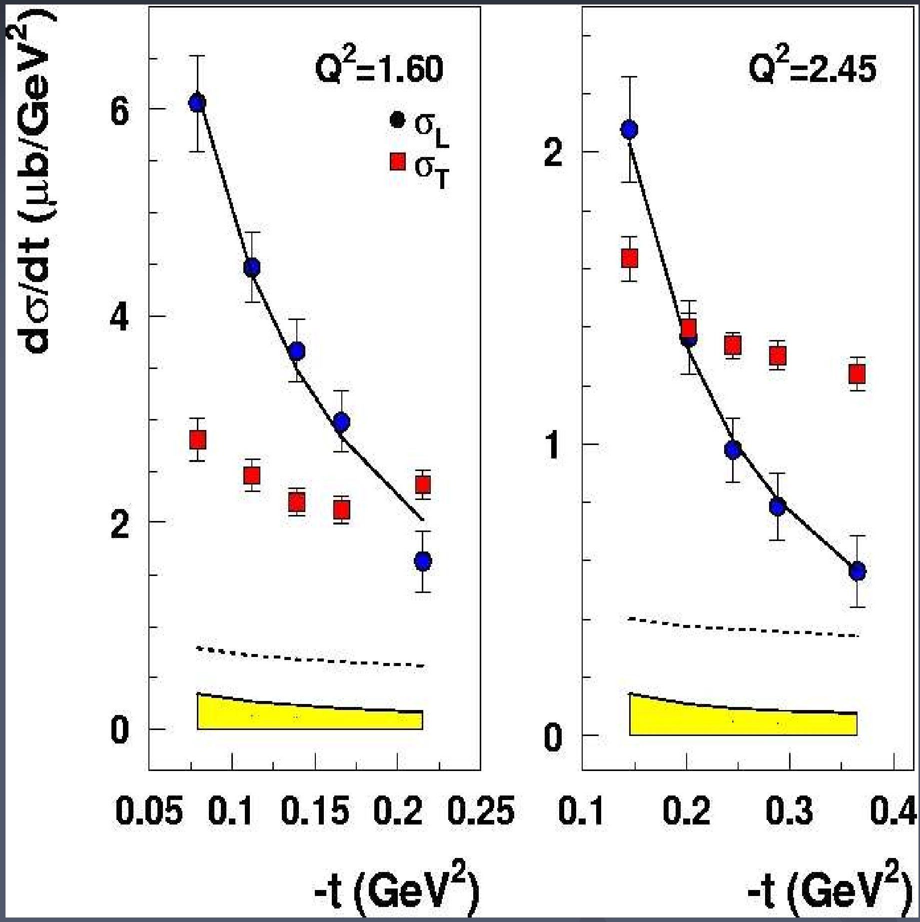
VGL Regge Model:

- Feynman propagator $\left(\frac{1}{t - m_\pi^2}\right)$ replaced by π and ρ Regge propagators.
  - Represents the exchange of a series of particles, compared to a single particle.
- Model parameters fixed from pion photoproduction.
- Free parameters: Λ_π, Λ_ρ (trajectory cutoff).

[Vanderhaeghen, Guidal, Laget, PRC **57**(1998)1454]

$$F_\pi = \frac{1}{1 + Q^2/\Lambda_\pi^2}$$

Fit of model to $\sigma_L(t)$ data gives F_π at each Q^2.



F_π-2 data: T. Horn *et al.*, PRL **97**(2006)192001.

