

# Measurement of the Charged Pion Form Factor at EIC

Garth Huber and Dave Gaskell  
(U. Regina and JLab)

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# pQCD and the Pion Form Factor

At large  $Q^2$ , pion form factor ( $F_\pi$ ) can be calculated using perturbative QCD (pQCD)

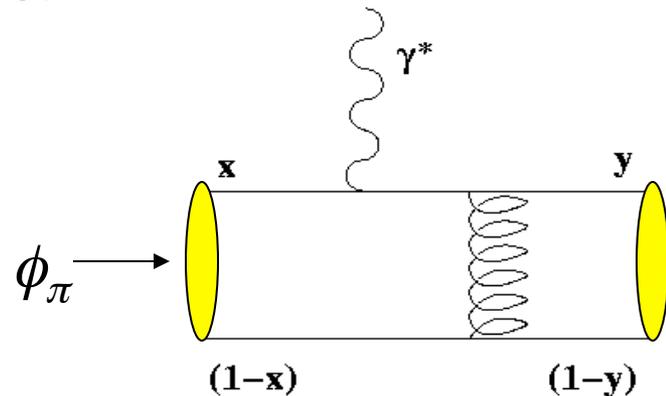
$$F_\pi(Q^2) = \frac{4}{3} \pi \alpha_s \int_0^1 dx dy \frac{2}{3} \frac{1}{xy Q^2} \phi(x) \phi(y)$$

at asymptotically high  $Q^2$ , only the hardest portion of the wave function remains

$$\phi_\pi(x) \xrightarrow[Q^2 \rightarrow \infty]{\sqrt{n_c}} \frac{3f_\pi}{\sqrt{n_c}} x(1-x)$$

and  $F_\pi$  takes the very simple form

$$F_\pi(Q^2) \xrightarrow[Q^2 \rightarrow \infty]{} \frac{16\pi\alpha_s(Q^2)f_\pi^2}{Q^2}$$



where  $f_\pi=93$  MeV is the  $\pi^+ \rightarrow \mu^+ \nu$  decay constant.

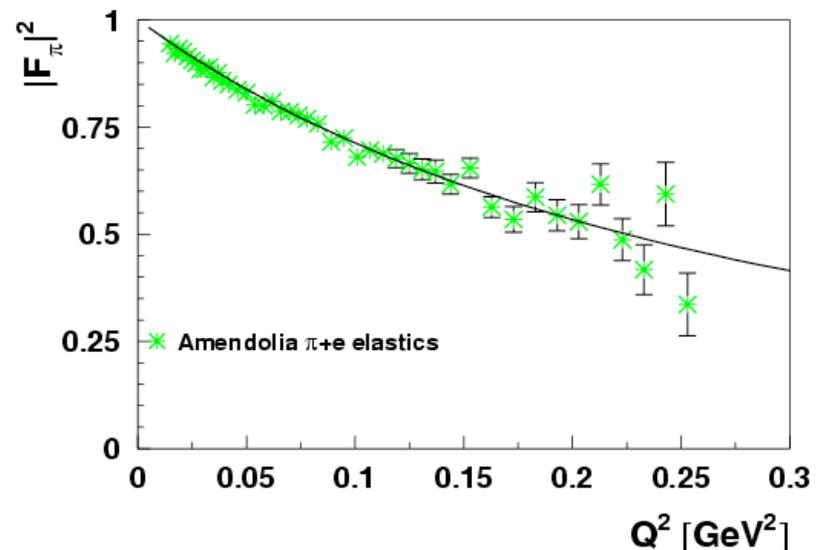
G.P. Lepage, S.J. Brodsky, Phys.Lett. **87B**(1979)359.

# Measurement of $\pi^+$ Form Factor – Low $Q^2$

- At low  $Q^2$ ,  $F_\pi$  can be measured **directly** via high energy elastic  $\pi^-$  scattering from atomic electrons
  - CERN SPS used 300 GeV pions to measure form factor up to  $Q^2 = 0.25 \text{ GeV}^2$   
[Amendolia et al, NPB277, 168 (1986)]
  - These data used to extract the pion charge radius

$$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

- Maximum accessible  $Q^2$  roughly proportional to pion beam energy
  - $Q^2=1 \text{ GeV}^2$  requires 1000 GeV pion beam

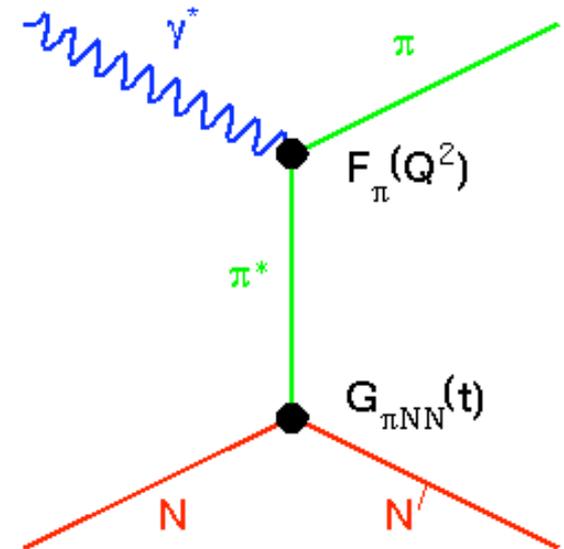


# Measurement of $\pi^+$ Form Factor – Larger $Q^2$

- At larger  $Q^2$ ,  $F_\pi$  must be measured indirectly using the “pion cloud” of the proton via  $p(e, e' \pi^+) n$ 
  - At small  $-t$ , the pion pole process dominates the longitudinal cross section,  $\sigma_L$
  - In Born term model,  $F_\pi^2$  appears as,

