Measurement of the Charged Pion Form Factor at EIC

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pQCD and the Pion Form Factor

At large $Q^2$, pion form factor ($F_\pi$) can be calculated using perturbative QCD (pQCD)

$$F_\pi(Q^2) = \frac{4}{3} \pi \alpha_s \int_0^1 dx dy \frac{2}{3} \frac{1}{xyQ^2} \phi(x)\phi(y)$$

at asymptotically high $Q^2$, only the hardest portion of the wave function remains

$$\phi_\pi(x) \xrightarrow{Q^2 \to \infty} \frac{3 f_\pi}{\sqrt{n_c}} x(1 - x)$$

and $F_\pi$ takes the very simple form

$$F_\pi(Q^2) \xrightarrow{Q^2 \to \infty} \frac{16 \pi \alpha_s(Q^2) f_\pi^2}{Q^2}$$

where $f_\pi = 93$ MeV is the $\pi^+ \to \mu^+ \nu$ decay constant.

At low $Q^2$, $F_\pi$ can be measured *directly* via high energy elastic $\pi^-$ scattering from atomic electrons

- CERN SPS used 300 GeV pions to measure form factor up to $Q^2 = 0.25$ GeV$^2$
  
  [Amendolia et al, NPB277, 168 (1986)]

- These data used to extract the pion charge radius

$$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

- Maximum accessible $Q^2$ roughly proportional to pion beam energy
  
  - $Q^2=1$ GeV$^2$ requires 1000 GeV pion beam
Measurement of $\pi^+$ Form Factor – Larger $Q^2$

- At larger $Q^2$, $F_\pi$ must be measured indirectly using the “pion cloud” of the proton via $p(e,e'\pi^+)n$.
  - At small $-t$, the pion pole process dominates the longitudinal cross section, $\sigma_L$.
  - In Born term model, $F_\pi^2$ appears as:

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t - m_\pi^2)} g_{\pi NN}^2(t) F_\pi^2(Q^2, t)$$

- Drawbacks of this technique:
  - Isolating $\sigma_L$ experimentally challenging.
  - Theoretical uncertainty in form factor extraction.
**VGL Regge Model**

- Feynman propagator replaced by $\pi$ and $\rho$ Regge propagators.
  - Represents the exchange of a series of particles, compared to a single particle.
- Model parameters fixed from pion photoproduction.
- Free parameters: $\Lambda_\pi$, $\Lambda_\rho$ (trajectory cutoff).

\[ F_\pi(Q^2) = \frac{1}{1 + Q^2 / \Lambda_\pi^2} \]

\[ \Lambda_\pi^2 = 0.513, 0.491 \text{ GeV}^2, \quad \Lambda_\rho^2 = 1.7 \text{ GeV}^2 \]
Unpolarized Pion Cross Section

\[ 2\pi \frac{d^2 \sigma}{dtd\phi} = \epsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT}}{dt} \cos \phi + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi \]

\( t = \) four-momentum transferred to nucleon
\( = (\text{mass})^2 \) of struck virtual pion

\( W = \) total energy in virtual photon-target center of mass

\( Q^2 = -(\text{mass})^2 \) of virtual photon

\( \epsilon = \) virtual photon polarization, \( 0 \rightarrow 1 \)

\( \phi = \) azimuthal angle between reaction plane and scattering plane

**L-T separation required to extract \( \sigma_L \)**
L-T Separation in an e-p Collider

\[ \varepsilon = \frac{2(1 - y)}{1 + (1 - y)^2} \] where the fractional energy loss \( y \approx \frac{Q^2}{x s_{tot}} \)

- Systematic uncertainties in \( \sigma_L \) are magnified by \( 1/\Delta\varepsilon \).
  - desire \( \Delta\varepsilon > 0.2 \).

- \( \varepsilon \approx 1 \) is simple to access.
  - 5 GeV (e-) on 50 GeV (p) typically assumed, but the exact energies are almost immaterial.

- To access \( \varepsilon < 0.8 \), one needs \( y > 0.5 \).
  - This can only be accessed with small \( s_{tot} \), i.e. low proton collider energies (5-15 GeV).
Scattered electron detection requirements

- **High $\varepsilon \approx 1$ measurements** (5 GeV $e^-$ on 50 GeV $p$):
  - Scattered electron angles of 20°-60° (wrt incident electron beam).

- **Low $\varepsilon$ measurements** (2-6 GeV $e^-$ on 5-15 GeV $p$):
  - In some cases, need to detect scattered electrons up to 135°.

- **Resolution requirements:**
  $$\frac{\delta P}{P} \approx 3 \times 10^{-3} \quad \delta \theta \approx 1 \text{mr}.$$
Recoil detector requirements

• Easiest way to assure exclusivity of the $p(e,e'\pi^+)n$ reaction is by detecting the recoil neutron.

• Parallel-kinematics measurements (e.g. pion form factor and QCD scaling tests):
  – Neutrons are emitted at small angle ($\theta<0.35^\circ$), with momentum typically about 80% of the proton beam.
  – Current discussions for mEIC detector envision neutron/hadron detector relatively close to the interaction region after an “ion dipole”, and/or very far away → I’ll come back to this
Kinematic Reach (Pion Form Factor)

Assumptions:
- **High $\varepsilon$:** $5(e^-)$ on $50(p)$.
- **Low $\varepsilon$** proton energies as noted.
- $\Delta\varepsilon\sim0.22$.
- Scattered electron detection over $4\pi$.
- **Recoil neutrons detected at $\theta<0.35^\circ$** with high efficiency.
- Statistical unc: $\Delta\sigma_L/\sigma_L\sim5\%$.
- Systematic unc: $6%/\Delta\varepsilon$.
- Approximately one year at $L=10^{34}$.