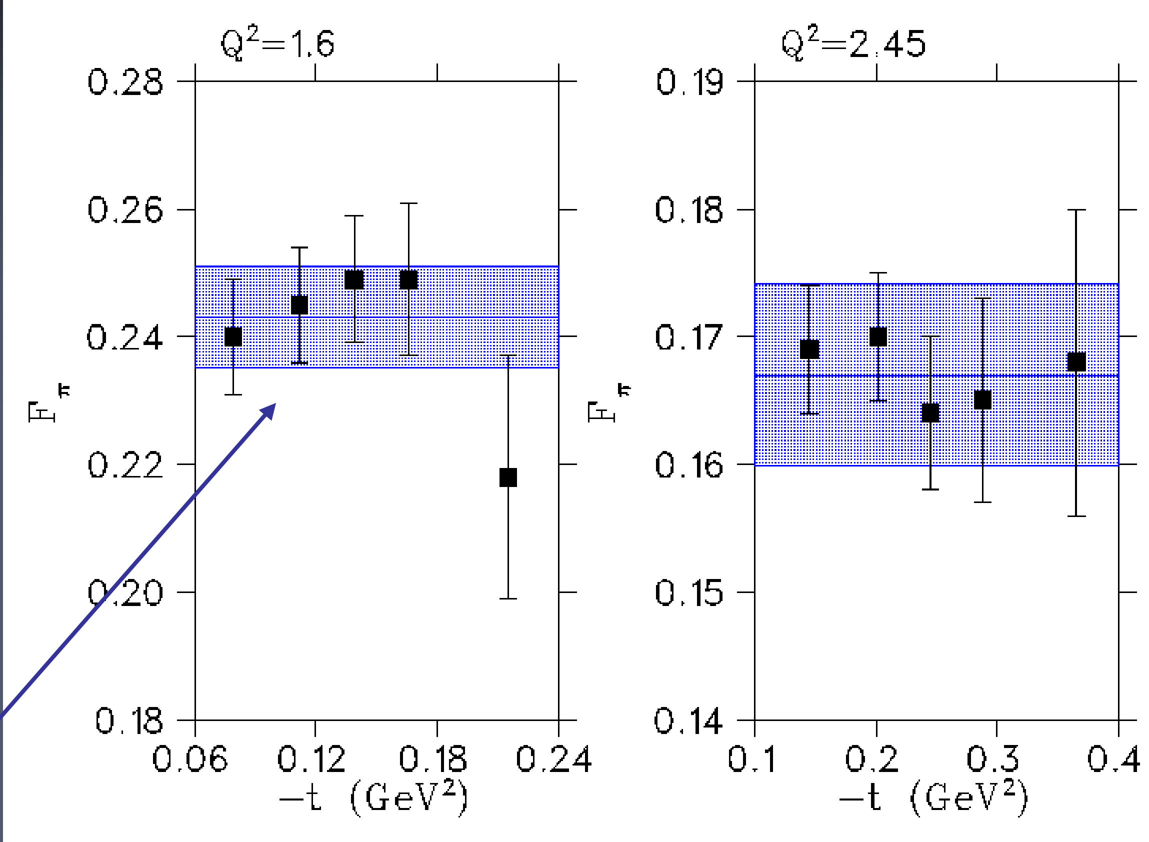
Error bars indicate statistical and random (pt-pt) systematic uncertainties in quadrature.
Yellow band indicates the correlated (scale) and partly correlated (t-corr) systematic uncertainties.

Λ_π^2=0.513, 0.491 GeV², Λ_ρ^2=1.7 GeV².

F_π-2 $p(e,e'\pi^+)n$ model check

- To check whether the VGL Regge model properly accounts for:
  - π^+ production mechanism.
  - spectator nucleon.
  - other off-shell (*t*-dependent) effects.

  extract F_π values for each *t*-bin separately, instead of one value from fit to 5 *t*-bins.



Published F_π error band based on fit to all *t*-bins.

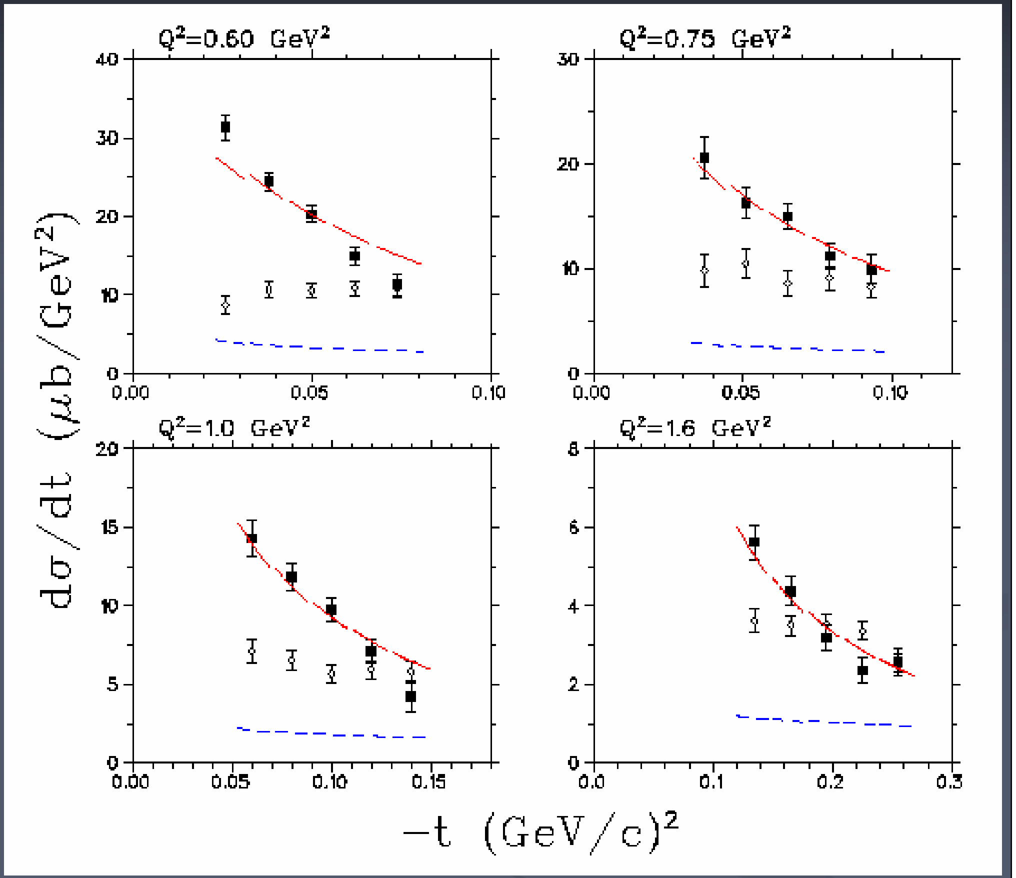
Only Statistical Uncertainties Shown.

