

Recent experimental results for the π^+ electric form factor from Jefferson Lab

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Jefferson Lab F_π Collaboration

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Scientific Motivation

- The pion form factor is a topic of fundamental importance to our understanding of hadronic structure.
- Pions are the lightest QCD system ($q\bar{q}$).
 - all hadronic structure models use the π^+ as a test case.
 - “the positronium atom of QCD”.

At large Q^2 , F_π is presumably given by pQCD

$$F_\pi(Q^2) = \frac{4\pi C_F \alpha_s(Q^2)}{Q^2} \left| \sum_{n=0}^{\infty} a_n \left(\log \left(\frac{Q^2}{\Lambda^2} \right) \right)^{-\gamma_n} \right|^2 \left[1 + O(\alpha_s(Q^2), m/Q) \right]$$

which in the $Q^2 \rightarrow \infty$ limit becomes

$$F_\pi(Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{16\pi\alpha_s(Q^2)f_\pi^2}{Q^2}$$

where $f_\pi^2 = 93 \text{ MeV}$ is the $\pi^+ \rightarrow \mu^+ \nu$ decay constant.

F_π is the clearest test case for study of transition between non-perturbative and pQCD regions.

- What is the structure of the π^+ at all Q^2 ?
 - at what value of Q^2 will the pQCD contributions dominate?
- A difficult question to answer, as both “hard” and “soft” components (such as gluonic effects) must be taken into account.
 - non-perturbative hard components of higher twist strongly cancel soft components, even at modest Q^2 .
[Braun et al., PRD 61(2000)073004]
 - the situation for nucleon form factors is even more complicated.
- Many model calculations exist, but ultimately...
 - Reliable $F_\pi(Q^2)$ data are needed to delineate the role of hard versus soft contributions at intermediate Q^2 .
- A program of study unique to Jefferson Lab.

Determination of F_π via Pion Electroproduction

At low $Q^2 < 0.3 \text{ GeV}^2$, the π^+ form factor can be measured exactly using high energy π^+ scattering from atomic electrons.

\Rightarrow 300 GeV pions at CERN SPS. [Amendolia et al., NP B277(1986)168]

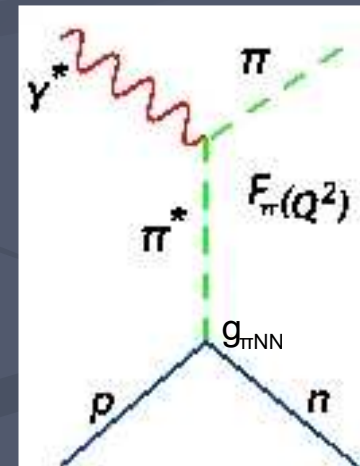
\Rightarrow Provides an accurate measure of the π^+ charge radius.

$$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

To access higher Q^2 , one must employ the $p(e, e' \pi^+)n$ reaction.

- t -channel process dominates σ_L at small $-t$.
- In the Born term model:

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t - m_\pi^2)} g_{\pi NN}^2(t) F_\pi^2(Q^2, t)$$

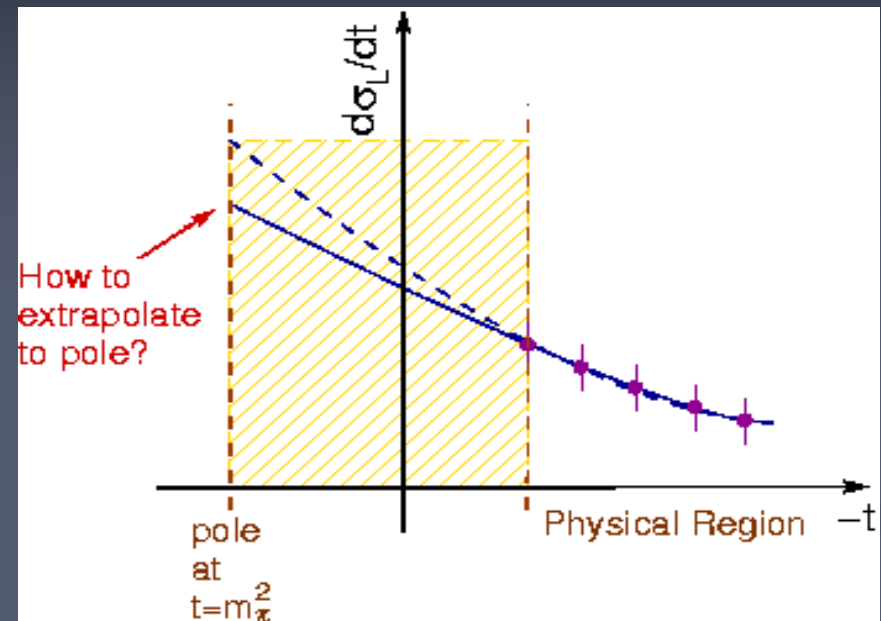


Extraction of form factor from σ_L data

$p(e, e' \pi^+) n$ data are obtained some distance from the $t=m_\pi^2$ pole.