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t - m_\pi^2)} g_{\pi NN}^2(t) F_\pi^2(Q^2, t)$$

- Drawbacks of this technique
  - Isolating  $\sigma_L$  experimentally challenging
  - Theoretical uncertainty in form factor extraction



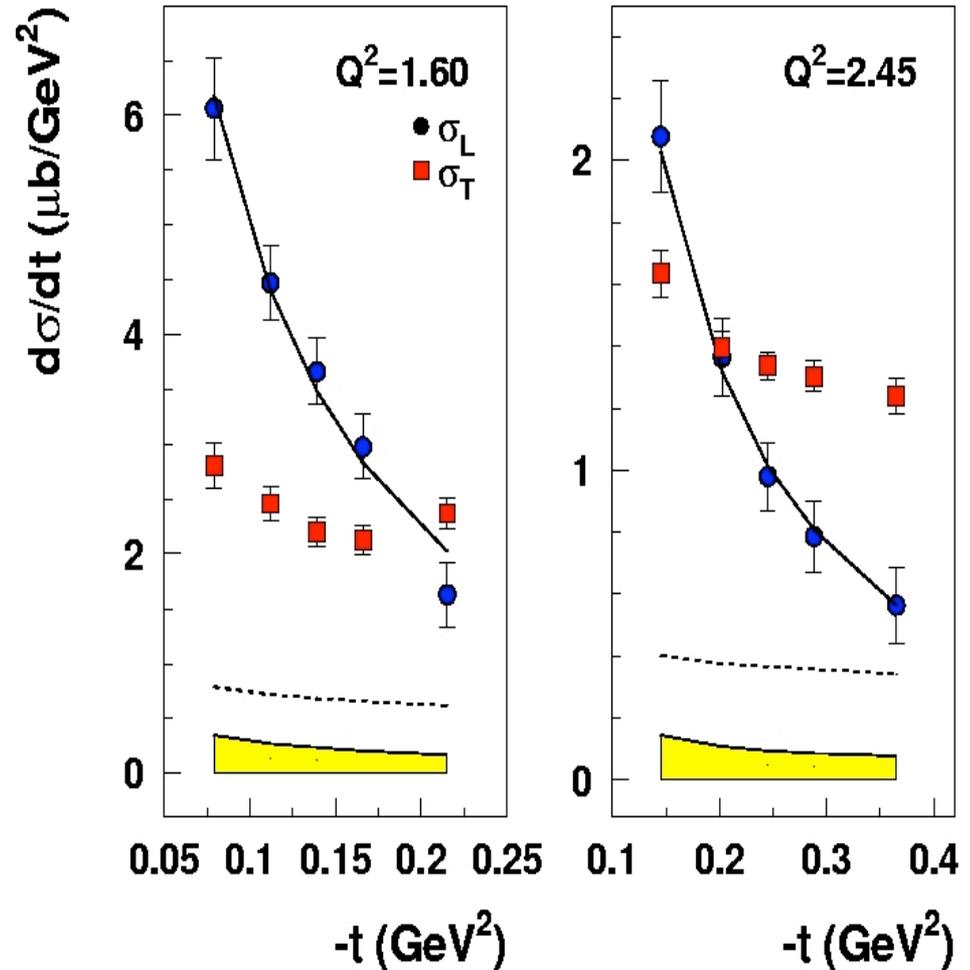
# $F_\pi$ Extraction from JLab data

Horn et al, PRL97, 192001,2006

## VGL Regge Model

- Feynman propagator replaced by  $\pi$  and  $\rho$  Regge propagators.
  - Represents the exchange of a series of particles, compared to a single particle.
- Model parameters fixed from pion photoproduction.
- Free parameters:  $\Lambda_\pi$ ,  $\Lambda_\rho$  (trajectory cutoff).

$$F_\pi(Q^2) = \frac{1}{1 + Q^2 / \Lambda_\pi^2}$$



$\Lambda_\pi^2=0.513, 0.491 \text{ GeV}^2$ ,  $\Lambda_\rho^2=1.7 \text{ GeV}^2$

# Unpolarized Pion Cross Section

$$2\pi \frac{d^2\sigma}{dt d\phi} = \epsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT}}{dt} \cos\phi + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi$$

