Meson and Nucleon Form Factors

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Science at the Luminosity Frontier: JLab @ 22 GeV
December 9, 2024

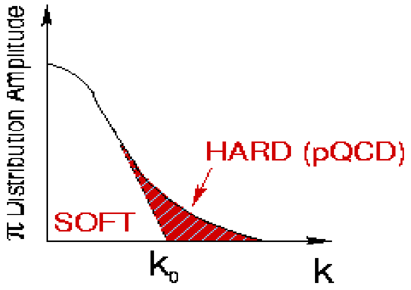
Supported by:
NSERC CRSNG
SAPIN-2021-00026

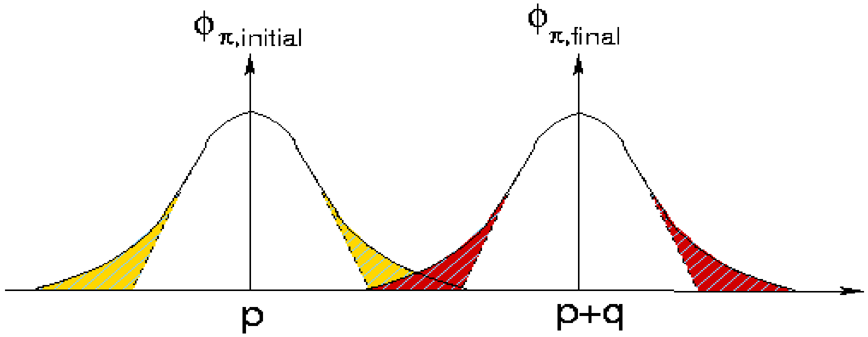
Meson Form Factors

Simple $q\bar{q}$ valence structure of mesons presents the ideal testing ground for our understanding of bound quark systems.

In quantum field theory, the form factor is the overlap integral:

$$F_\pi(Q^2)=\int \phi_\pi^*(p)\phi_\pi(p+q)dp$$





The meson wave function can be separated into φ_π^{soft} with only low momentum contributions ($k<k_0$) and a hard tail φ_π^{hard}.

While φ_π^{hard} can be treated in pQCD, φ_π^{soft} cannot.

From a theoretical standpoint, the study of the Q^2-dependence of the form factor focuses on finding a description for the hard and soft contributions of the meson wave-function.

A program of study unique to Hall C (until completion of EIC)

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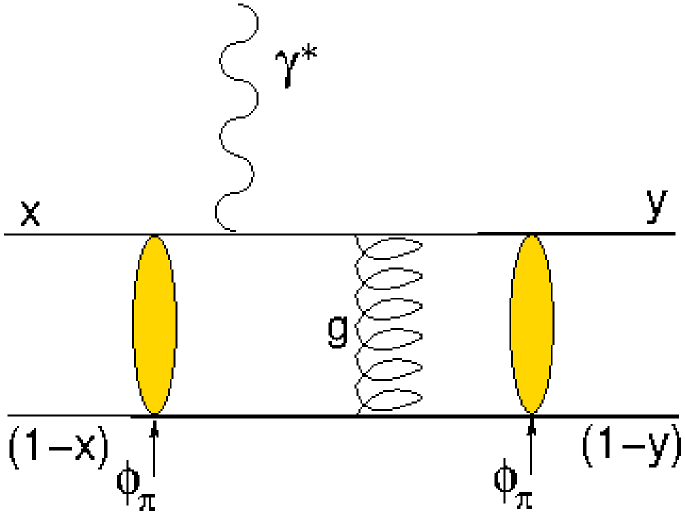
pQCD and the Charged Pion Form Factor

At large Q^2, perturbative QCD (pQCD) can be used

$$F_\pi(Q^2)=\frac{4\pi C_F\alpha_S(Q^2)}{Q^2}\left|\sum_{n=0}^{\infty}a_n\left(\log\left(\frac{Q^2}{\Lambda^2}\right)\right)^{-\gamma_n}\right|^2\left[1+O\left(\alpha_s(Q^2),\frac{m}{Q}\right)\right]$$

at asymptotically high Q^2, only the hardest portion of the wave function remains

$$\phi_\pi(x)\xrightarrow[Q^2\to\infty]{}\frac{3f_\pi}{\sqrt{n_c}}x(1-x)$$



and F_π takes the very simple form

$$F_\pi(Q^2)\xrightarrow[Q^2\to\infty]{}\frac{16\pi\alpha_s(Q^2)f_\pi^2}{Q^2}$$

where f_π=92.4 MeV is the $\pi^+\to\mu^+\nu$ decay constant.

G.P. Lepage, S.J. Brodsky, Phys.Lett. **87B**(1979)359

This prediction only relies on asymptotic freedom in QCD, *i.e.* $(\partial\alpha_S/\partial\mu)<0$ as $\mu\to\infty$

Pion Form Factor at Finite Q^2

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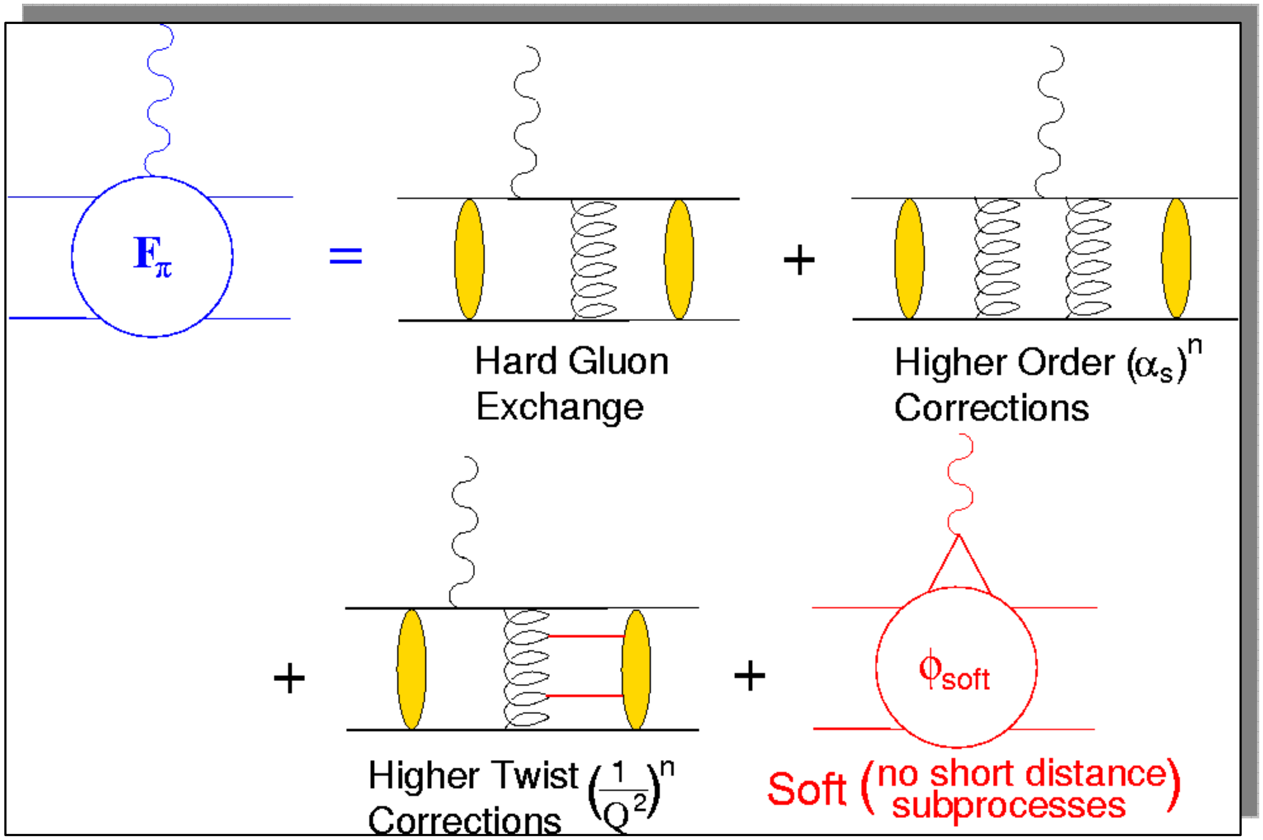
- At finite momentum transfer, higher order terms contribute.
- Calculation of higher order, “hard” (short distance) processes difficult, but tractable.