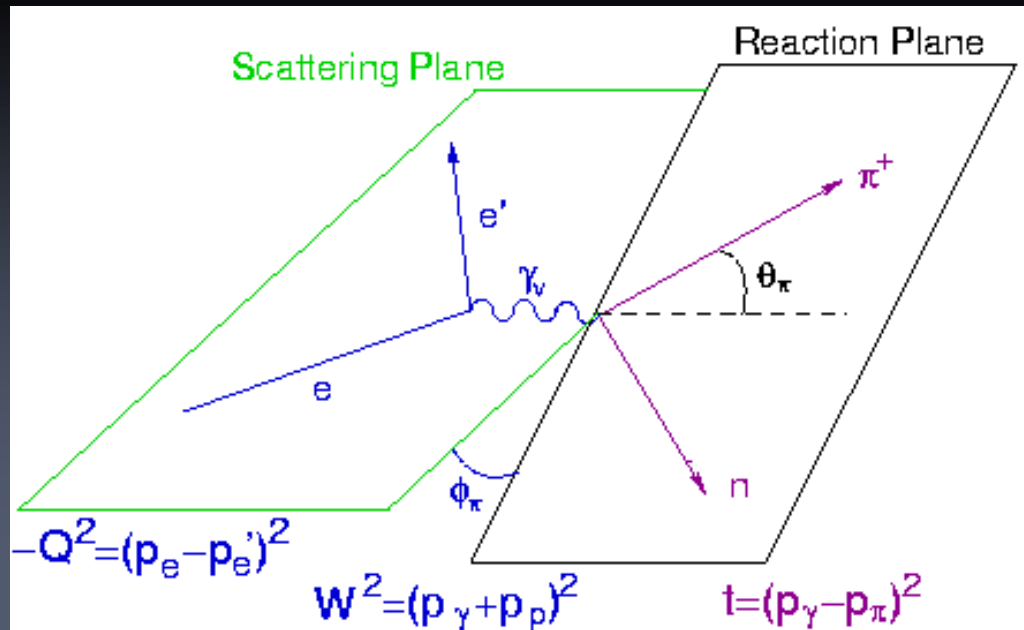
- No reliable phenomenological extrapolation possible.

A more reliable approach is to use a model incorporating the π^+ production mechanism and the 'spectator' nucleon to extract F_π from σ_L .



Method check:

- It would be of great value to verify that $F_\pi(Q^2)$ values extracted from electroproduction data are in good agreement with those determined from $e-\pi$ scattering data
- We have proposed a more stringent version of this test as part of the "JLab 12 GeV" experimental program.



$$2\pi \frac{d^2\sigma}{dt d\phi} = \varepsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon |\varepsilon + 1|} \frac{d\sigma_{LT}}{dt} \cos \phi + \varepsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi$$

1. Need to take data at smallest available $-t$, so σ_L has maximum contribution from the π^+ pole.
 - For given Q^2 , higher W allows smaller $|t_{min}|$.
 - reduced model uncertainty in F_π extraction.
2. Extraction of F_π requires t dependence of σ_L to be known.
 - Only three of Q^2 , W , t , θ_π are independent.
 - Vary θ_π to measure t dependence.
 - Since non-parallel data needed, LT and TT must also be determined.

$F_{\pi-1}$ and $F_{\pi-2}$ Experiments at JLab

$F_{\pi-2}$ Goals:

- Extension of our earlier $F_{\pi-1}$ to the highest Q^2 possible with JLab 6 GeV electron beam.
- Higher W above resonance region.
- Repeat $Q^2=1.60 \text{ GeV}^2$ closer to $t=m_{\pi}$ pole.
 - reduced model uncertainties.

Exp	Q^2 (GeV^2)	W (GeV)	$ t_{\min} $ (GeV^2)	E_e (GeV)
$F_{\pi-1}$	0.6-1.6	1.95	0.03-0.150	2.445-4.045
$F_{\pi-2}$	1.6,2.45	2.22	0.093,0.189	3.779-5.246

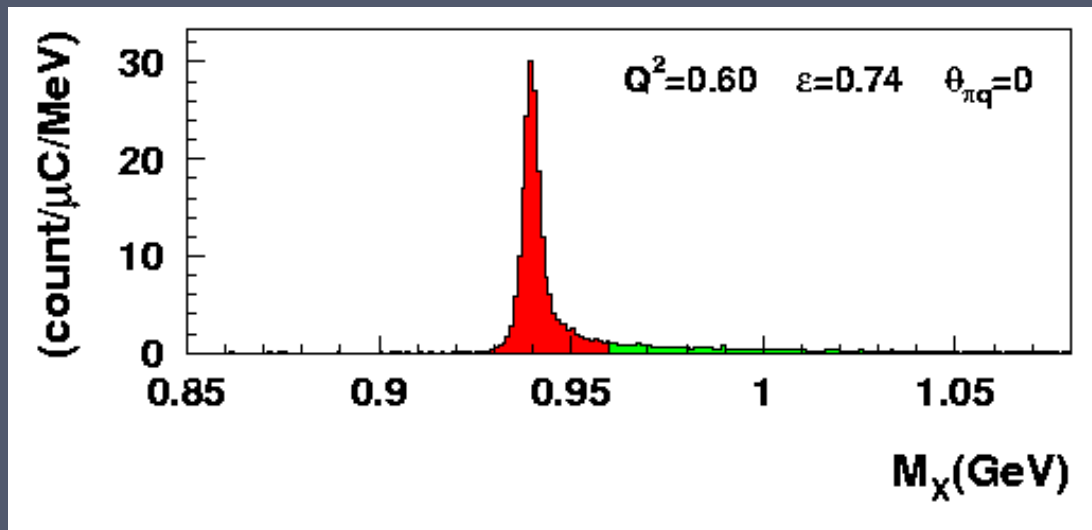
Experiment:

- Extract F_{π} via L/T/LT/TT Rosenbluth separation in $p(e,e'\pi^+)n$.
- Coincidence measurement between charged pions in HMS and electrons in SOS.
- Data acquired: $F_{\pi-1}$: 1997, $F_{\pi-2}$: 2003.