**$t$  = four-momentum transferred to nucleon**

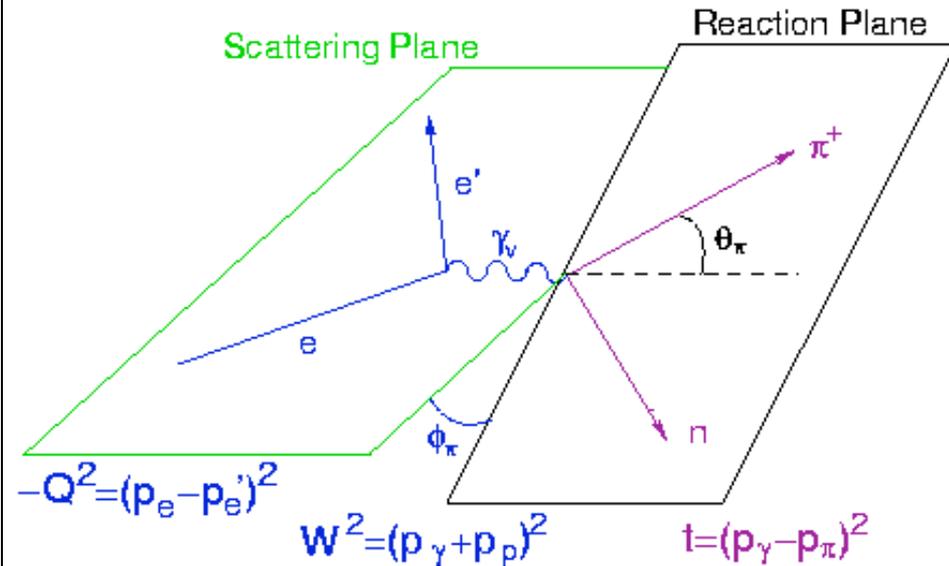
**= (mass)<sup>2</sup> of struck virtual pion**

$W$  = total energy in virtual photon-target center of mass

**$Q^2$  = -(mass)<sup>2</sup> of virtual photon**

$\epsilon$  = virtual photon polarization,  $0 \rightarrow 1$

$\phi$  = azimuthal angle between reaction plane and scattering plane



***L-T separation required to extract  $\sigma_L$***

# L-T Separation in an e-p Collider

$$\varepsilon = \frac{2(1-y)}{1+(1-y)^2} \quad \text{where the fractional energy loss } y \approx \frac{Q^2}{xs_{tot}}$$

- Systematic uncertainties in  $\sigma_L$  are magnified by  $1/\Delta\varepsilon$ .
  - desire  $\Delta\varepsilon > 0.2$ .
- $\varepsilon \approx 1$  is simple to access.
  - 5 GeV ( $e^-$ ) on 50 GeV ( $p$ ) typically assumed, but the exact energies are almost immaterial.
- To access  $\varepsilon < 0.8$ , one needs  $y > 0.5$ .
  - **This can only be accessed with small  $s_{tot}$ , i.e. low proton collider energies (5-15 GeV).**

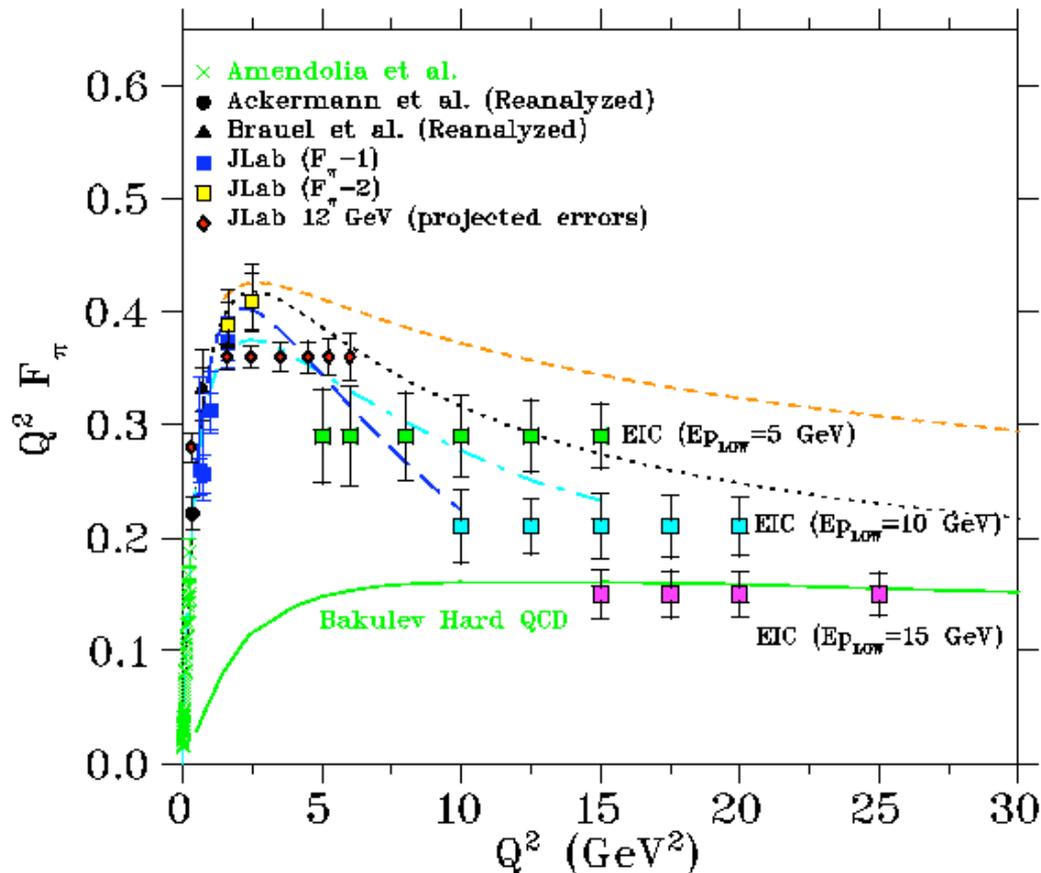
# Scattered electron detection requirements