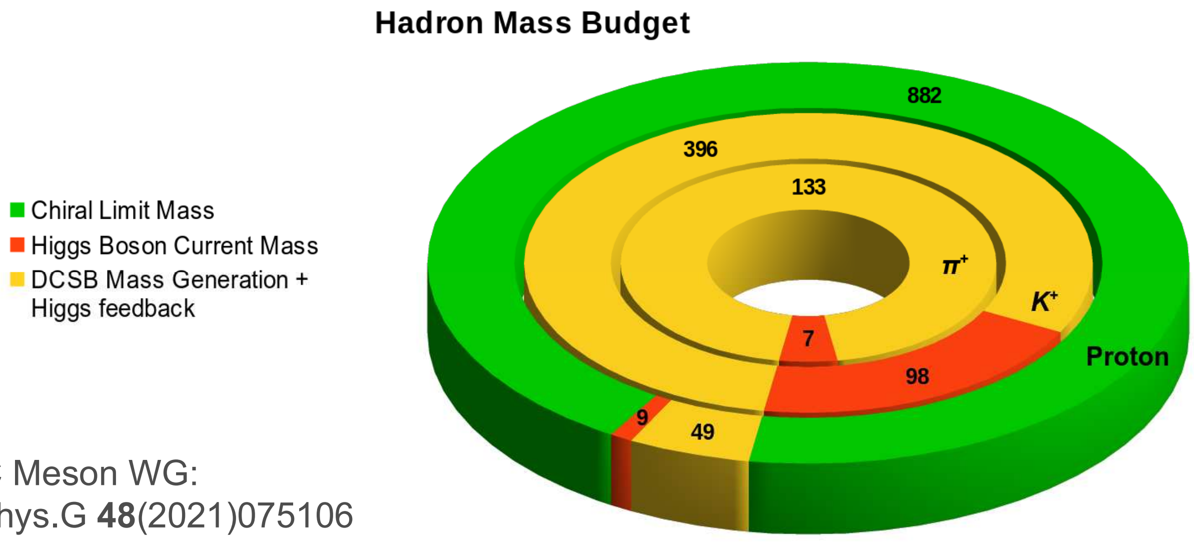


Q^2F_π should behave like $\alpha_s(Q^2)$ even for moderately large Q^2.

→ Pion form factor seems to be best tool for experimental study of nature of the quark-gluon coupling constant renormalization.

[A.V. Radyushkin, JINR 1977, arXiv:hep–ph/0410276]

Contrasts in Hadron Mass Budgets



EIC Meson WG:
J.Phys.G **48**(2021)075106

Stark Differences between proton, K^+, π^+ mass budgets

- Due to Emergent Hadronic Mass (EHM), Proton mass large in absence of quark couplings to Higgs boson (chiral limit).
- Conversely, and yet still due to EHM and DCSB, *K* and π are massless in chiral limit (i.e. they are Goldstone bosons of QCD).
- The mass budgets of these crucially important particles demand interpretation.
- Equations of QCD stress that any explanation of the proton's mass is incomplete, unless it simultaneously explains the light masses of QCD's Goldstone bosons, the π and *K*.

Synergy: Emergent Mass and π^+ Form Factor

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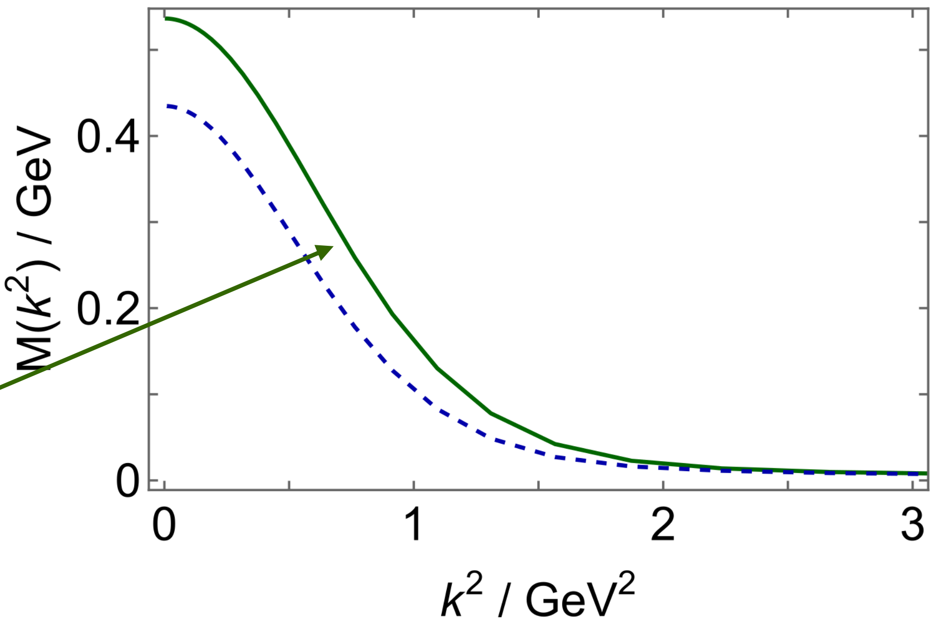
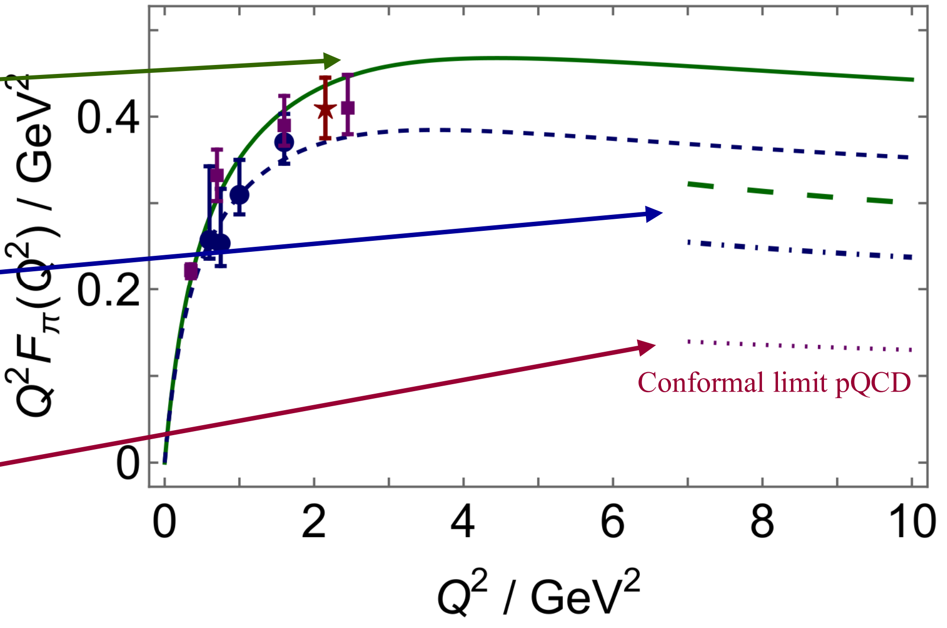
At empirically accessible energy scales, π^+ form factor is sensitive to emergent mass scale in QCD

- **Two dressed–quark mass functions distinguished by amount of DCSB**
  - DCSB emergent mass generation is 20% stronger in system characterized by solid green curve, which is more realistic case



- **$F_\pi(Q^2)$ obtained with these mass functions**
  - r_π=0.66 fm with solid green curve
  - r_π=0.73 fm with solid dashed blue curve
- **$F_\pi(Q^2)$ predictions from QCD hard scattering formula, obtained with related, computed pion PDAs**
- **QCD hard scattering formula, using conformal limit of pion's twist–2 PDA**

$$\phi_\pi^{cl}(x) = 6x(1-x)$$



Chen, et al., PRD **98**(2018)091505(R); Aguilar et al, EPJA **55**(2019)190

Why Meson Form Factors?