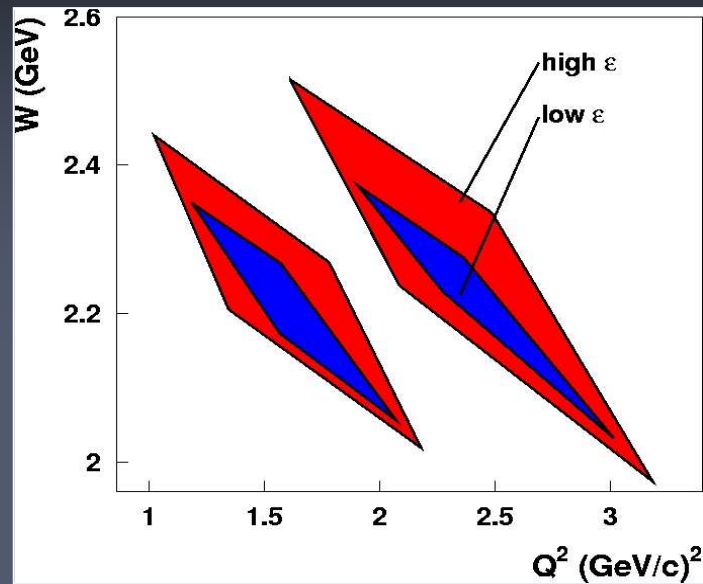
$p(e,e'\pi^+)n$ Event Selection

- Coincidence measurement between charged pions in HMS and electrons in SOS.
- π^+ detected in HMS – Aerogel Cerenkov and Coincidence time for PID.
- Electrons in SOS – identified by Cerenkov /Calorimeter.
- After PID cuts, almost no random coincidences remain.

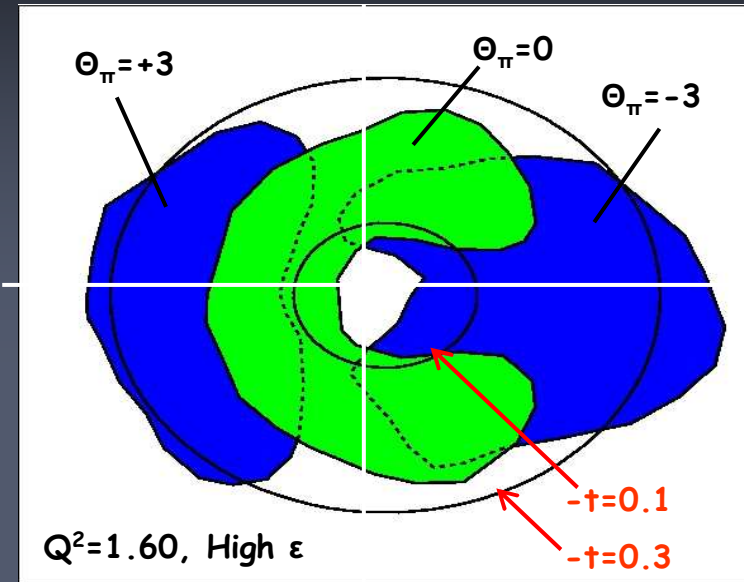


Missing mass cut
assures exclusivity.

$F_{\pi-2}$ Kinematic Coverage



- Overlapping data at high and low ϵ are required for L/T separation.
- Diamond cuts define common (W, Q^2) coverage at both ϵ .



Radial coordinate $(-t)$ Azimuthal coordinate (φ) .

- Measurements over $0 < \varphi < 2\pi$ are required to determine LT, TT contributions versus $-t$.
- HMS settings $\pm 3^\circ$ left and right of the q -vector are used to obtain good φ -coverage over a range of $-t$.
- Technique demands good knowledge of spectrometer acceptances.

Magnetic Spectrometer Calibrations

- Over-constrained $p(e, e'p)$ reaction and $e+^{12}\text{C}$ reactions used to calibrate spectrometer acceptances, momenta, offsets, etc.
 - Beam energy and spectrometer momenta determined to $<0.1\%$.
 - Spectrometer angles to ~ 0.5 mr.
 - Agreement with published $p+e$ elastics cross sections $<2\%$.
- Per data t -bin ($F_{\pi-2}$):
 - Typical statistical error: 1-2%.
 - Uncorrelated syst. unc. in σ_{UNS} common to all t bins: 1.8(1.9)%.
 - Additional uncorrelated unc. also uncorrelated in t : 1.1(0.9)%.
 - Total correlated uncertainty: 3.5%.
- Uncorrelated uncertainties in σ_{UNS} are amplified by $1/\Delta\varepsilon$ in L-T separation.
- Scale uncertainty propagates directly into separated cross section.

Experimental Cross Section Determination

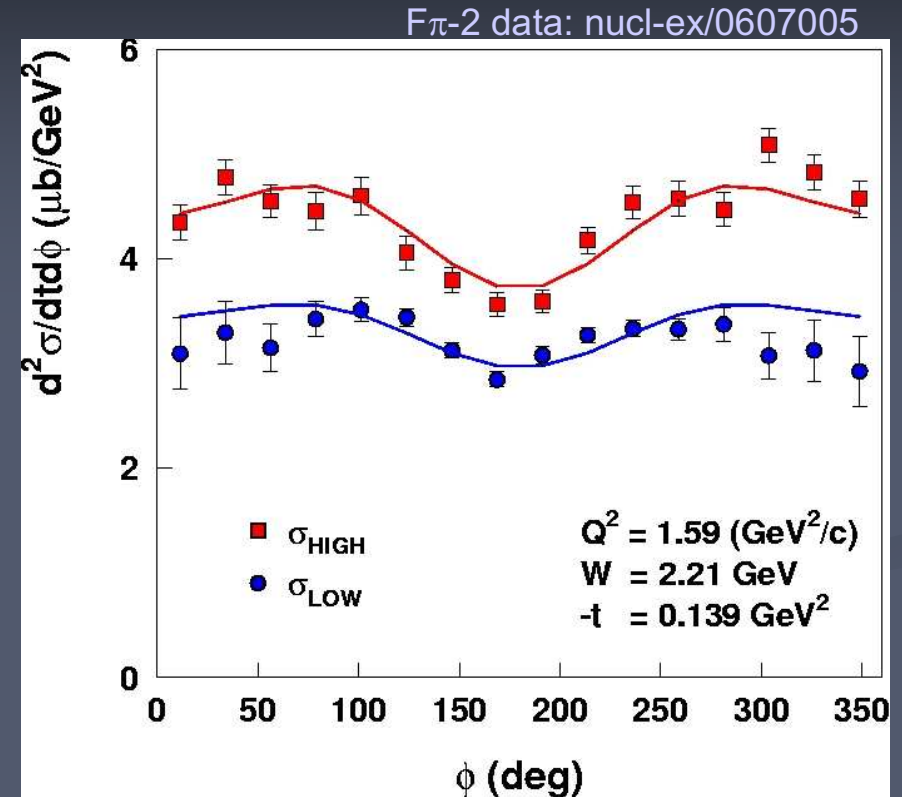
- Compare experimental yields to Monte Carlo of the experiment:

- $p(e, e'\pi^+)n$ model based on pion electroproduction data.
- Radiative effects, pion decay, energy loss, multiple scattering
- COSY model for spectrometer optics.

$$\left(\frac{d\sigma(\bar{W}, \bar{Q}^2, t, \phi)}{dt} \right)_{\text{exp}} = \frac{\langle Y_{\text{exp}} \rangle}{\langle Y_{\text{MC}} \rangle} \left(\frac{d\sigma(\bar{W}, \bar{Q}^2, t, \phi)}{dt} \right)_{\text{MC}}$$

- Extract σ_L by simultaneous fit using measured azimuthal angle (ϕ_π) and knowledge of photon polarization (ϵ).

$$2\pi \frac{d^2\sigma}{dt d\phi} = \epsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\epsilon |\epsilon + 1|} \frac{d\sigma_{LT}}{dt} \cos \phi + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi$$



Only Statistical Uncertainties Shown.

After σ_L is determined, a model is required to extract $F_\pi(Q^2)$

Model incorporates π^+ production mechanism and spectator neutron effects:

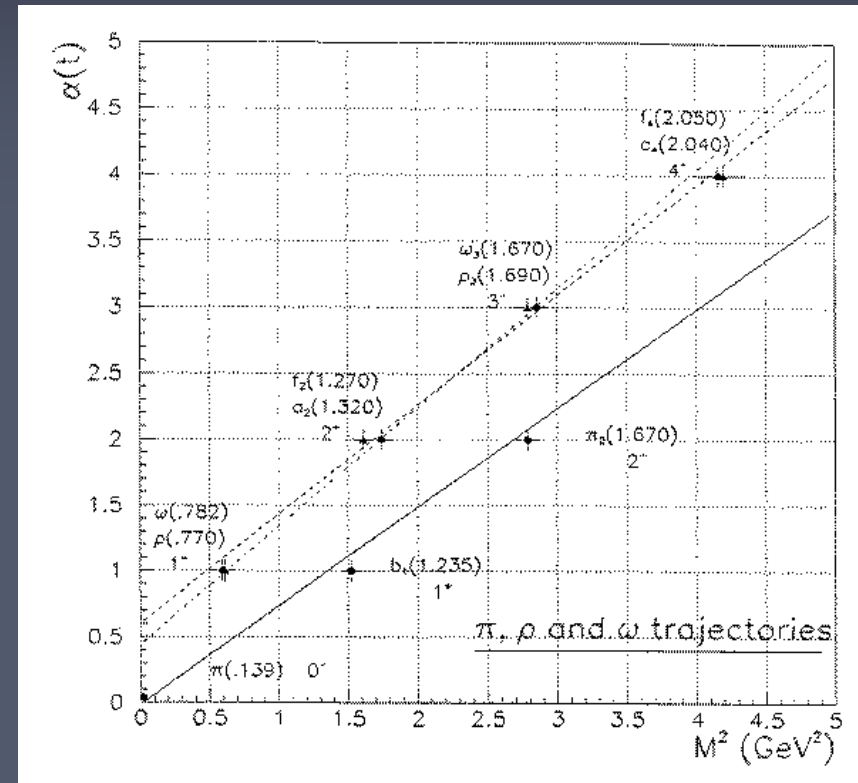
- The experimentalist would like to use a variety of models to extract $F_\pi(Q^2)$ from the electroproduction data, so that the model dependence can be better understood.
 - The VGL model is the only reliable model available for our use at present.
 - It would be useful to have additional models for the form factor extraction.
- Our philosophy remains to publish our experimentally measured $d\sigma_L/dt$, so that updated values of $F_\pi(Q^2)$ could be extracted in the future.

The experimental $F_\pi(Q^2)$ result is not permanently “locked in” to a specific model.

$F_{\pi-1}$ and $F_{\pi-2}$ used the VGL Regge Model to extract $F_{\pi}(Q^2)$ from the σ_L data

- Feynman propagator $\left(\frac{1}{t - m_{\pi}^2} \right)$ replaced by π and ρ Regge propagators.
 - Represents the exchange of a series of particles, compared to a single particle.
- Model parameters fixed from pion photoproduction.
- Free parameters: Λ_{π} , Λ_{ρ} (trajectory cutoffs).
- ρ exchange does not significantly influence σ_L at small $-t$.
- Pion form factor is a free parameter in the model, parameterized as:

$$F_{\pi} = \frac{1}{1 + Q^2 / \Lambda_{\pi}^2}$$



[Vanderhaeghen, Guidal, Laget, PRC 57(1998)1454]