- **High  $\epsilon \approx 1$  measurements** (5 GeV  $e^-$  on 50 GeV  $p$ ):
  - Scattered electron angles of  $20^\circ$ - $60^\circ$  (wrt incident electron beam).
- **Low  $\epsilon$  measurements** (2-6 GeV  $e^-$  on 5-15 GeV  $p$ ):
  - In some cases, need to detect scattered electrons up to  $135^\circ$ .
- **Resolution requirements:**  
 $\delta P/P \approx 3 \times 10^{-3}$      $\delta \theta \approx 1 \text{ mr.}$

# Recoil detector requirements

- **Easiest way to assure exclusivity of the  $p(e, e' \pi^+) n$  reaction is by detecting the recoil neutron.**
- **Parallel-kinematics measurements (e.g. pion form factor and QCD scaling tests):**
  - Neutrons are emitted at small angle ( $\theta < 0.35^\circ$ ), with momentum typically about 80% of the proton beam.
  - Current discussions for mEIC detector envision neutron/hadron detector relatively close to the interaction region after an “ion dipole”, and/or very far away  $\rightarrow$  I’ll come back to this

# Kinematic Reach (Pion Form Factor)

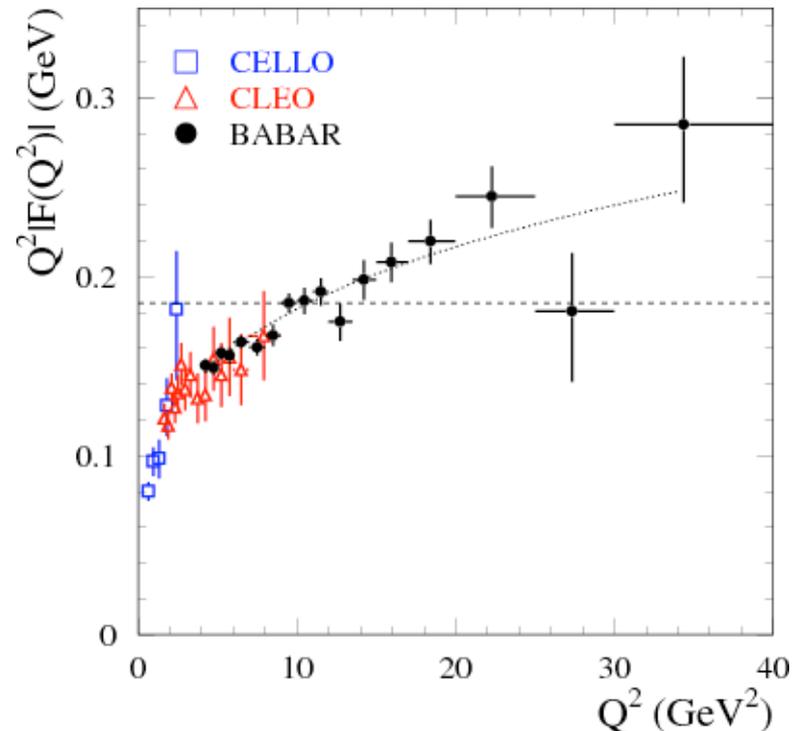
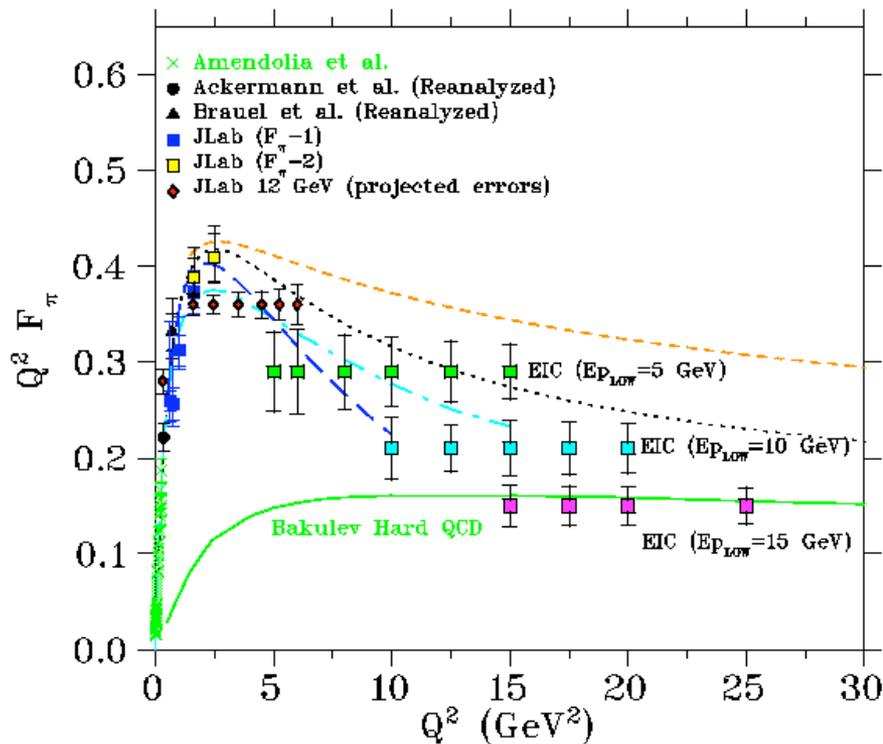


## Assumptions:

- High  $\epsilon$ :  $5(e^-)$  on  $50(p)$ .
- Low  $\epsilon$  proton energies as noted.
- $\Delta\epsilon \sim 0.22$ .
- Scattered electron detection over  $4\pi$ .
- Recoil neutrons detected at  $\theta < 0.35^\circ$  with high efficiency.
- Statistical unc:  $\Delta\sigma_L / \sigma_L \sim 5\%$
- Systematic unc:  $6\% / \Delta\epsilon$ .
- Approximately one year at  $L = 10^{34}$ .