- Deficiencies in the model may show up as a *t*-dependence in the extracted $F_\pi(Q^2)$ values.
- The resulting F_π values are insensitive (<2%) to the *t*-bin used.
- Lends confidence in the applicability of the VGL model to the kinematical regime of the F_π-2 data, and the validity of the extracted $F_\pi(Q^2)$ values.

Fitting the VGL model to the F_π-1 data

F_π-1 data: V. Tadevosyan *et al.*, nucl-ex/0607007.

- F_π-1 data were acquired in 1997, when maximum beam energy available was 4 GeV.
- Experimental data constrained to W≈1.95 GeV.
- σ_L: t-dependence of VGL model is significantly flatter than the data.
- σ_T: model strongly underestimates data for any value of Λ_ρ^2 used.



Error bars indicate statistical and random (pt-pt) systematic uncertainties added in quadrature.
In addition, there is an overall systematic uncertainty of ~6%, mainly from the t-correlated, ε-uncorrelated systematic uncertainty.

Λ_π^2=0.393, 0.373, 0.412, 0.458 GeV^2

Λ_ρ^2=1.5 GeV^2.

huberg@uregina.ca

Possible effect of resonances at W≈1.95 GeV

The deficiencies in the description of the F_π-1 data by the VGL model may be due to contributions from resonances

→ No such terms are included in the Regge model.

→ Could enhance the strength in σ_T and contribute to σ_L.

- **The discrepancy is strongest at the lowest Q^2.**
- **At higher Q^2 the resonance form factor is expected to reduce resonance contributions.**

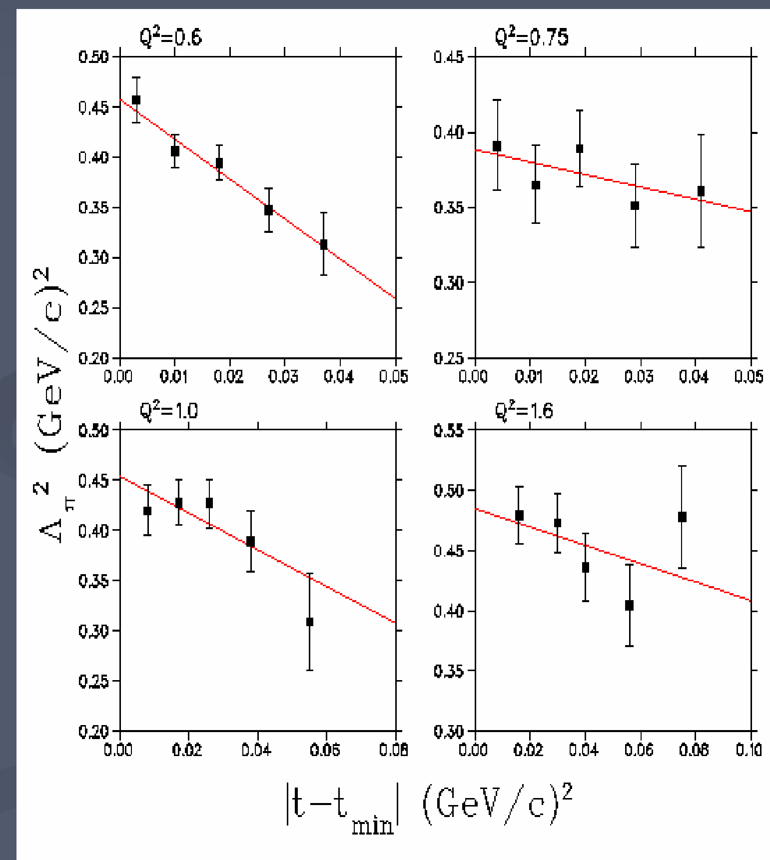
Our analysis assumes that the contribution of the `missing background' is small at the kinematic endpoint *tmin*.

