Exploring the Electromagnetic Structure of the Charged Pion and Kaon

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The pion is attractive as a QCD laboratory:
- Simple, 2 quark system

Electromagnetic form factor can be calculated exactly at very large momentum transfer (small distances).
- For moderate $Q^2$, it remains a theoretical challenge.
  - “the positronium atom of QCD”

Downside for experimentalists:
- No “free” pion targets.
- Measurements at large momentum transfer difficult.
Measurement of $F_\pi$ via Electroproduction

Above $Q^2>0.3$ GeV$^2$, $F_\pi$ is measured indirectly using the “pion cloud” of the proton via pion electroproduction $p(e,e'\pi^+)n$

$$|p\rangle = |p\rangle_0 + |n\pi^+\rangle + ...$$

- At small $-t$, the pion pole process dominates the longitudinal cross section, $\sigma_L$
- In Born term model, $F_\pi^2$ appears as

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t-m_\pi^2)} g^2_{\pi NN}(t) F_\pi^2(Q^2,t)$$

**Drawbacks of this technique:**
1. Isolating $\sigma_L$ experimentally challenging.
2. The $F_\pi$ values are in principle dependent upon the model used, but this dependence is expected to be reduced at sufficiently small $-t$. 
• 2 $F_\pi$ experiments have been carried out at JLab
  (spokespersons H. Blok, G. Huber, D. Mack)
  • $F_\pi$ -1: $Q^2=0.6$-1.6 GeV$^2$ with 4 GeV beam, 1997-2001.
\[
2\pi \frac{d^2\sigma}{dtd\phi} = \varepsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon (\varepsilon + 1)} \frac{d\sigma_{LT}}{dt} \cos\phi + \varepsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi
\]

Virtual-photon polarization:
\[
\varepsilon = \left(1 + 2 \left(\frac{E_e - E_{e'}}{E_e} \right)^2 + \frac{Q^2}{Q^2} \tan^2 \theta_{e'} \right)^{-1}
\]

- **L-T separation required to separate** $\sigma_L$ **from** $\sigma_T$.
- **Need to take data at smallest available** $-t$, **so** $\sigma_L$ **has maximum contribution from the** $\pi^+$ **pole.**
**$F_\pi$ Extraction from JLab data**

- Model is required to extract $F_\pi$ from $\sigma_L$
- JLab $F_\pi$ experiments used the VGL Regge model [Vanderhaeghen, Guidal, Laget, PRC 57, 1454 (1998)]
  - Propagator replaced by $\pi$ and $\rho$ Regge trajectories
  - Most parameters fixed by photoproduction data
  - 2 free parameters: $\Lambda_\pi$, $\Lambda_\rho$
  - At small $-t$, $\sigma_L$ only sensitive to $\Lambda_\pi$

\[
F_\pi(Q^2) = \frac{1}{1 + Q^2 / \Lambda_\pi^2}
\]

Horn et al, PRL97, 192001, 2006

The model of: T.K. Choi, K.J. Kong, B.G. Yu [arXiv: 1508.00969] may soon become available as a second way to extract $F_\pi$ from data.
**F\_π-2 VGL p(e,e’π\^+)n model check**

- To check whether VGL Regge model properly accounts for:
  - \(\pi^+\) production mechanism.
  - spectator nucleon.
  - other off-shell (t-dependent) effects.

extract \(F_\pi\) values for each \(t\)-bin separately, instead of one value from fit to all \(t\)-bins.

![Error band based on fit to all \(t\)-bins.]

- Deficiencies in model may show up as \(t\)-dependence in extracted \(F_\pi(Q^2)\) values.
- Resulting \(F_\pi\) values are insensitive (<2%) to \(t\)-bin used.
- Lends confidence in applicability of VGL model to the kinematical regime of the JLab data, and the validity of the extracted \(F_\pi(Q^2)\) values.

G. M. Huber et al., PRC 78(2008)045203.

Only statistical and \(t\)-uncorrelated systematic uncertainties shown.
\( \pi^-/\pi^+ \) data to check \( t \)-channel dominance

- \( \pi^+ \) \( t \)-channel diagram is purely isovector (G-parity conservation).

\[
R_L = \frac{\sigma_L[n(e,e'\pi^-)p]}{\sigma_L[p(e,e'\pi^+)n]} = \left| A_V - A_S \right|^2 / \left| A_V + A_S \right|^2
\]

- Isoscalar backgrounds (such as \( b_1(1235) \) contributions to \( t \)-channel) will dilute ratio.

- Qualitatively in agreement with our \( F_\pi \)-1 analysis:
  - We found evidence for small additional contribution to \( \sigma_L \) at \( W=1.95 \) GeV not taken into account by the VGL model.
  - We found no evidence for this contribution at \( W=2.2 \) GeV.

![Graphs showing \( \sigma_L/\sigma_L(e^+e^-) \) and \( \sigma_T/\sigma_T(e^+e^-) \) vs. \( -t \) for different \( Q^2 \) and \( W \) values.]

**Vranckx-Ryckebusch Model:**
- VR extend VGL with hard DIS process of virtual photons off nucleons.

[PRC 89(2014)025203]

\( R_L = 0.8 \) consistent with \( |A_S/A_V| < 6\% \).
Current Experimental Status

JLab results in a region of $Q^2$ where model calculations begin to diverge.

Bethe-Salpeter/Dyson-Schwinger:
- B-S equation is conventional formalism for relativistic bound states.
- D-S expansion in terms of dressed quark propagators, consistent w/ confinement.
- Model parameters fixed from $f_\pi$ and $m_\pi$, then $r_\pi$ and $F_\pi$ predicted.