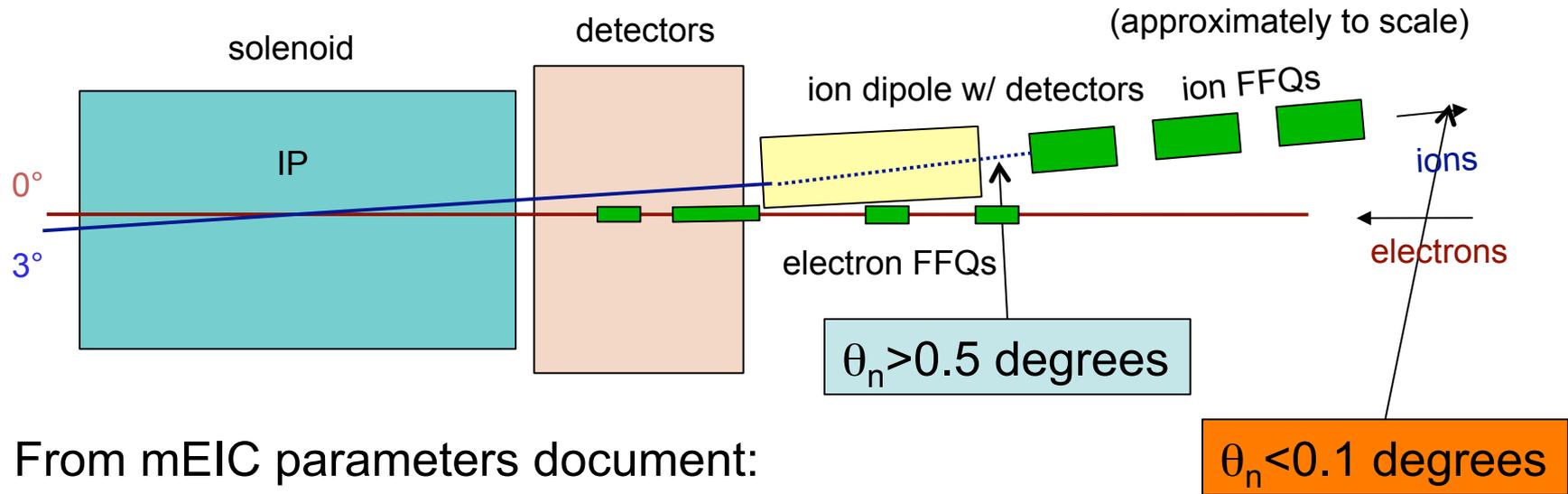
Excellent potential to study the **QCD transition** nearly over the whole range from the **strong QCD** regime to the **hard QCD** regime.

# Kinematic Reach (Pion Form Factor)



$Q^2$  reach comparable to that of recent  $\gamma\gamma \rightarrow \pi^0$  transition form factor measurements from Babar

# $F_\pi$ Compatible with mEIC?



From mEIC parameters document:

→  $E_e = 3-11$  GeV (mostly ok)

→  $E_p = 20-60$  GeV (not ok for low  $\varepsilon$  at lowest  $Q^2$ )