→ Fit VGL model to each *t*-bin separately, yielding $\Lambda_\pi^2(Q^2,t)$.

→ Λ_π^2 decreases with $-t$, presumably due to background not included in the model.

→ Linear fit of Λ_π^2 to t_{min} yields `best estimate' of F_π at each Q^2.

→ We assign an additional `model uncertainty' to account for the possibility that this assumption may be inappropriate.

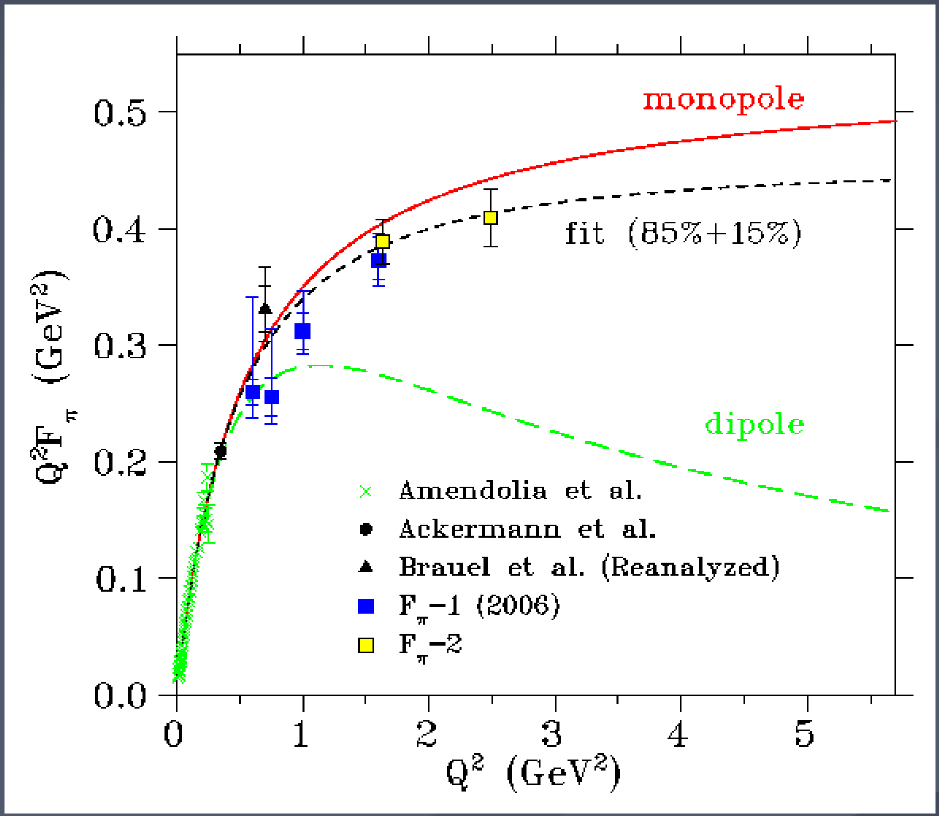


F_π-1 data: V. Tadevosyan *et al.*, nucl-ex/0607007.

Jefferson Lab Experimental Results

- ***Q*²=1.60 GeV² point:** to check model dependence of form factor extraction.
  - F_π-1 (W=1.95 GeV) & F_π-2 (W=2.22 GeV) agree to ~4%, indicating reliability of analyses.
  - F_π-2 point is 30% closer to pion pole, with significantly reduced uncertainties.
- ***Q*²=2.45 GeV² point:** deviates from 0.657fm charge radius curve by ~1σ.
  - Monopole curve reflects soft (VMD) physics at low Q^2.
  - A significant deviation would indicate the increased role of π^+ hard wave function components at moderate Q^2.

F_π-1: nucl-ex/0607007.
F_π-2: PRL **97**(2006)192001.