- **The π^+ form factor is our best hope of observing experimentally QCD's transition from soft QCD to hard QCD**
  - This transition is expected to occur at a much lower Q^2 than for the proton
- ***K*$^+$ form factor:**
  - How does meson structure change when *s* quark is substituted for *d* quark?
  - At what Q^2 will the K^+ to π^+ form factor ratio converge to the value predicted by QCD?
- **The normalization of π^+ and K^+ form factors at high Q^2 is sensitive to quark and gluon energy contributions to emergent hadronic mass**
  - A comparison of π^+ and K^+ form factors over a wide range of Q^2 will provide unique information relevant to our understanding of hadronic mass generation

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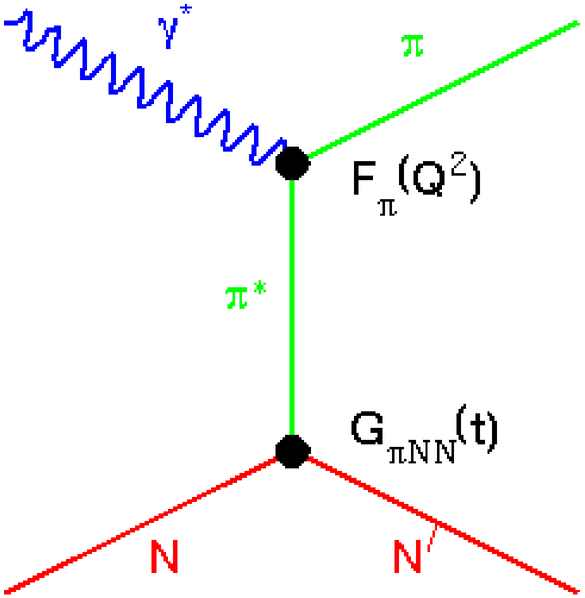
Measurement of π^+ Form Factor – Larger Q^2

At larger Q^2, F_π must be measured indirectly using the "pion cloud" of the proton via pion electroproduction $p(e,e'\pi^+)n$

$$|p\rangle = |p\rangle_0 + |n\pi^+\rangle + \ldots$$

- At small $-t$, the pion pole process dominates the longitudinal cross section, σ_L
- In Born term model, F_π^2 appears as,

$$\boxed{\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t-m_\pi^2)} g_{\pi NN}^2(t) F_\pi^2(Q^2,t)}$$



Drawbacks of this technique

1. Isolating σ_L experimentally challenging
2. Theoretical uncertainty in form factor extraction.

K^+ pole is further in the unphysical region, uncertainties will be larger

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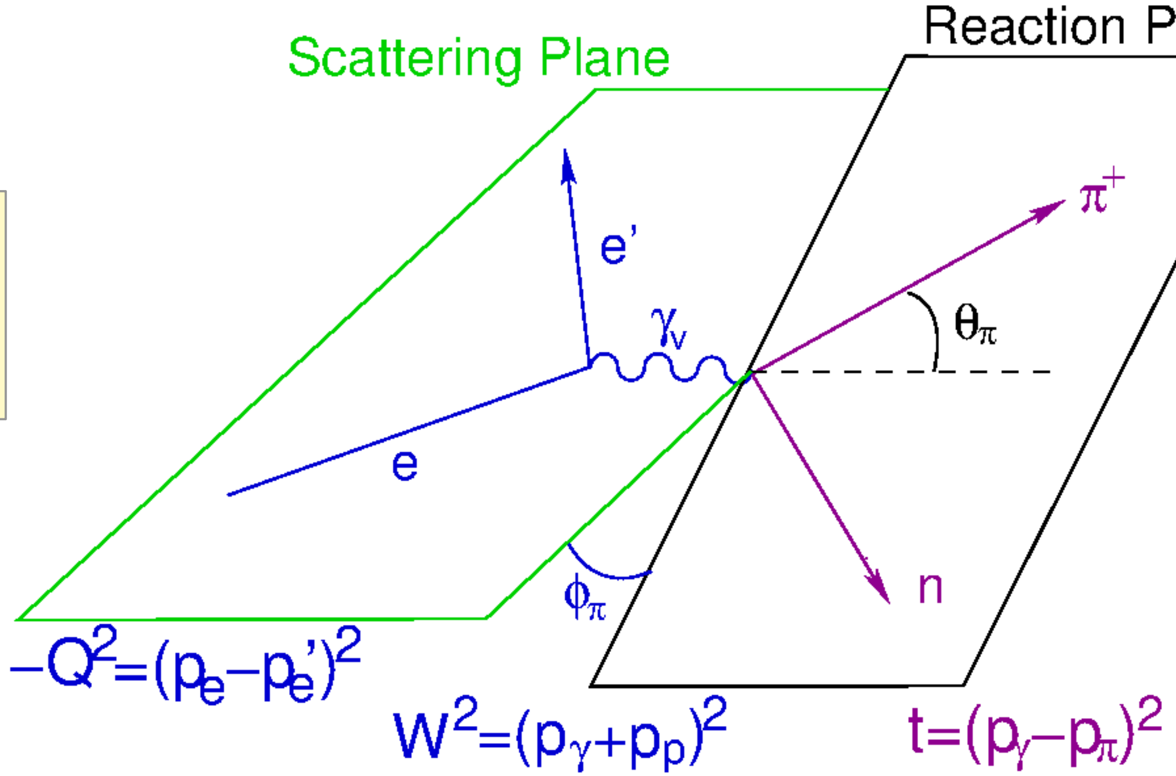
$$2\pi\frac{d^2\sigma}{dt\,d\phi}=\varepsilon\frac{d\sigma_L}{dt}+\frac{d\sigma_T}{dt}+\sqrt{2\varepsilon(\varepsilon+1)}\frac{d\sigma_{LT}}{dt}\cos\phi+\varepsilon\frac{d\sigma_{TT}}{dt}\cos 2\phi$$

Virtual-photon polarization:

$$\varepsilon=\left(1+2\frac{(E_e-E_{e'})^2+Q^2}{Q^2}\tan^2\frac{\theta_{e'}}{2}\right)^{-1}$$



- L-T separation required to separate σ_L from σ_T
- Need to take data at smallest available $-t$, so σ_L has maximum contribution from the π^+ pole
- Need to measure t–dependence of σ_L at fixed Q^2,W

L/T–separation error propagation

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Error in $d\sigma_L/dt$ is magnified by $1/\Delta\varepsilon$, where $\Delta\varepsilon=(\varepsilon_{Hi}-\varepsilon_{Low})$

→ To keep magnification factor <5x, need $\Delta\varepsilon>0.2$, preferably more!

$$\frac{d^2\sigma}{dt\,d\phi}=\varepsilon\frac{d\sigma_L}{dt}+\frac{d\sigma_T}{dt}+\sqrt{2\varepsilon(\varepsilon+1)}\,\frac{d\sigma_{LT}}{dt}\cos\phi_\pi+\varepsilon\frac{d\sigma_{TT}}{dt}\cos 2\phi_\pi$$

$$\frac{\Delta\sigma_L}{\sigma_L}=\frac{1}{(\varepsilon_1-\varepsilon_2)}\left(\frac{\Delta\sigma}{\sigma}\right)\sqrt{(R+\varepsilon_1)^2+(R+\varepsilon_2)^2}\qquad \text{where } R=\frac{\sigma_T}{\sigma_L}$$

$$\frac{\Delta\sigma_T}{\sigma_T}=\frac{1}{(\varepsilon_1-\varepsilon_2)}\left(\frac{\Delta\sigma}{\sigma}\right)\sqrt{\varepsilon_1^{\,2}\left(1+\frac{\varepsilon_2}{R}\right)^2+\varepsilon_2^{\,2}\left(1+\frac{\varepsilon_1}{R}\right)^2}$$

The relevant quantities for F_π extraction are R and $\Delta\varepsilon$

$$\frac{d\sigma_L}{dt}\propto\frac{-tQ^2}{(t-m_\pi^2)}\,g^2_{\pi NN}(t)\,F_\pi^2(Q^2,t)$$

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Experimental Issues

What is being measured?