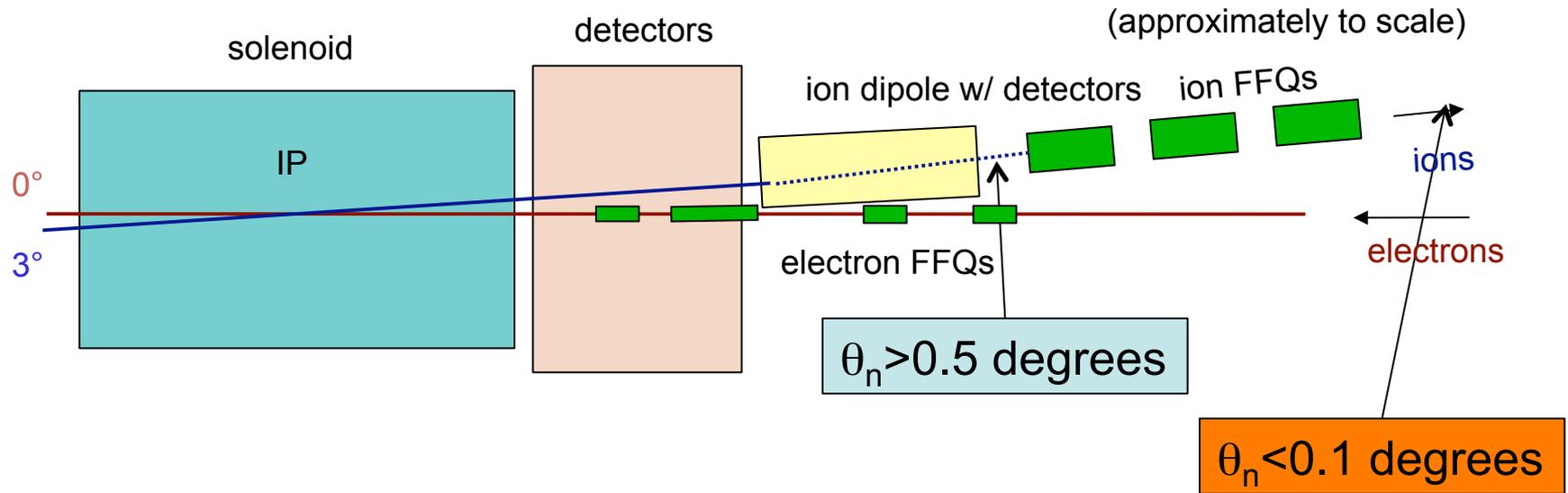
## ***Recoil neutron detection:***

→ There will be a “dead zone” in which recoil neutrons cannot be detected → 0.1 to 0.5 degrees likely not accessible<sup>1</sup>

→ Low  $\varepsilon$  points require neutron detection between  $\theta_n = 0.2-0.3$  for  $Q^2$  below 12.5 GeV<sup>2</sup>

<sup>1</sup>*Rolf Ent, private communication*

# $F_\pi$ Compatible with mEIC?



Kinematics may be adjusted to accommodate nominal (m)EIC parameters depending on ability to detect neutrons at VERY small angles  
→ In general, increasing  $W$  allows  $\epsilon=0.8$  for nominal mEIC energies

▪ ***This pushes neutrons very far forward***

→ Example – shift  $W$  from 10 to 10.5 GeV at  $Q^2=10$  GeV<sup>2</sup> allows us to use 3 GeV e on 20 GeV p for  $\epsilon=0.8$ ; ( $\theta_n=0.01$  degrees)

▪ But at large  $\epsilon$ ,  $\theta_n$  becomes 0.005 degrees

# Extract $\sigma_L$ with no L-T separation?

In principle possible to extract  $R=\sigma_L/\sigma_T$  using polarization degrees of freedom

In parallel kinematics  
(outgoing meson along  $\vec{q}$ )  $\longrightarrow$   $\frac{R_L}{R_T} = \frac{1}{\epsilon} \left( \frac{1}{\chi_z} - 1 \right)$

$$\chi_z = \frac{1}{P_e \sqrt{1 - \epsilon^2}} P_z$$

$\chi_z$  = z-component of proton  
“reduced” recoil polarization in  
 $H(e, e'p)\pi^0$

*Schmieden and Titator [Eur. Phys. J. A **8**, 15-17 (2000)]*

A similar relation holds for pion production from a polarized target if we re-define  $\chi_z$

$$\chi_z = \frac{1}{2P_e P_T \sqrt{1 - \epsilon^2}} A_z$$

$A_z$  = target double-spin asymmetry

# Isolating $\sigma_L$ with Polarization D.O.F

$$\sigma_{pol} \sim P_e P_p \sqrt{(1 - \epsilon^2)} A_z$$

Nominal, high energies,  $\epsilon$  very close to 1.0  $\rightarrow$  destroys figure of merit for this technique

$\rightarrow$  If we can adjust  $\epsilon$  to 0.9 then  $\sqrt{(1 - \epsilon^2)} \rightarrow 0.44$

$\rightarrow \epsilon = 0.95$   $\sqrt{(1 - \epsilon^2)} \rightarrow 0.31$

Example: At  $Q^2 = 5$ , lowest  $s$  of 3 GeV  $e^-$  on 20 GeV  $p$  results in the smallest  $\epsilon = 0.947$  (for which neutron is still easily detectable)

Additional issue:  $A_z$  = component of  $p$  polarization parallel to  $q$   $\rightarrow$  proton polarization direction ideally tunable at IP