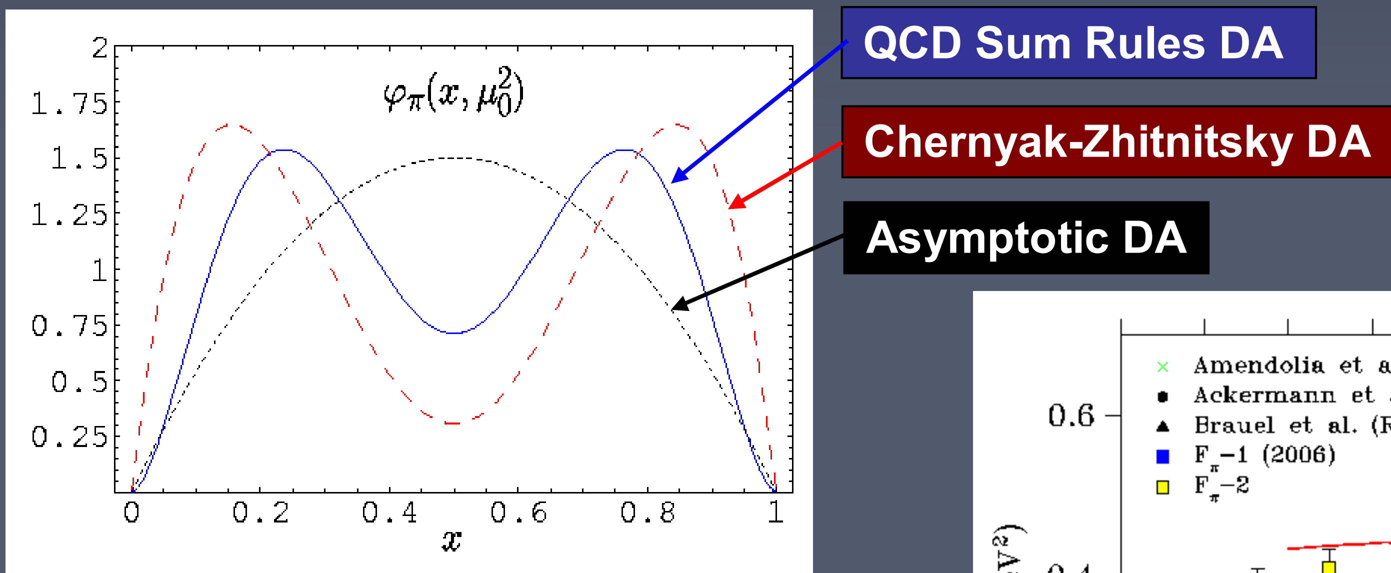


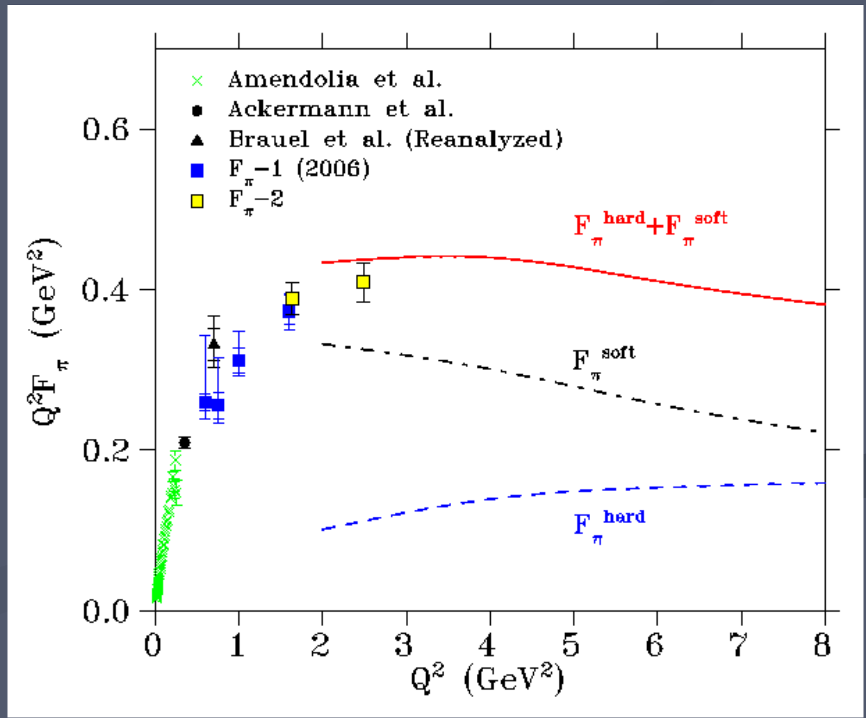
S.R. Amendolia, et al., Nucl. Phys. **B277** (1986) 168.
H. Ackermann, et al, Nucl. Phys. **B137** (1978) 294.
P. Brauel, et al., Z. Phys. **C3** (1979) 101.

pQCD LO+NLO Calculation

A.P. Bakulev, K. Passek-Kumericki, W. Schroers, & N.G. Stefanis, PRD **70** (2004) 033014.