- Scattered electron and π^+/K^+ in coincidence with the two high performance spectrometers in Hall C
  - High momentum, forward angle (5.5°) meson detection is required, with good Particle ID to separate π^+, K^+, p
  - Good momentum resolution required to reconstruct crucial kinematics, such as M_{miss}, Q^2, W, t
  - Need to measure the longitudinal cross section $d\sigma_L/dt$ needed for form factor extraction

The role of 22 GeV electrons?

- Allows access to higher Q^2
- Expanded range of virtual photon polarization $\Delta\varepsilon=(\varepsilon_{HI}-\varepsilon_{LO})$, leading to reduced errors in the extraction of $d\sigma_L/dt$
  - Uncertainty in $\sigma_L \sim 1/\Delta\varepsilon$, desire $\Delta\varepsilon>0.2$, preferably larger

Upgrade Scenarios Considered

Phase 1: higher energy beam, keep HMS+SHMS largely as is, with relatively small DAQ and PID upgrades

- See what can be accomplished in "cost effective approach"
- Goal: to extend kinematic range of L/T–separated measurements beyond what is possible with JLab 11 GeV beam

Phase 2: Replace HMS with a new Very High Momentum Spectrometer (VHMS) to enable measurements utilizing full 22 GeV beam energy

- See what extra physics can be obtained for significantly larger investment

Hall C instrumentation has been optimized for specifically such studies

Phase 1 Scenario: π^+ Form Factor

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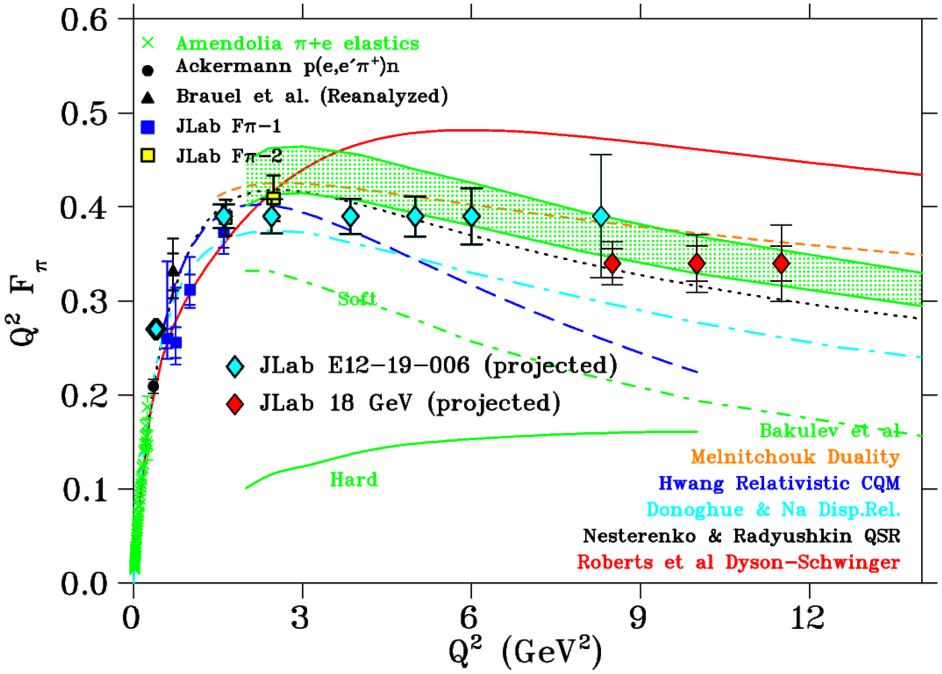
- 7.2 GeV/c HMS & 11.0 GeV/c SHMS allow a lot of kinematic flexibility, **with no major upgrades**
  - Success depends on good K^+/π^+ separation in SHMS at high momenta, likely requires a modest aerogel detector upgrade
  - Experiment could be done as soon as beam energy is available!
  - Maximum beam energy and higher Q^2 reach constrained by sum of HMS+SHMS maximum momenta



- F_π assumes same statistics as acquired in PionLT experiment
- Inner error bar is projected statistical and systematic error
- Outer error bar also includes a model uncertainty in the form factor extraction, added in quadrature

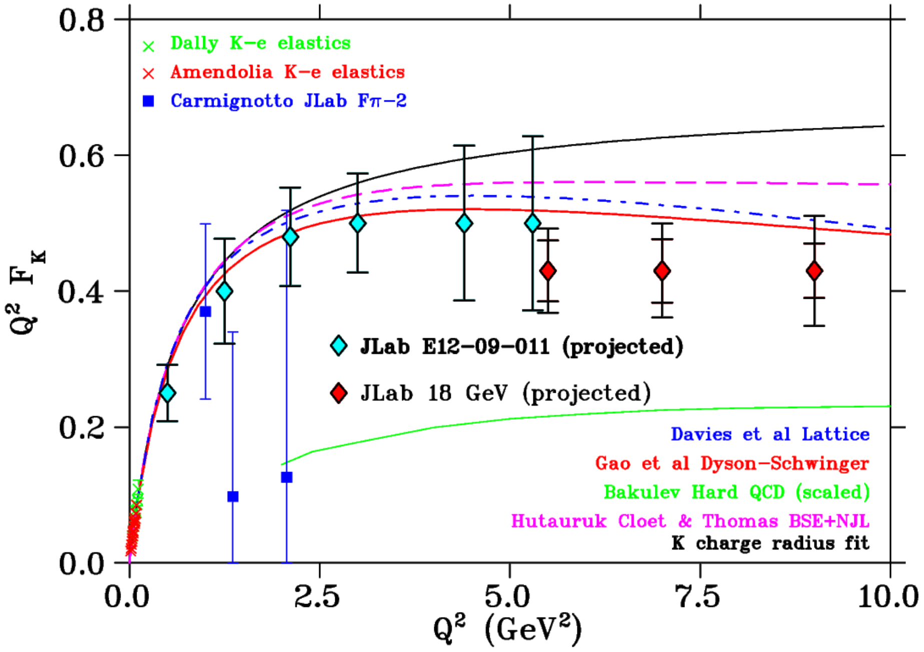
	10.6 GeV	18.0 GeV	Improvement in $\delta F_\pi/F_\pi$
Q^2=8.5	Δε=0.22	Δε=0.40	17.9%→4.6%
Q^2=10.0	New high quality F_π data		
Q^2=11.5	Larger F_π extraction uncertainty due to higher $-t_{min}$		

- Since quality L–T separations are impossible at EIC (can't access ε<0.95) this extension of L–T separated data considerably increases F_π data set overlap between JLab and EIC

Phase 1 Scenario: K^+ Form Factor

- 7.2 GeV/c HMS & 11.0 GeV/c SHMS allow a lot of kinematic flexibility
- Maximum beam energy and higher Q^2 reach constrained by sum of HMS+SHMS maximum momenta
- Success depends on good K^+/π^+ separation in SHMS at high momenta, likely requires a modest aerogel detector upgrade
- Counting rates are roughly 10x lower than pion form factor measurement