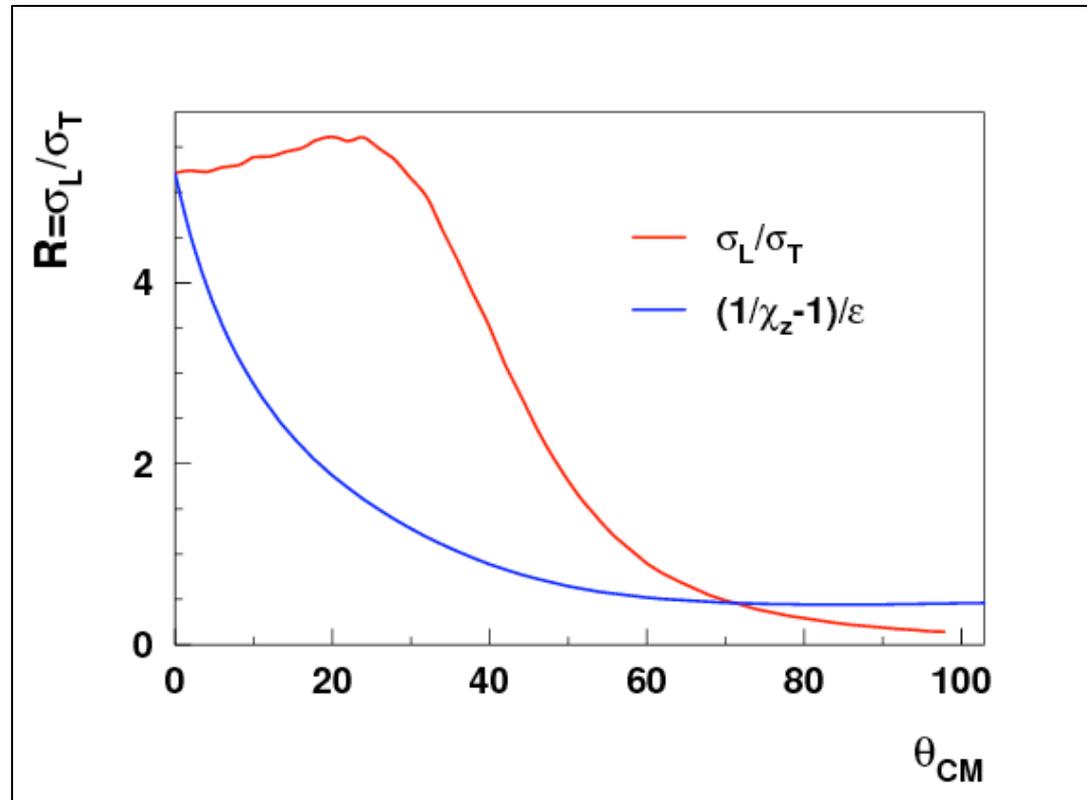
# Parallel Kinematics

Polarization relation for extracting  $\sigma_L/\sigma_T$  only applies in parallel kinematics – how quickly does this relation break down away from  $\theta_{CM} = 0$ ?

MAID2007

$Q^2 = 5 \text{ GeV}^2$

$W = 1.95 \text{ GeV}$



# L/T Extraction

Extraction via this technique requires strict cuts on  $\theta_{\text{CM}}$

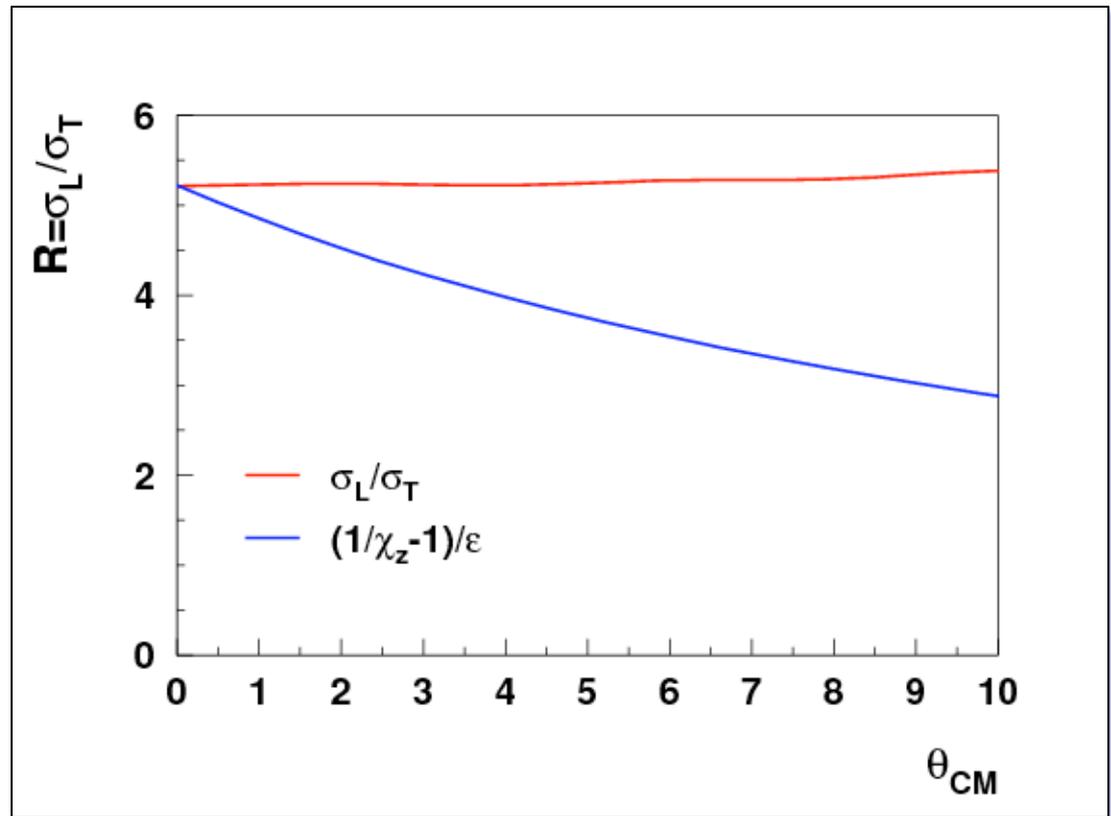
**$Q^2=5 \text{ GeV}^2$ , (3 on 20):**

→ 1 degree CM cut  
corresponds to  $\sim 30$  mrad  
in the lab

**$Q^2=25 \text{ GeV}^2$ , (5 on 50):**

→ 1 degree CM cut  
corresponds to 20 mrad  
in the lab

At 1 degree, polarization  
observable already  $\sim 15\%$   
different from true value  
→ very tight cuts will be  
needed (0.1 degrees?)