Constituent Quark Model:
[C-W. Hwang, Phys.Rev.D 64(2001)034001]
- Relativistic constituent quarks and effective interaction on the light front
- Consistent treatment of quark spins.
- Wave function parameters determined from $f_\pi$ and $\pi^0\rightarrow\gamma\gamma$ decay width, then charge and transition FF’s and $\pi^0$ branching ratios predicted.

For details see: G.M. Huber et al., PRC 78(2008)045203.

Dispersion Relation with QCD Constraint:
- Uses constraints posed by causality and analyticity to relate the timelike and spacelike domains of the pion form factor on the complex plane.
- Additional constraints, such as behavior of $F_\pi$ in asymptotic region, imposed.
12 GeV era – Hall C with SHMS and HMS

**SHMS:**
- 11 GeV/c Spectrometer
- Partner of existing 7 GeV/c HMS

**MAGNETIC OPTICS:**
- Point-to-Point QQQD for easy calibration and wide acceptance.
- Horizontal bend magnet allows acceptance at forward angles (5.5°)

**Detector Package:**
- Drift Chambers
- Hodoscopes
- Cerenkovs
- Calorimeter
- All derived from existing HMS/SOS detector designs

**Well-Shielded Detector Enclosure**

**Rigid Support Structure**
- Rapid & Remote Rotation
- Provides Pointing Accuracy & Reproducibility demonstrated in HMS

**SHMS:**
- dQQQD

**SHMS (New)**

**HMS:**
- QQQD

**HMS (Exists)**

**SHMS = Super High Momentum Spectrometer**

**HMS = High Momentum Spectrometer**
JLab 12 GeV upgrade will allow measurement of $F_\pi$ up to $Q^2 = 6 \rightarrow 8.5$ GeV$^2$

No other facility worldwide can perform this measurement.

New overlap point at $Q^2=1.6$ will be closer to pole to constrain $-t_{min}$ dependence.

New low $Q^2$ point will provide best comparison of the electroproduction extraction of $F_\pi$ vs elastic $\pi^+e^-$ data.

Approved with “A” scientific rating and identified by JLab PAC41 as “high impact”. (E12-06-101: GH, D. Gaskell, spokespersons)

Extension to $Q^2=8.5$ GeV$^2$ submitted to PAC44 (GH, D. Gaskell, T. Horn, spokespersons)
The Charged Kaon – a second QCD test case

- In the hard scattering limit, pQCD predicts that the $\pi^+$ and $K^+$ form factors will behave similarly

\[
\frac{F_K(Q^2)}{F_\pi(Q^2)} \xrightarrow{Q^2 \to \infty} f_K^2
\]

- It is important to compare the magnitudes and $Q^2$-dependences of both form factors.
**Measurement of $K^+$ Form Factor**

- Similar to $\pi^+$ form factor, elastic $K^+$ scattering from electrons used to measure charged kaon for factor at low $Q^2$
  
  [Amendolia et al, PLB 178, 435 (1986)]

- Can “kaon cloud” of the proton be used in the same way as the pion to extract kaon form factor via $p(e,e'K^+)$△?

- Kaon pole further from kinematically allowed region.

- Can we demonstrate that the “pole” term dominates the reaction mechanism?
Isolate Exclusive Final States via Missing Mass

\[ M_X = \sqrt{(E_{\text{det}} - E_{\text{init}})^2 - (p_{\text{det}} - p_{\text{init}})^2} \]

- SHMS+HMS missing mass resolution expected to be very good.
- Spectrometer coincidence acceptance allows for simultaneous studies of \( \Lambda \) and \( \Sigma^0 \) channels.
- Kaon-pole dominance test through
  \[ \frac{\sigma_L(\gamma^* p \rightarrow K^+ \Sigma^0)}{\sigma_L(\gamma^* p \rightarrow K^+ \Lambda^0)} \]
- Should be similar to ratio of \( g_{pK\Lambda}^2 / g_{pK\Sigma}^2 \) coupling constants if t-channel exchange dominates.

Simulation at \( Q^2=2.0 \text{ GeV}^2 \), \( W=3.0 \) and high \( \varepsilon \)
Projected Uncertainties for $K^+$ Form Factor

- First measurement of $F_K$ well above the resonance region.

- Measure form factor to $Q^2=3$ GeV$^2$ with good overlap with elastic scattering data.
  - Limited by $-t<0.2$ GeV$^2$ requirement to minimize non-pole contributions.

- Data will provide an important second $qar{q}$ system for theoretical models, this time involving a strange quark.

Scheduled as an early SHMS commissioning experiment: LT-separation. (E12-09-011: T. Horn, G. Huber and P. Markowitz, spokespersons)
## Hall C Meson Form Factors Timeline

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
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<tbody>
<tr>
<td>SHMS superconducting magnet installation and testing</td>
<td>Until Sept, 2016</td>
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<tr>
<td>SHMS front detector installation</td>
<td>July – Aug, 2016</td>
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<tr>
<td>SHMS commissioning with beam</td>
<td>Dec 6 – 21, 2016</td>
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<td>First physics-quality runs in Hall C</td>
<td>Feb 11 – May 7, 2017</td>
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<td>First p(e,e’K+) run</td>
<td>May 31 – June 20, 2017</td>
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### Data Reconstruction Software (hcana)
- Z. Ahmed (PDF), completed

### SHMS Detector Checkout & Commissioning
- W. Li (Ph.D.), S. Basnet (M.Sc.), work underway

### p(e,e’K+)Λ Kaon Form Factor
- L/T commissioning experiment (2017 – 2018)

### Pion Form Factor and π+ QCD-Scaling Experiments
- Interleaved run-plans (2018 – 2020)

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