1. Pion distribution amplitude consistent to 1σ level with CLEO πγ transition data.
2. Analytic perturbation theory at the parton amplitude level.

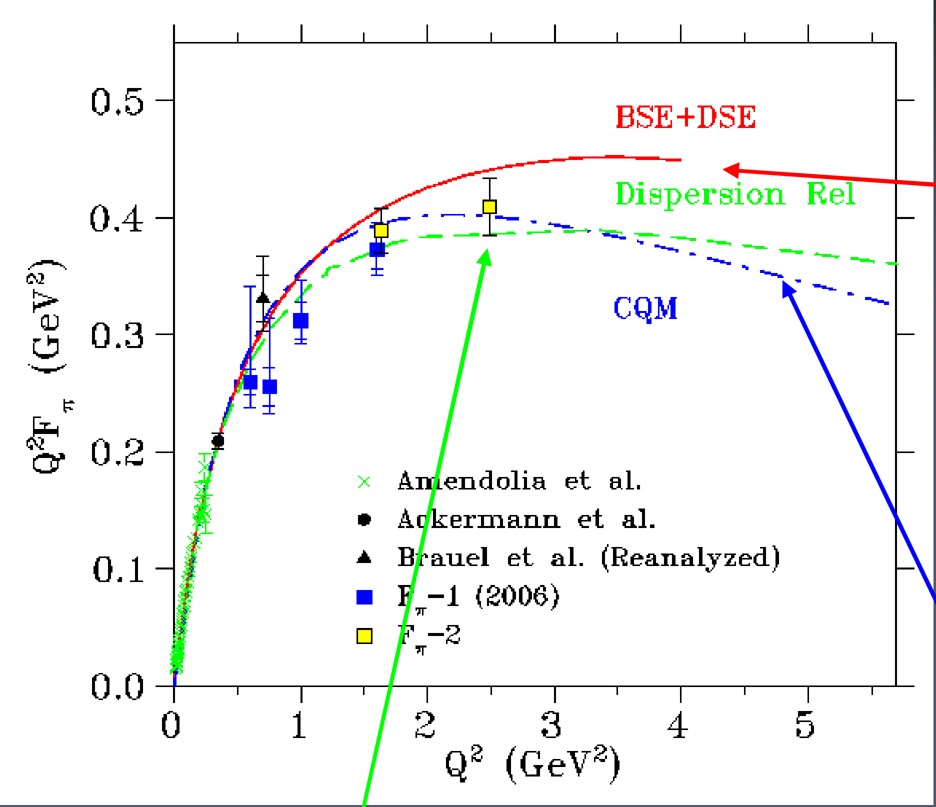




Hard component is only slightly larger than that calculated with asymptotic DA in all considered schemes.

→ Soft contribution from local quark-hadron duality model needed to describe data.

Comparison with QCD-based Models



Bethe-Salpeter/Dyson-Schwinger

[P. Maris and P. Tandy, Phys.Rev.C 62(2000)055204]

- B-S equation is conventional formalism for relativistic bound states.
- D-S expansion in terms of dressed quark propagators, consistent w/ confinement.
- Model parameters fixed from f_π and m_π, then r_π and F_π predicted.

Constituent Quark Model

[C-W. Hwang, Phys.Rev.D 64(2001)034001]

- Relativistic constituent quarks and effective interaction on the light front
- Consistent treatment of quark spins.
- Wave function parameters determined from f_π and $\pi^0 \rightarrow \gamma\gamma$ decay width, then charge and transition FF's and π^0 branching ratios predicted.