# Summary

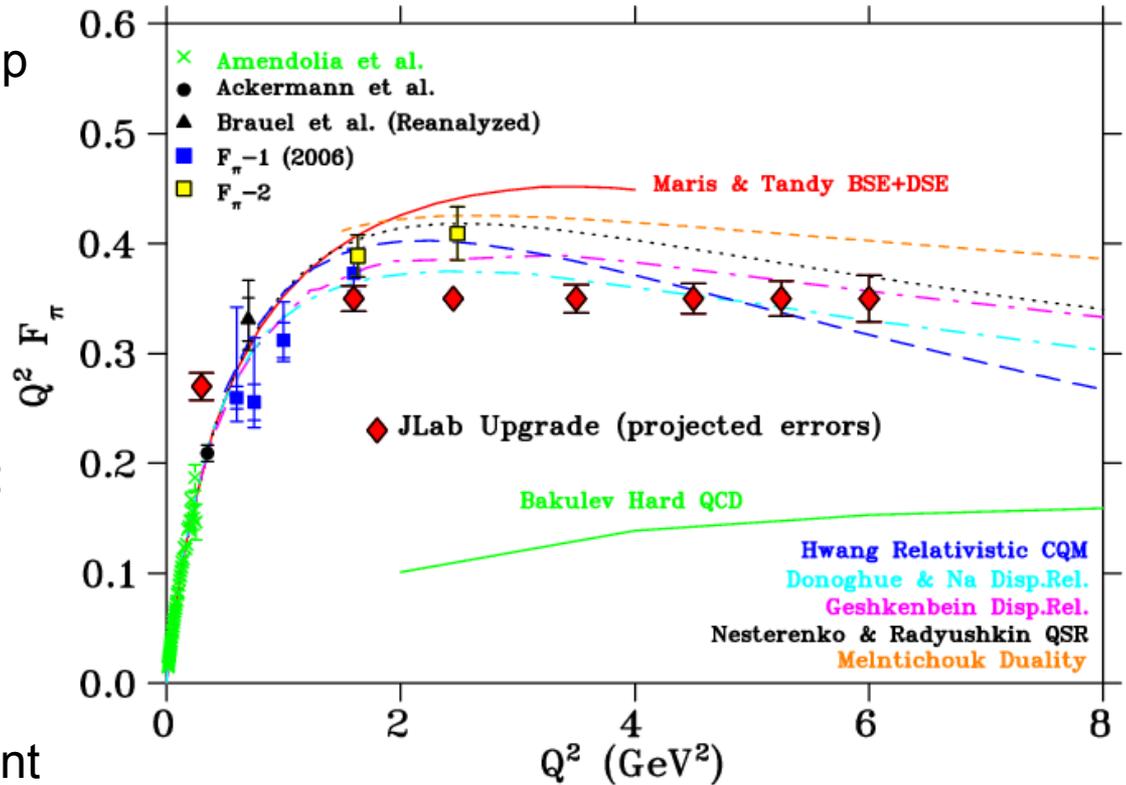
- Measurement of  $F_\pi$  at EIC will be challenging
  - Use of L-T separation made easier with energies outside of “nominal”
  - Reduction of neutron detection “dead zone” would also be beneficial
  - Extreme forward neutron detection ( $<0.01$  degrees) would alleviate both of the above
  - ***Another option: measure away from  $-t_{min}$  so neutron angle  $> 0.5$  degrees  $\rightarrow$  phase space for this is quite small and  $-t$  pretty large ( $-t \sim 0.2$ )***
- Measurement using polarization degrees of freedom seems, at first glance, feasible not impossible
  - Very tight cuts on pion angle will be required
  - More detailed studies required  $\rightarrow$  a model incorporating all response functions needed to simulate how close to parallel we must be

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Extra

# $F_{\pi^+}(Q^2)$ after JLAB 12 GeV Upgrade

- JLab 12 GeV upgrade will allow measurement of  $F_{\pi}$  up to  $6 \text{ GeV}^2$ 
  - Will we see the beginning of the transition to the perturbative regime?
- Additional point at  $Q^2=1.6 \text{ GeV}^2$  will be closer to pole: will provide another constraint on  $-t_{min}$  dependence
- $Q^2=0.3 \text{ GeV}^2$  point will be best direct test of agreement with elastic  $\pi^+e$  data



# Low $\varepsilon$ $F_\pi$ Kinematics

$Q^2$	$P_p$	$P_e$	$-t$	$\varepsilon$	$\theta_n$
5	5	2	0.047	0.78	0.32
6	5	3	0.031	0.80	0.19
8	5	3	0.052	0.77	0.28
10	5	4	0.042	0.75	0.19
10	10	5	0.008	0.80	0.02
12.5	5	4	0.062	0.72	0.26
12.5	10	4	0.013	0.64	0.02
15	5	4	0.085	0.69	0.32
15	10	5	0.018	0.78	0.04
15	15	6	0.006	0.79	0.01
17.5	10	5	0.024	0.77	0.04
17.5	15	6	0.008	0.78	0.01
20	10	5	0.030	0.75	0.05
20	15	6	0.010	0.77	0.01
25	15	6	0.015	0.76	0.02