Extraction of $F_\pi(Q^2)$ from the F_π -2 σ_L data

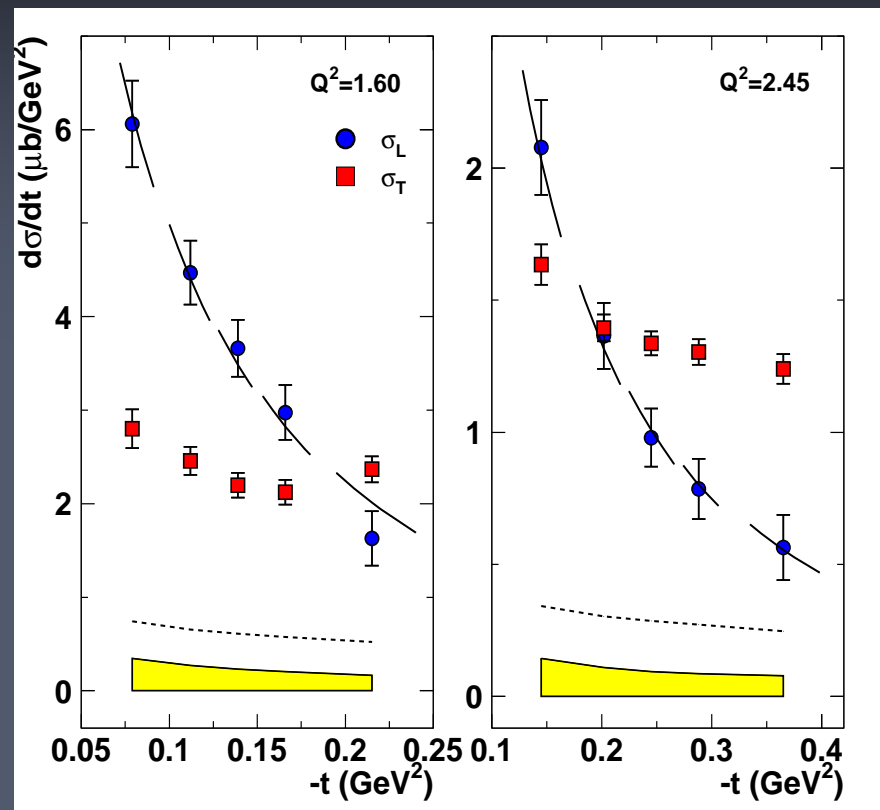
Q^2	W	$-t_{\min}$
1.60	2.22	0.079
2.45	2.22	0.145

- The VGL model fits the t -dependence of the σ_L cross sections acceptably well.
- Single parameter fit of VGL model to data yields F_π values at each Q^2 .

$$F_\pi = \frac{1}{1 + Q^2 / \Lambda_\pi^2}$$

Fit of model to $\sigma_L(t)$ data gives F_π at each Q^2 .

- Resulting F_π values are insensitive ($\sim 1\%$) to the t -bin used in the fit.



Error bars indicate statistical and random (pt-pt) systematic uncertainties added in quadrature. Yellow band indicates the correlated (scale) and partly correlated (t-corr) systematic uncertainties.

$$\Lambda_\pi^2 = 0.513, 0.491 \text{ GeV}^2, \Lambda_\rho^2 = 1.7 \text{ GeV}^2.$$

Results from the F_{π} -1 Experiment

- The F_{π} -1 experiment acquired data in 1997, and first results were published in
J. Volmer et al., PRL **86**(2001)1713.
- We have recently completed the reanalysis of these data with careful inspection of all steps, and special attention to systematic uncertainties.
- The method used to extract the form factor from the measured σ_L was also critically reconsidered.

“The data on the pion form factor seem to be evolving.

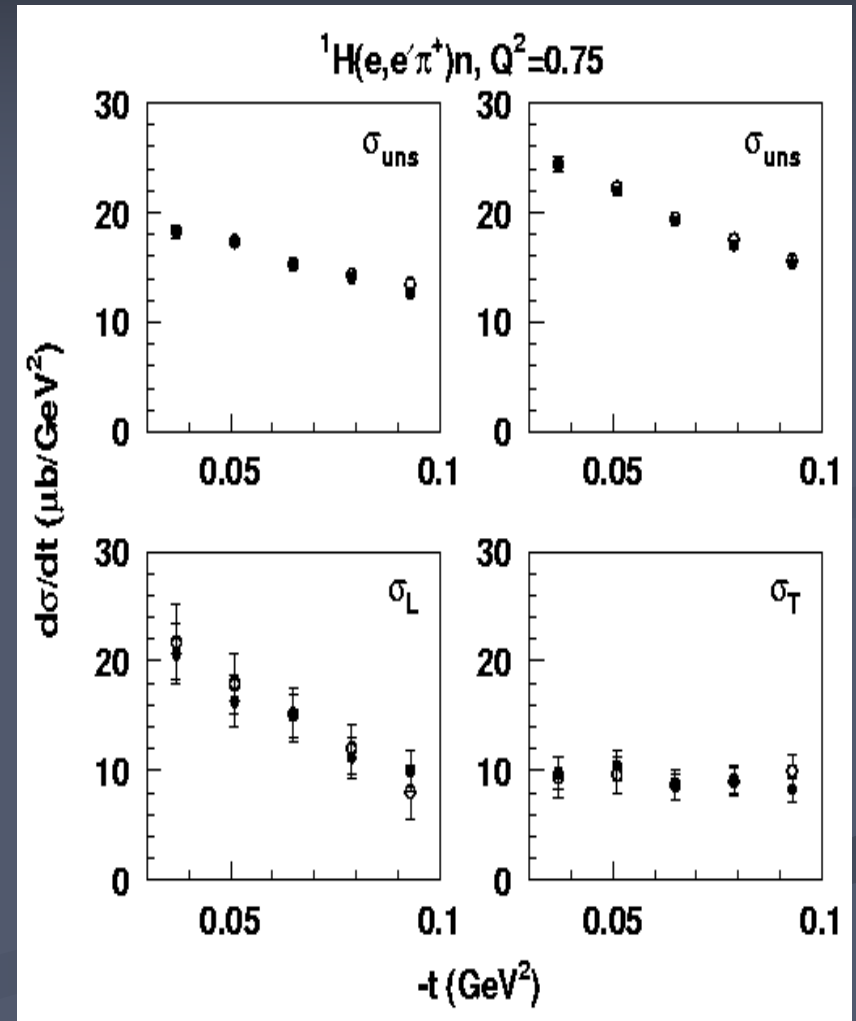
Why was the reanalysis thought to be necessary, and why is it thought to be an improvement by those who’ve done it?”