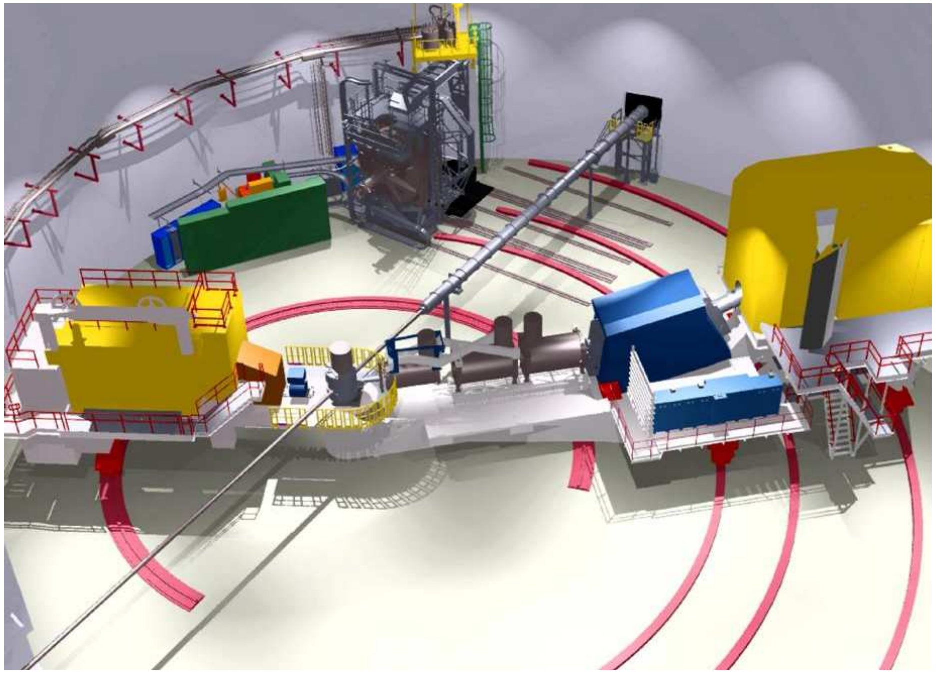
Disperson Relation with QCD Constraint:

[B.V. Geshkenbein, Phys.Rev.D 61(2000)033009]

- Uses constraints posed by **causality** and **analyticity** to relate the timelike and spacelike domains of the pion form factor on the complex plane.
- Additional constraints, such as behavior of F_π in asymptotic region, imposed.

Experimental Hall C

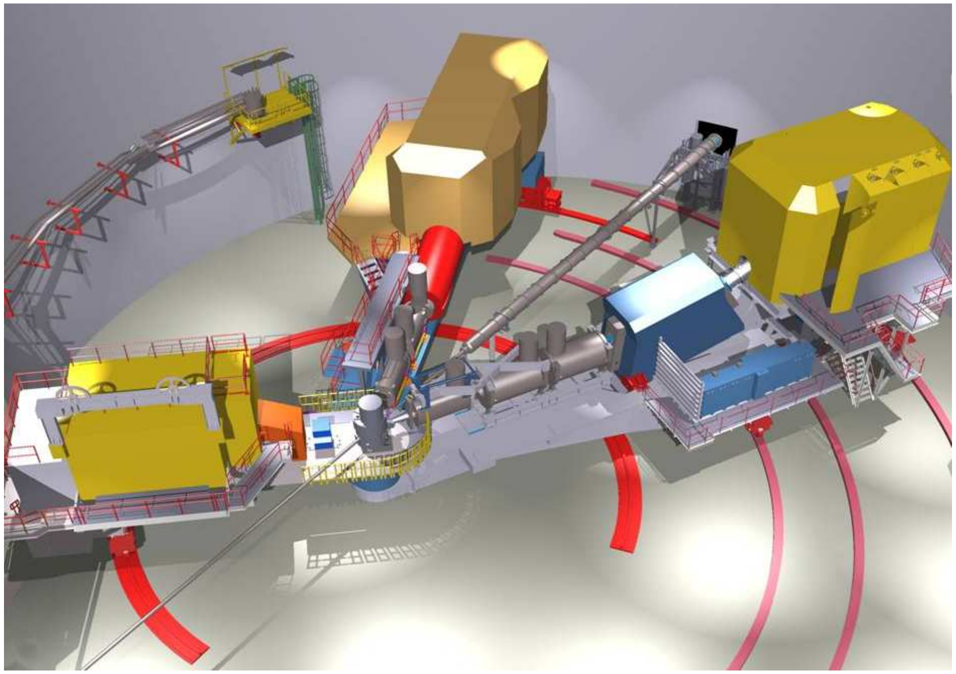
At the present 6 GeV Beam Energy



Hall C's High Momentum Spectrometer, Short Orbit Spectrometer and specialized equipment for studying:

- The strange quark content of the proton.
- Form factors of simple quark systems.
- The transition from hadrons to quarks.
- Nuclei with a strange quark embedded.