	10.6 GeV	16.0 GeV	Improvement in $\delta F_K/F_K$
Q^2=5.5	$\Delta\varepsilon$=0.33	$\Delta\varepsilon$=0.40	17.9%→10.4%
Q^2=7.0	New high quality F_K data		
Q^2=9.0	Larger F_K extraction uncertainty due to higher $-t_{min}$		



- Projected running times extremely long
- F_K errors uncertain, as E12–09–011 analysis not yet completed
- **F_K feasibility studies at EIC are ongoing, but we already know that such measurements there are exceptionally complex.**
- **JLab measurements likely a complement to those at EicC.**

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Phase 2 Scenario: π^+ Form Factor

- **Replace HMS with VHMS for π^+, use SHMS for e'**
  - Assume θ_{min}=5.5°, θ_{open}=15.0°
  - VHMS: ΔΩ, ΔP/P similar SHMS
- P_{VHMS}=15.0 GeV/c is sufficient, constrained by max beam energy
- θ_{VHMS}~5.5° allows improved Δε, but does not affect maximum Q^2 reach
- θ_{SHMS}<12.0°, P_{SHMS}>9.0 not used
- Dramatic increase in upper Q^2 11.5 → 15.0 GeV^2
- Error bars for Q^2=8.5–11.5 GeV^2 substantially decrease due to smaller $-t_{min}$ (better $R=\sigma_T/\sigma_L$) and shorter running times
- Q^2=15.0 GeV^2 point would be "expensive" in terms of running time, but it would likely have very high scientific priority

- **Feasible scenario for Phase 2 Upgrade**

p(e,e'π^+)n Kinematics

E_{beam}	θ_{SHMS} (e')	P_{SHMS} (e')	$\theta_{q(VHMS)}$ (π^+)	P_{VHMS} (π^+)	Time FOM
Q^2=8.5 W=4.18 $-t_{min}$=0.15 Δε=0.28					
17.0	21.39	3.63	5.55	13.29	20.5
22.0	12.15	8.63	7.62	13.29	1.8
Q^2=10.0 W=4.08 $-t_{min}$=0.21 Δε=0.30					
17.0	24.49	3.27	5.52	13.62	53.3
22.0	13.46	8.27	7.85	13.62	4.3
Q^2=11.5 W=3.95 $-t_{min}$=0.29 Δε=0.31					
17.0	27.34	3.03	5.55	13.82	124.8
22.0	14.66	8.03	8.12	13.82	9.3
Q^2=13.0 W=3.96 $-t_{min}$=0.35 Δε=0.25					
18.0	27.55	3.18	5.54	14.63	209.5
22.0	16.49	7.18	7.69	14.63	24.4
Q^2=15.0 W=3.73 $-t_{min}$=0.52 Δε=0.26					
18.0	30.24	3.06	5.73	14.66	560
22.0	17.88	7.06	8.07	14.66	65.7

JLab L–T Separations in the EIC Era

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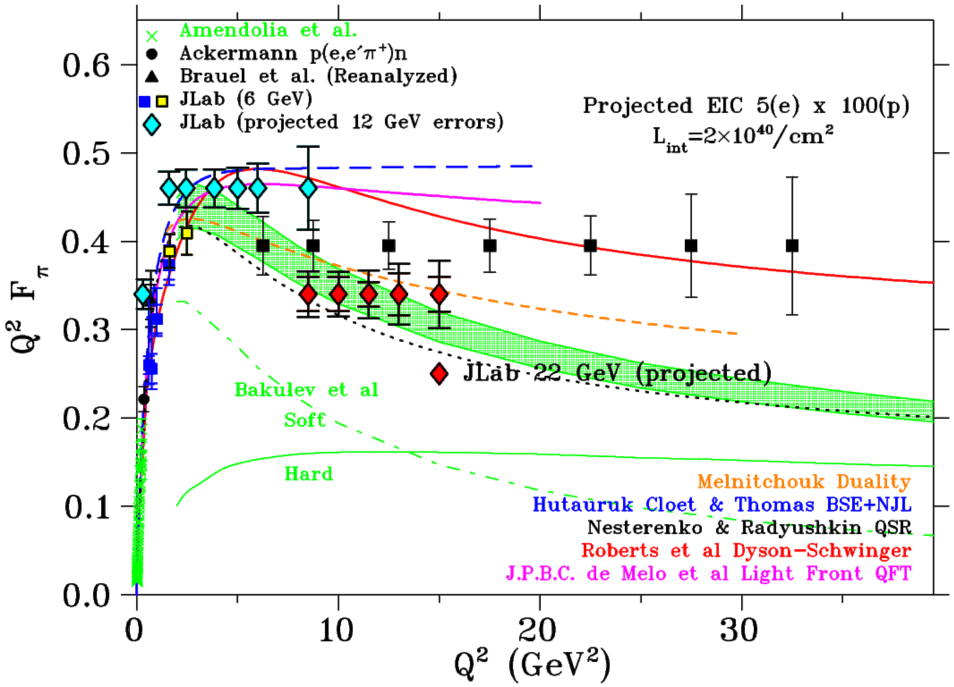


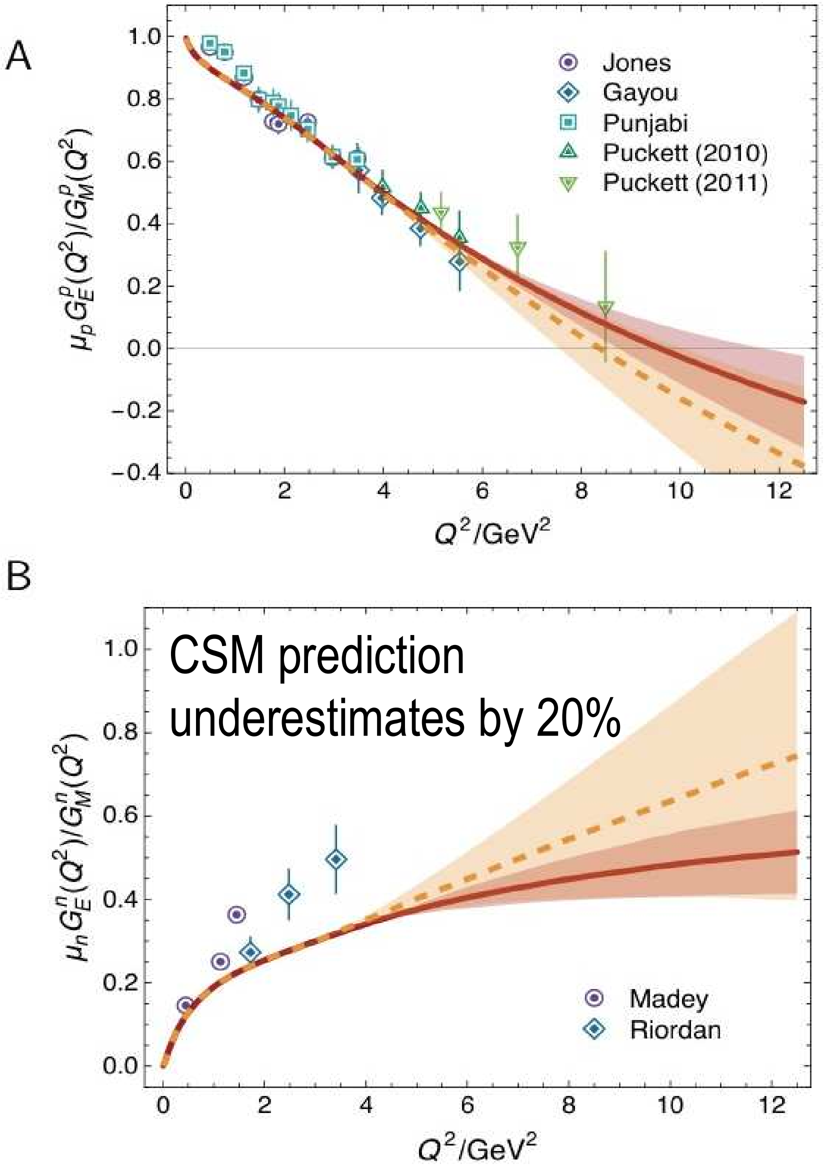
JLab 22 GeV			EIC 5x100		
Q^2	W	$-t_{min}$	Q^2	W	$-t_{min}$
8.5	4.18	0.15	5.9	7.71	0.02
10.0	4.08	0.21	8.5	8.06	0.02
11.5	3.95	0.29	11.7	8.53	0.02
13.0	3.96	0.35	16.9	8.88	0.03
15.0	3.73	0.52	22.5	9.03	0.05