-- Craig Roberts, GHP organizer

$F_{\pi}-1$ Cross section Analysis

- **Updated version of our previous analysis.**
 - Adjustments made to cuts and efficiencies; small mistake in θ calculation corrected.
 - **The new cross sections should be considered more reliable.**
 - More direct L/T/LT/TT separation algorithm developed here was also used in the $F_{\pi}-2$ analysis.
- **The unseparated cross sections differ slightly from the older values but are in most cases within the total uncertainty quoted there.**
 - The differences from the older analysis are magnified in the $\Delta\varepsilon$ separation.
- **On average σ_L is 6% smaller than the older values and σ_T is 3% larger.**
 - These differences are largest at $Q^2=1.0$, where σ_L is 14% smaller and σ_T is 10% larger.

OPEN: 2001 analysis.
SOLID: 2006 analysis.

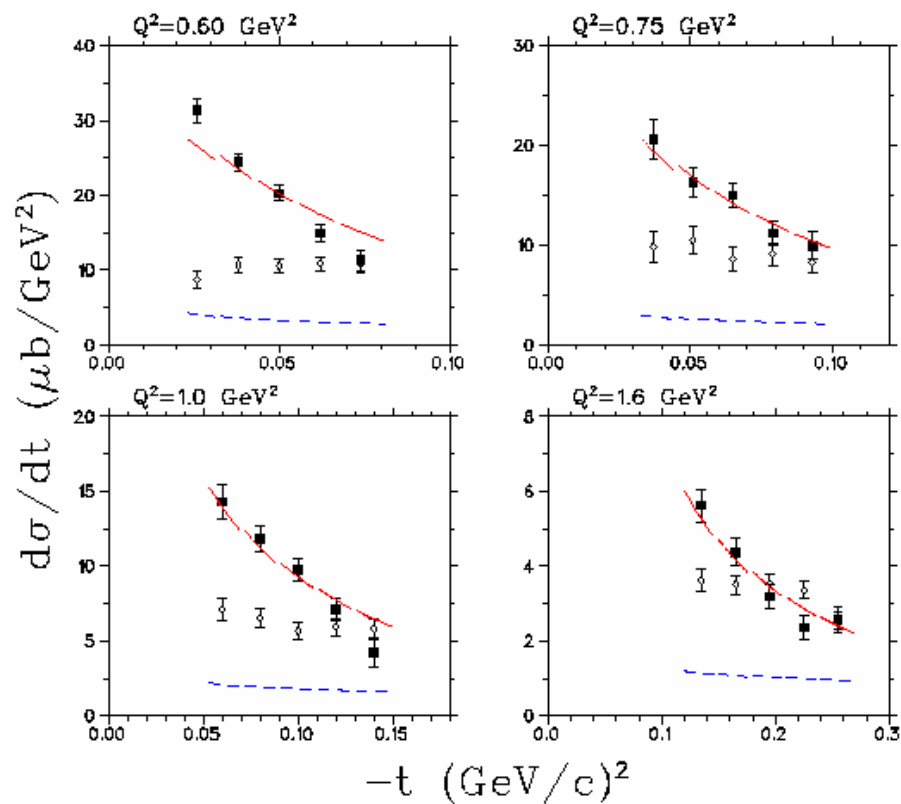


σ_{UNS} : All statistical & syst. uncertainties.
 $\sigma_{\text{L,T}}$: Statistical & ε -uncorrelated unc. only.

Fitting the VGL model to the F_{π^-1} data

Q^2	W	$-t_{min}$
0.60	1.95	0.026
0.75	1.95	0.037
1.00	1.95	0.060
1.60	1.95	0.135

- F_{π^-1} data were acquired in 1997, when maximum beam energy available was 4 GeV.
- Experimental data constrained to $W < 2$ GeV.
- σ_L : t -dependence of VGL model is significantly flatter than data.
- σ_T : model strongly underestimates data for any value of Λ_p^2 used.



Error bars indicate statistical and random (pt-pt) systematic uncertainties added in quadrature. In addition, there is an overall systematic uncertainty of $\sim 6\%$, mainly from the t -correlated, ε -uncorrelated systematic uncertainty.

$$\Lambda_{\pi}^2 = 0.393, 0.373, 0.412, 0.458 \text{ GeV}^2$$

$$\Lambda_p^2 = 1.5 \text{ GeV}^2.$$

Since these data have been taken at relatively low $\sqrt{s} \approx 1.95$ GeV, the deficiencies in the description of the data by the VGL model may be due to contributions from resonances, enhancing the strength in σ_T .

No such terms are included in the Regge model.

As in the case of σ_T , the discrepancy between the σ_L data and model is attributed to resonance contributions.

- The discrepancy is strongest at the lowest Q^2 .
- At higher Q^2 the resonance form factor is expected to reduce resonance contributions.

Since virtually nothing is known about the L/T character of resonances at $\sqrt{s} = 1.95$ GeV and their influence on σ_L , the questions are:

1. How to determine $F_\pi(Q^2)$ from these data?
2. What is the associated ‘model uncertainty’ in doing so?

2001 Form Factor Extraction Method

F_π lower limit:

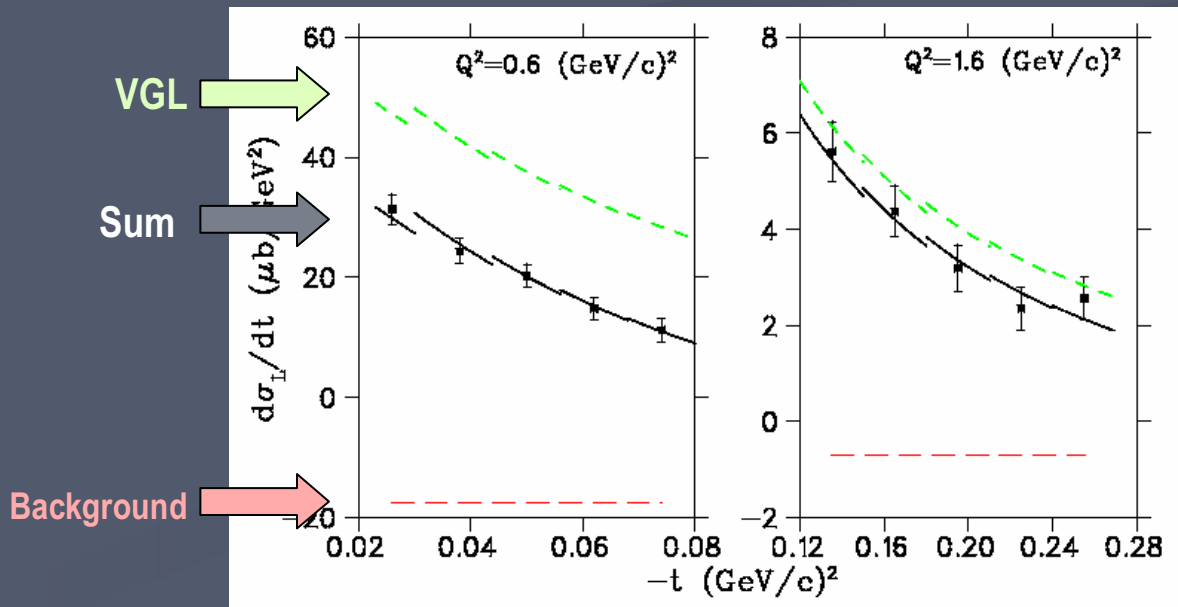
- Since the data are steeper in $-t$ than the VGL calculation, fit Λ_π^2 at the lowest $-t$ bin.

F_π best estimate:

- The average of upper and lower limits.
- 'Model uncertainty':
 - $\frac{1}{2}$ their difference.

F_π upper limit:

- Assume the 'background' effectively yields a negative contribution to σ_L , constant in t .
- Two parameter fit: VGL and background.



Criticisms Raised

The 2001 analysis makes some very special choices about the missing 'background'.

- Flat σ_L background \rightarrow based on σ_T being nearly flat with t .
- Negative interfering cross section \rightarrow may not be realistic.

These choices require a special choice of magnitude and phase for interfering amplitude which may not be reasonable to assume.

Since the 2001 publication, we have looked at the nature of the VGL model's σ_L discrepancy in more detail.

- Code modified by VGL to allow a constant complex background amplitude to be added to the Regge amplitude.
 - \rightarrow Fit background amplitude and Λ_π^2 together to data.
- Study supports an interfering amplitude whose phase does not necessarily result in a net negative cross section contribution to σ_L as previously assumed.

Determining F_π by fitting the interfering amplitude is not viable.

2006 Form Factor Extraction Method

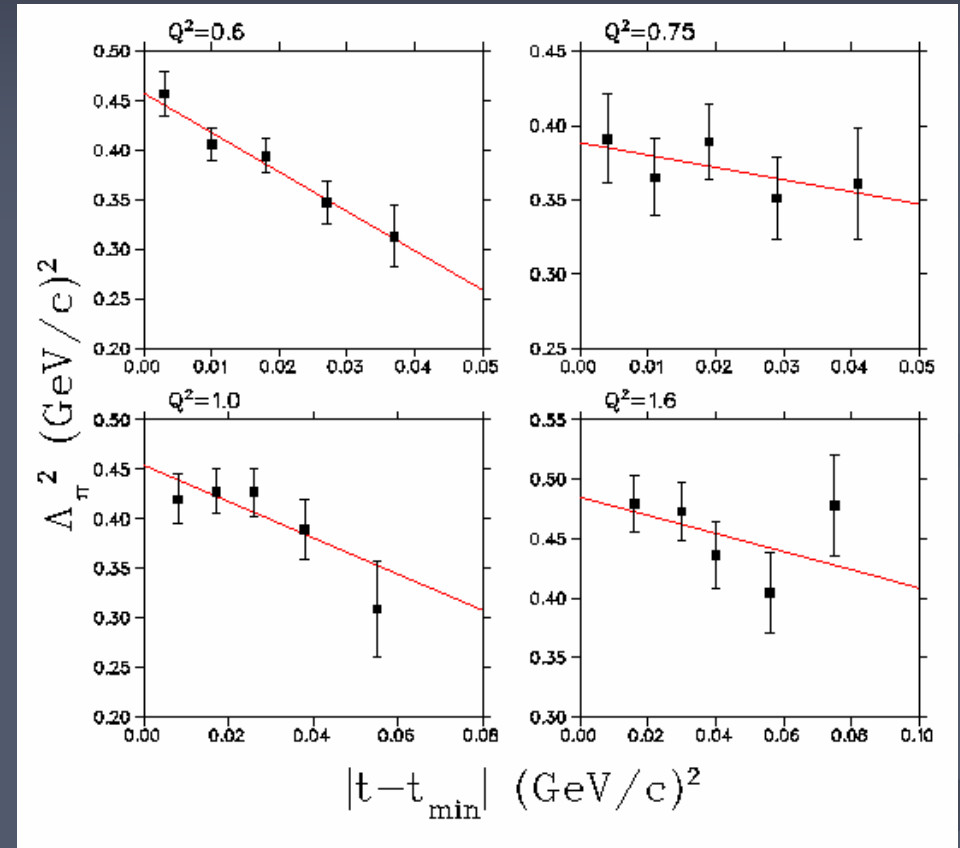
Guiding principle of new method is to minimize assumptions to the greatest extent possible.

Single assumption:

- The contribution of the background is 'small' at the kinematic endpoint t_{min} .
- Still a 'special choice' but more defensible than earlier assumptions.