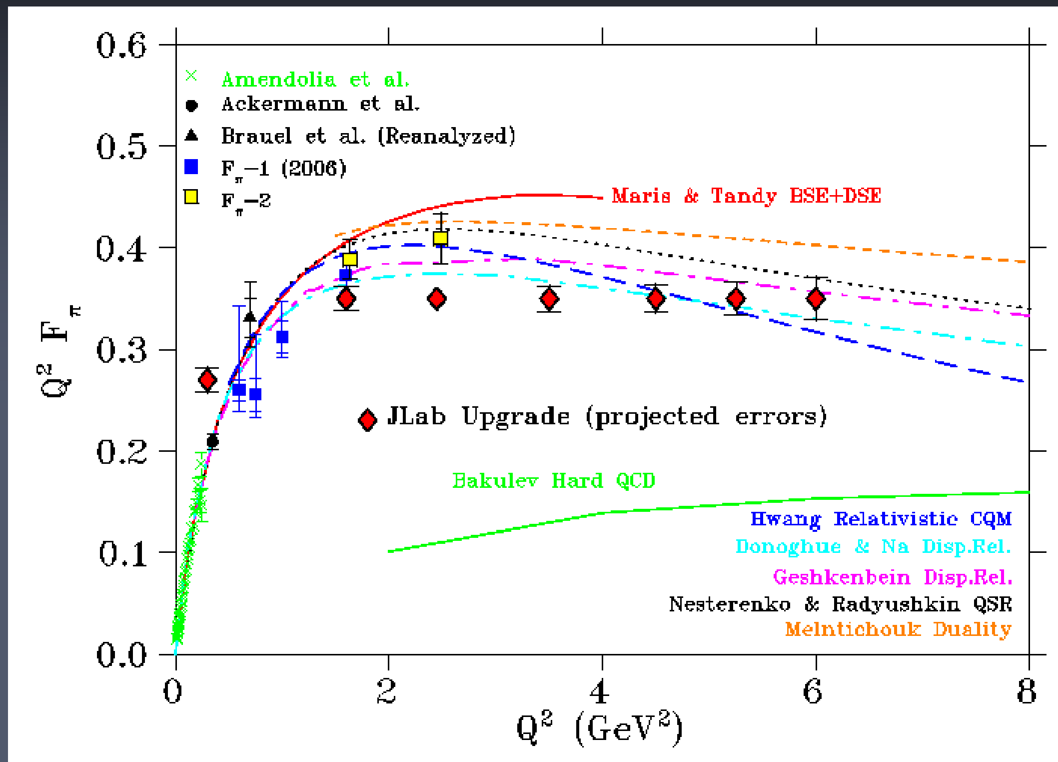
After the 12 GeV Upgrade



Add a Super- High Momentum (12 GeV) Spectrometer for studying:

- Super-fast (high x_B) quarks.
- Form factors of simple quark systems.
- The transformation of quarks into hadrons.
- Quark-quark correlations.

Proposed F_π Measurements with JLab Upgrade



JLab Proposal 12-06-101

11 GeV electron beam and SHMS spectrometer with θ=5.5° capability will allow:

- Best test to date of electroproduction method at Q^2=0.30 GeV^2.
  - → compare F_π from very low $-t$ electroproduction to upper limit of exact elastic data.
- Stringent test of model-dependence in F_π extraction.
  - → New $Q2$=1.60, 2.45 GeV2 data at W≈3 GeV to compare with same at W=2.2 GeV.
- Precision data up to Q^2=6 GeV^2 to study the transition to hard QCD.

Summary

- **F_π a good observable to study transition from soft to hard QCD.**
- **Ambitious experimental program at Jefferson Lab now yielding high quality data.**

  This success is due in large part to:
  - Continuous electron beam provided by the JLab superconducting linac.
  - Magnetic spectrometers and detectors with well-understood properties.
  - The hard work of many people.
- **Highest Q^2 JLab results show Q^2F_π still increasing, but ~1σ below monopole parameterization of charge radius.**
  - Still far from pQCD prediction.
- **Studies of F_π at higher electron beam energy will allow us to reach the kinematic range where hard contributions are larger.**
  - Planned measurement at JLab after upgrade to Q^2=6 GeV^2.
  - Planned tests of the F_π extraction method from electroproduction data also an important part of the program.
- **Look forward to continued development of QCD techniques for the non-perturbative light quark sector.**