- **Quality L/T-separations impossible at EIC (can't access $\varepsilon<0.95$)**
  - High *W* can be accessed, so $-t_{min}$ is low
  - Projected T/L ratio: ~0.05 at $-t_{min}$, to 0.5 at $-t=0.3$ GeV^2
  - Model must be used to correct for σ_T contribution
  - Model must be validated from other data, adds systematic uncertainty

- **JLab will remain ONLY source of quality L–T separated data!**
  - $-t_{min}$ is higher, but true L/T separation is performed
  - Overlap of F_π data set between JLab and EIC needed to constrain EIC model uncertainty

Nucleon charge/magnetic FF ratios

- **Proton electric form factor possesses a zero**
  - **$Q^2 = 8.86^{+1.93}_{-0.86}$ GeV^2**
- **Neutron electric form factor is positive definite**
  - **$G_E^n(Q^2) > G_E^p(Q^2)$ on $Q^2 > 4.7$ GeV^2**
  - On this domain, electric form factor of charge-neutral neutron is larger than that of charge-one proton
  - A remarkable, non-intuitive result!
- Verification of this is within JLab reach
  - perhaps already in Gen-II data?
- Curves are Continuum Schwinger Model (CSM)



CSM prediction underestimates by 20%

FIG. 6. Panel A: $\mu_p G_E^p/G_M^p$. Panel B: $\mu_n G_E^n/G_M^n$. SPM I – dashed orange curve within like-coloured band; and SPM II – solid red curve within like-coloured band. Data: proton – Refs. [20–24]; and neutron – Refs. [87, 97].

Courtesy of Craig Roberts cdroberts@nju.edu.cn

Flavor Separation of Charge, Magnetic FF

- **Isospin symmetry limit:**
  - Behaviors of $\mu_p G_E^p/G_M^p$ and $\mu_p G_E^n/G_M^n$ are correlated (e_u=2/3, e_d=–1/3)
  - $G_E^p = e_u G_E^u + e_d G_E^d$
  - $G_E^n = e_u G_E^d + e_d G_E^u$
- **G_E^p possesses a zero because**
  - although remaining positive, G_E^u/G_M^p falls steadily with increasing Q^2
  - while G_E^d/G_M^p>0 and approximately constant
- **G_E^n predicted to NOT exhibit a zero at high Q^2 because**
  - e_u>0, G_E^d/G_M^p large & positive
  - $|e_d G_E^u|$ always < $e_u G_E^{pd}$

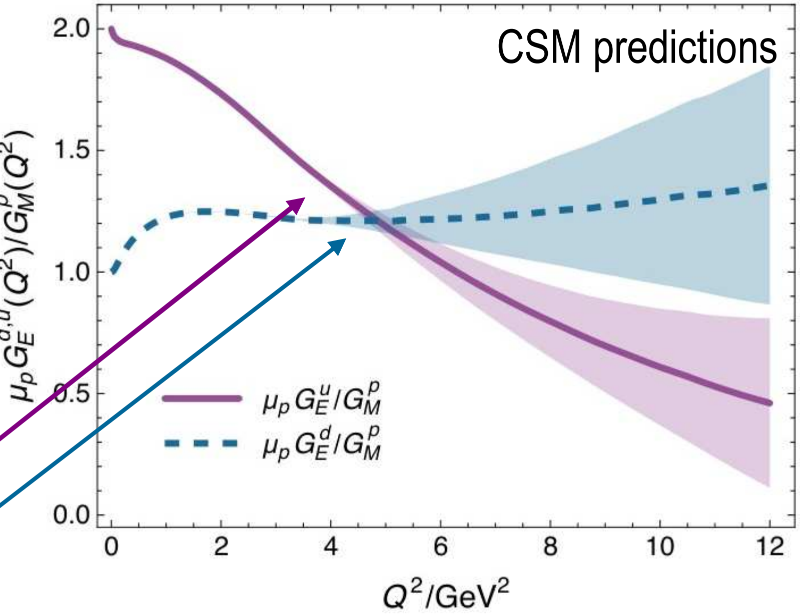
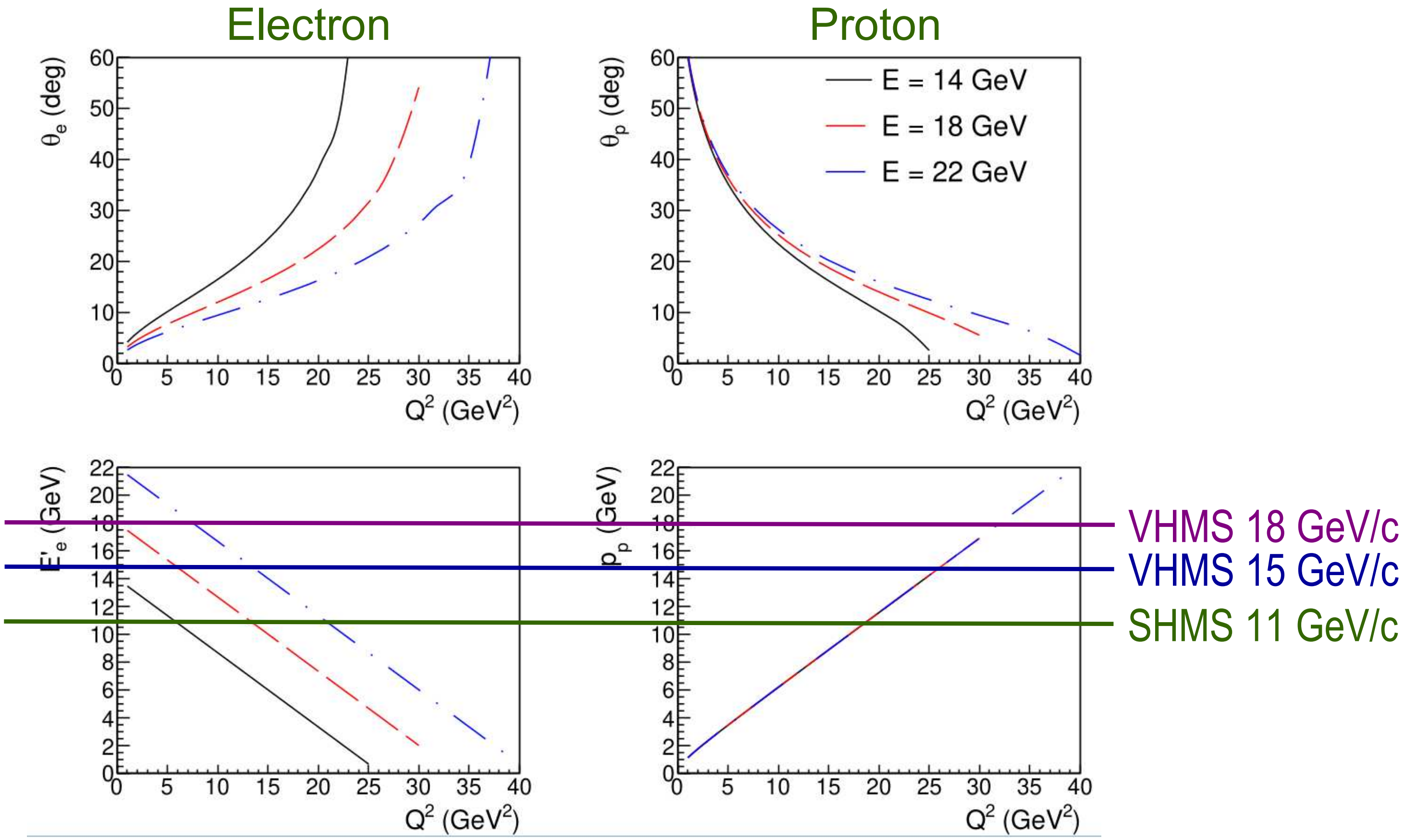


Figure 8: Flavour separation of the charge and magnetisation form factors, with each function normalised by G_M^p in order to highlight their differing Q^2-dependence.