F_π best estimate:

- Fit VGL model to each t -bin separately, yielding $\Lambda_\pi^2(Q^2, t)$.
- Λ_π^2 decreases with $-t$, presumably due to background not included in the model.
- Linear fit of Λ_π^2 to t_{min} yields 'best estimate' of F_π at each Q^2 .



2006 Model Uncertainty Calculation

- The results obtained by extrapolating Λ_π^2 to t_{\min} will correspond to the true F_π values within the context of the VGL model only if the 'background' vanishes at t_{\min} .
- The 'model uncertainty' represents our best estimate of the uncertainty caused by the fact that this assumption may not be correct.

Two estimation methods used:

1. Fit a flat negative background cross section in addition to VGL model, just as in the 2001 analysis.
 - uncertainty assigned as $\frac{1}{2}$ difference with t_{\min} extrapolation result.
 - uncertainty drops from 15% at $Q^2=0.6$ to 2% at $Q^2=1.6$.
2. Fit flat background amplitude in addition to VGL model.
 - resulting $\chi^2=1$ variation in Λ_π^2 is taken as the model uncertainty.
 - comparable magnitude to #1, but not as strongly Q^2 -dependent.

The larger of these two estimates is taken as the 'model uncertainty'.

Analysis Check

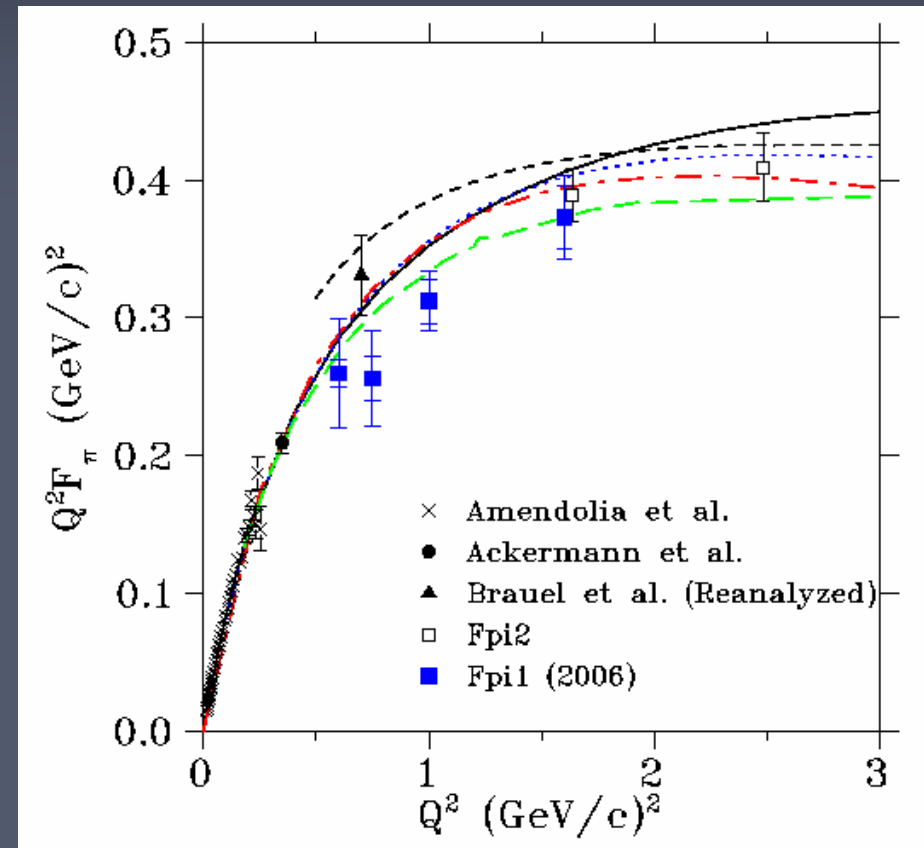
- Other published data at higher W indicate that the discrepancy with the t -dependence of the VGL calculation is smaller.
- As a check of our technique, we reanalyzed the DESY σ_L data at $Q^2=0.70 \text{ GeV}^2$, $W=2.19 \text{ GeV}$.
[Brauel et al., Z. Phys. C3(1979)101]
 - These L/T-separated data are of comparable quality to our JLab data.
- The resulting difference between our 2006 and 2001 analysis methods is only +0.4%.
- Our F_π analysis methods are robust when the background contribution is small, as appears to be the case at this higher W .
 - Consistent with our F_π -2 experience at $W=2.22 \text{ GeV}$.

$F_{\pi-2}$ and revised $F_{\pi-1}$ Results

$F_{\pi-1}$: nucl-ex/0607007.

$F_{\pi-2}$: nucl-ex/0607005.

- New $F_{\pi-1}$ analysis is our best estimate of F_{π} from these data.
 - Error bars are experimental (inner) and exp.+model (outer).
 - Still doing last checks on the model uncertainties.
- Data point at $Q^2=1.60 \text{ GeV}^2$ to check model dependence of form factor extraction.
 - $F_{\pi-1}$ ($W=1.95 \text{ GeV}$) and $F_{\pi-2}$ ($W=2.22 \text{ GeV}$) agree to $\sim 4\%$, indicating reliability of analyses.
 - $F_{\pi-2}$ point is 30% closer to pion pole, with significantly reduced uncertainties.



S.R. Amendolia, et al., Nucl. Phys. **B277** (1986) 168.

H. Ackermann, et al, Nucl. Phys. **B137** (1978) 294.

P. Brauel, et al., Z. Phys. **C3** (1979) 101.

Future work

We are planning a number of initiatives to further improve the $F_\pi(Q^2)$ data set and to reduce model-dependent uncertainties.

1. Study of non-pole backgrounds via π^0 production.
2. Is it valuable to acquire new $0.5 < Q^2 < 1$ GeV² data at higher W ?
3. Eventually extend measurements to $Q^2 = 6$ GeV² with the JLab 12 GeV upgrade.
4. Precise comparison of electroproduction and elastic scattering form factor results at $Q^2 = 0.30$ GeV².