Excellent potential to study the **QCD transition** nearly over the whole range from the **strong QCD** regime to the **hard QCD** regime.
Kinematic Reach (Pion Form Factor)

Q^2 reach comparable to that of recent $\gamma\gamma\rightarrow\pi^0$ transition form factor measurements from Babar
$F_\pi$ Compatible with mEIC?

From mEIC parameters document:
\[ E_e = 3-11 \text{ GeV} \text{ (mostly ok)} \]
\[ E_p = 20-60 \text{ GeV} \text{ (not ok for low } \varepsilon \text{ at lowest } Q^2 \text{)} \]

Recoil neutron detection:
\[ \text{There will be a “dead zone” in which recoil neutrons cannot be detected } \rightarrow \theta_n > 0.5 \text{ degrees likely not accessible}^1 \]
\[ \text{Low } \varepsilon \text{ points require neutron detection between } \theta_n = 0.2-0.3 \text{ for } Q^2 \text{ below 12.5 GeV}^2 \]

$^1$Rolf Ent, private communication
Kinematics may be adjusted to accommodate nominal (m)EIC parameters depending on ability to detect neutrons at VERY small angles

→ In general, increasing $W$ allows $\varepsilon=0.8$ for nominal mEIC energies

- *This pushes neutrons very far forward*

→ Example – shift $W$ from 10 to 10.5 GeV at $Q^2=10$ GeV$^2$ allows us to use 3 GeV e on 20 GeV p for $\varepsilon=0.8$; ($\theta_n=0.01$ degrees)

- But at large $\varepsilon$, $\theta_n$ becomes 0.005 degrees
Extract $\sigma_L$ with no L-T separation?

In principle possible to extract $R = \sigma_L / \sigma_T$ using polarization degrees of freedom.

In parallel kinematics (outgoing meson along $\vec{q}$)

$$\frac{R_L}{R_T} = \frac{1}{\epsilon} \left( \frac{1}{\chi_z} - 1 \right)$$

$$\chi_z = z\text{-component of proton } \text{"reduced" recoil polarization in } H(e,e'p)\pi^0$$


A similar relation holds for pion production from a polarized target if we re-define $\chi_z$

$$\chi_z = \frac{1}{2P_e P_T \sqrt{1 - \epsilon^2}} A_z$$

$A_z = \text{target double-spin asymmetry}$
Isolating $\sigma_L$ with Polarization D.O.F

$$\sigma_{pol} \sim P_e P_p \sqrt{(1 - \epsilon^2)} A_z$$

Nominal, high energies, $\epsilon$ very close to 1.0 $\rightarrow$ destroys figure of merit for this technique
$\rightarrow$ If we can adjust $\epsilon$ to 0.9 then $\sqrt{(1 - \epsilon^2)} \rightarrow 0.44$
$\rightarrow$ $\epsilon = 0.95$ $\sqrt{(1 - \epsilon^2)} \rightarrow 0.31$

Example: At $Q^2 = 5$, lowest $s$ of 3 GeV $e^-$ on 20 GeV $p$
results in the smallest $\epsilon = 0.947$ (for which neutron is still easily detectable)

Additional issue: $A_z = \text{component of } p \text{ polarization parallel to } q \rightarrow \text{proton polarization direction ideally tunable at IP}$
Parallel Kinematics

Polarization relation for extracting $\sigma_L/\sigma_T$ only applies in parallel kinematics – how quickly does this relation break down away from $\theta_{CM} = 0$?

$Q^2 = 5$ GeV\(^2\)
$W = 1.95$ GeV
Extraction via this technique requires strict cuts on $\theta_{\text{CM}}$

$Q^2=5 \text{ GeV}^2$, (3 on 20):
$\rightarrow$ 1 degree CM cut corresponds to $\sim 30$ mrad in the lab

$Q^2=25 \text{ GeV}^2$, (5 on 50):
$\rightarrow$ 1 degree CM cut corresponds to 20 mrad in the lab

At 1 degree, polarization observable already $\sim 15\%$ different from true value
$\rightarrow$ very tight cuts will be needed (0.1 degrees?)
Summary

- Measurement of $F_\pi$ at EIC will be challenging
  - Use of L-T separation made easier with energies outside of “nominal”
  - Reduction of neutron detection “dead zone” would also be beneficial
  - Extreme forward neutron detection (<0.01 degrees) would alleviate both of the above
    - *Another option: measure away from $-t_{\text{min}}$ so neutron angle > 0.5 degrees* → *phase space for this is quite small and $-t$ pretty large ($-t \sim 0.2$)*

- Measurement using polarization degrees of freedom seems, at first glance, feasible not impossible
  - Very tight cuts on pion angle will be required
  - More detailed studies required → a model incorporating all response functions needed to simulate how close to parallel we must be
Extra
\( F_{\pi^+}(Q^2) \) after JLAB 12 GeV Upgrade

- JLab 12 GeV upgrade will allow measurement of \( F_\pi \) up to 6 GeV\(^2\)
  - Will we see the beginning of the transition to the perturbative regime?
- Additional point at \( Q^2=1.6 \) GeV\(^2\) will be closer to pole: will provide another constraint on \(-t_{\text{min}}\) dependence
- \( Q^2=0.3 \) GeV\(^2\) point will be best direct test of agreement with elastic \( \pi^+e \) data
Low $\varepsilon F_\pi$ Kinematics

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