- **For nucleons, discovering QCD scaling and scaling violations requires Q^2>18 GeV^2**
- **Each feature is a sensitive expression of emergent phenomena in QCD**

Courtesy of Craig Roberts cdroberts@nju.edu.cn

Elastic *eN* Kinematics @ 14, 18, 22 GeV



- **Scattering angles well matched** to acceptances of a variety of JLab detectors (CLAS12, SoLID, HMS+SHMS, SBS+BigBite
- **High proton momentum is a challenge:**
    - SHMS could measure up to Q^2=18 GeV^2
    - VHMS could allow Q^2=26-32 GeV^2, depending on capability
    - Large acceptance spectrometers would have resolution challenges at high Q^2

Courtesy of Andrew Puckett andrew.puckett@uconn.edu

Elastic *eN* Count Rate Projections

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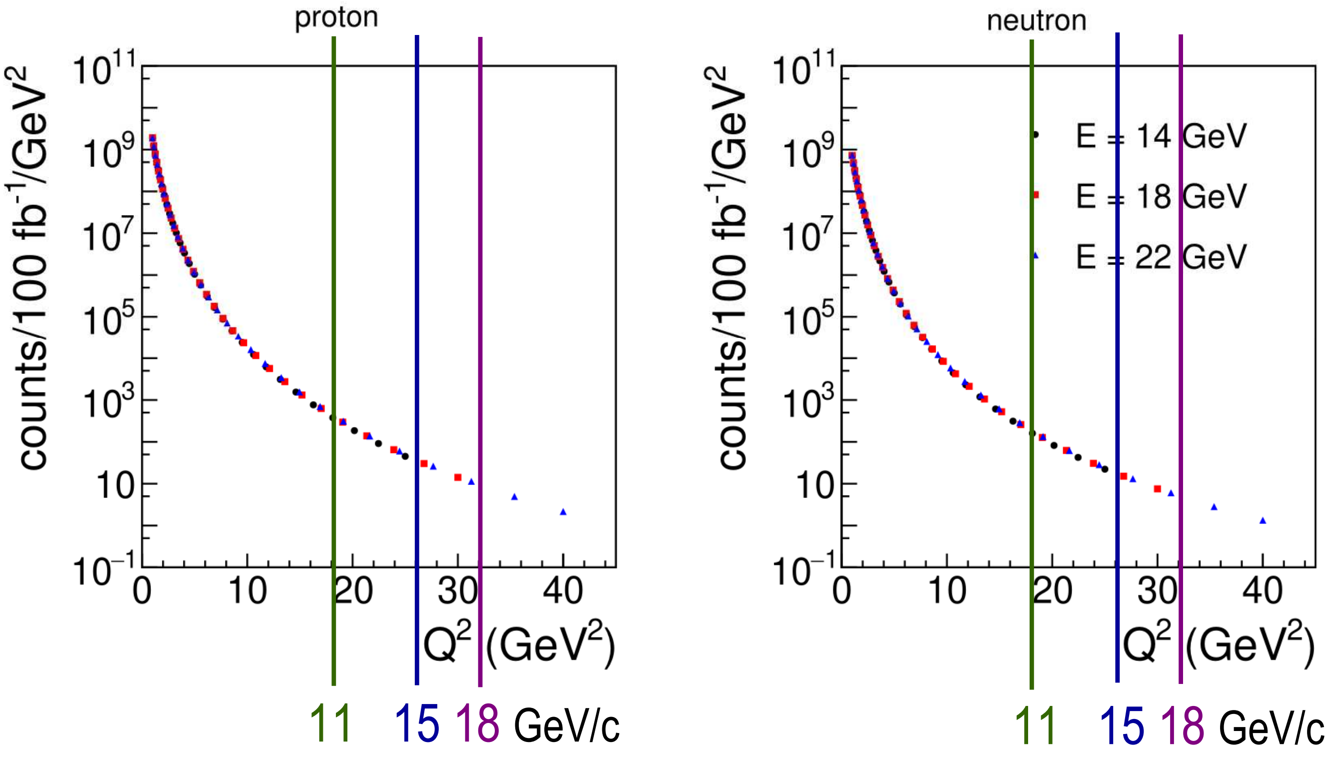

	Luminosity (cm^{-2}s^{-1}, e-N)	**fb^{-1} / day**
NH_3/ND_3 **polarized p/n**	10^{35}	8.64
^{3}He SEOP **polarized n**	10^{37}	8.64
LH_2/LD_2 **unpolarized**	$10^{38\text{-}39}$	8600 – 86000

- **Event rate per unit integrated luminosity for 14, 18, 22 GeV beam**
  - Assumes Q^2 bin width ≈ Q^2 spacing between points
  - Assumes 2π azimuthal acceptance
- **JLab provides lots of count rate up to maximum accessible Q^2**
  - EIC best case: 100 fb^{-1}/year

Courtesy of Andrew Puckett andrew.puckett@uconn.edu

Summary

- **Meson and nucleon form factors are fundamental structure observables that are intimately linked to many open questions in QCD**
  - QCD's transition from soft to hard degrees of freedom
  - Unraveling emergent phenomena in QCD

- **22 GeV upgrade enables a significant expansion of Q^2 reach**
  - $\pi+$ form factor up to Q^2=11.5(15) GeV^2 with SHMS(VHMS)
  - K^+ form factor up to Q^2=9 GeV^2 with SHMS
    - Unfortunately, running times are very long
  - Proton/Neutron up to Q^2=18(26-32) GeV^2

- **Understanding how QCD explains the emergence of hadron mass and structure requires investment in facilities that can deliver precision data on mesons, nucleons, and beyond**

F_π PionLT Projected Uncertainties

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$p(e,e'\pi^+)n$ Kinematics

E_{beam} (GeV)	ε	θ_{HMS} (e')	P_{HMS} (e')	$\theta_{q(SHMS)}$ (π^+)	P_{SHMS} (π^+)	LH_2 Run hrs	Online #Events /t-bin	$\delta\sigma_{UNS}$ (stat & est uncorrel syst unc)	$\delta\sigma_L$ (stat & est uncorrel+ correlated syst unc)	δF_π (stat & est syst unc / incl est model unc)
Q^2=5.0 W=2.95 $-t_{min}$=0.21 R(VR)=T/L=0.79										
8.0	0.22	44.37	1.10	6.17	6.72	64	3.9k	2.3%	10.8%	5.4% / 12.2%
10.6	0.60	20.57	3.72	10.47	6.72	36	6.4k	2.1%		
Q^2=6.0 W=3.19 $-t_{min}$=0.37 R(VR)=T/L=0.70										
9.2	0.18	47.04	1.03	5.06	8.04	166	4.1k	2.3%	10.7%	5.4% / 12.2%
10.6	0.40	28.18	2.40	7.65	8.04	97	6.4k	2.1%		
Q^2=8.5 W=2.79 $-t_{min}$=0.55 R(VR)=T/L=1.71										
9.2	0.15	58.53	0.97	5.44	7.91	615	2.5k	2.6%	35.8%	17.9% / 33.5%
10.6	0.38	34.11	2.34	8.67	7.91	101	1.9k	2.4%		

- σ_{UNS} errors include 0.6% pt-pt and 1.6% t-correlated syst unc from E12-06-101 proposal
- σ_L errors include 3.3% scale systematic uncertainty from E12-06-101 proposal

pion_sigl_errors_22summer.ods

Phase 1 Scenario: π^+ Form Factor

- 7.2 GeV/c HMS & 11.0 GeV/c SHMS allow a lot of kinematic flexibility, **with no major upgrades**
  - Experiment could be done as soon as beam energy is available!
  - Maximum beam energy and higher Q^2 reach constrained by sum of HMS+SHMS maximum momenta
  - Q^2=8.5 and 11.5 Time FOM similar to PionLT Q^2=6.0 and 8.5 points

$p(e,e'\pi^+)n$ Kinematics

E_{beam}	θ_{HMS} (e')	P_{HMS} (e')	$\theta_{q(SHMS)}$ (π^+)	P_{SHMS} (π^+)	Time FOM
Q^2=8.5 W=3.64 $-t_{min}$=0.24 $\Delta\varepsilon$=0.40					
13.0	34.30	1.88	5.29	10.99	64.7
18.0	15.05	6.88	8.94	10.99	2.2
Q^2=10.0 W=3.44 $-t_{min}$=0.37 $\Delta\varepsilon$=0.40					
13.0	37.78	1.83	5.56	10.97	122.7
18.0	16.39	6.83	9.57	10.97	4.5
Q^2=11.5 W=3.24 $-t_{min}$=0.54 $\Delta\varepsilon$=0.29					
14.0	31.73	2.75	7.06	10.96	82.4
18.0	17.70	6.75	10.05	10.96	8.8

	10.6 GeV	18.0 GeV	Improvement in $\delta F_\pi/F_\pi$
Q^2=8.5	$\Delta\varepsilon$=0.22	$\Delta\varepsilon$=0.40	17.9%→4.6%
Q^2=10.0	New high quality F_π data		
Q^2=11.5	Larger F_π extraction uncertainty due to higher $-t_{min}$		

- **Since quality L–T separations are impossible at EIC (can't access ε<0.95) this extension of L–T separated data considerably increases F_π data set overlap between JLab and EIC**

F_π Hall C Phase 1 (existing HMS+SHMS)

Garth Huber, huberg@uregina.ca

$p(e,e'\pi^+)n$ Kinematics

E_{beam} (GeV)	ε	θ_{HMS} (e')	P_{HMS} (e')	$\theta_{q(SHMS)}$ (π^+)	P_{SHMS} (π^+)	Time FOM	#Events /t-bin	$\delta\sigma_{UNS}$ (stat & est uncorrel syst unc)	$\delta\sigma_L$ (stat & est uncorrel+ correlated syst unc)	δF_π (stat & est syst unc / incl est model unc)
Q^2=8.5 W=3.64 $-t_{min}$=0.24 R(VR)=T/L=0.56										
13.0	0.25	34.30	1.88	5.29	10.99	64.7	3.2k	2.5%	9.1%	4.6% / 9.6%
18.0	0.65	15.05	6.88	8.94	10.99	2.2	4.8k	2.2%		
Q^2=10.0 W=3.44 $-t_{min}$=0.37 R(VR)=T/L=0.80										
13.0	0.24	37.78	1.83	5.56	10.97	122.7	2.8k	2.56%	11.1%	5.6% / 12.7%
18.0	0.64	16.39	6.83	9.57	10.97	4.5	4.2k	2.3%		
Q^2=11.5 W=3.24 $-t_{min}$=0.54 R(VR)=T/L=1.20										
14.0	0.22	31.73	2.75	7.06	10.96	82.4	2.0k	2.8%	16.3%	5.4% / 18.8%
18.0	0.63	17.70	6.75	10.05	10.96	8.8	2.0k	2.8%		

- σ_{UNS} errors include 0.6% pt-pt and 1.6% t-correlated syst unc from E12-06-101 proposal
- σ_L errors include 3.3% scale systematic uncertainty from E12-06-101 proposal

pion_sigl_errors_higherE.ods

Phase 1 Scenario: K^+ Form Factor

- 7.2 GeV/c HMS & 11.0 GeV/c SHMS allow a lot of kinematic flexibility
- Maximum beam energy and higher Q^2 reach constrained by sum of HMS+SHMS maximum momenta
- Success depends on good K^+/π^+ separation in SHMS at high momenta, likely requires a modest aerogel detector upgrade
- Counting rates are roughly 10x lower than pion form factor measurement

	10.6 GeV	16.0 GeV	Improvement in $\delta F_K/F_K$
Q^2=5.5	$\Delta\varepsilon$=0.33	$\Delta\varepsilon$=0.40	17.9%→10.4%
Q^2=7.0	New high quality F_K data		
Q^2=9.0	Larger F_K extraction uncertainty due to higher $-t_{min}$		

$p(e,e'K^+)\Lambda$ Kinematics

E_{beam}	θ_{HMS} (e')	P_{HMS} (e')	$\theta_{q(SHMS)}$ (K^+)	P_{SHMS} (K^+)	Time FOM
Q^2=5.5 W=3.56 $-t_{min}$=0.32 $\Delta\varepsilon$=0.40					
11.0	30.69	1.79	5.50	8.84	746
16.0	12.92	6.79	9.18	8.84	150
Q^2=7.0 W=3.90 $-t_{min}$=0.33 $\Delta\varepsilon$=0.29					
14.0	25.16	2.64	5.51	10.98	620
18.0	13.91	6.64	7.85	10.98	192
Q^2=9.0 W=3.66 $-t_{min}$=0.54 $\Delta\varepsilon$=0.30					
14.0	29.17	2.54	5.98	10.97	964
18.0	15.90	6.54	8.69	10.97	350

- **F_K feasibility studies at EIC are ongoing, but we already know that such measurements there are exceptionally complex.**
- **JLab measurements likely a complement to those at EicC.**

F_K Hall C Phase 1 (existing HMS+SHMS)

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$p(e,e'K^+)\Lambda$ Kinematics										
E_{beam} (GeV)	ε	θ_{HMS} (e')	P_{HMS} (e')	$\theta_{q(SHMS)}$ (K^+)	P_{SHMS} (K^+)	Time FOM	#Events /t-bin	$\delta\sigma_{UNS}$ (stat & est uncorrel syst unc)	$\delta\sigma_L$ (stat & est uncorrel+ correlated syst unc)	δF_π (stat & est syst unc / incl est model unc)
Q^2=5.5 W=3.56 $-t_{min}$=0.22 R(VR)=T/L=1.17										
11.0	0.36	30.69	1.79	5.50	8.84	746	3.0k	2.5%	20.8%	10.4% / 20.4%
16.0	0.64	12.92	6.79	9.18	8.84	150	3.0k	2.6%		
Q^2=7.0 W=3.90 $-t_{min}$=0.33 R(VR)=T/L=1.19										
14.0	0.34	25.16	2.64	5.51	10.98	620	2.5k	2.6%	21.6%	10.8% / 21.0%
18.0	0.63	13.91	6.64	7.85	10.98	192	2.5k	2.6%		
Q^2=9.0 W=3.66 $-t_{min}$=0.54 R(VR)=T/L=1.63										
14.0	0.32	29.17	2.54	5.98	10.97	964	2.0k	2.8%	28.1%	9.4% / 28.0%
18.0	0.62	15.60	6.54	8.69	10.97	350	2.0k	2.8%		

- σ_{UNS} errors include 0.6% pt-pt and 1.6% t-correlated syst unc from E12-06-101 proposal
- σ_L errors include 3.3% scale systematic uncertainty from E12-06-101 proposal

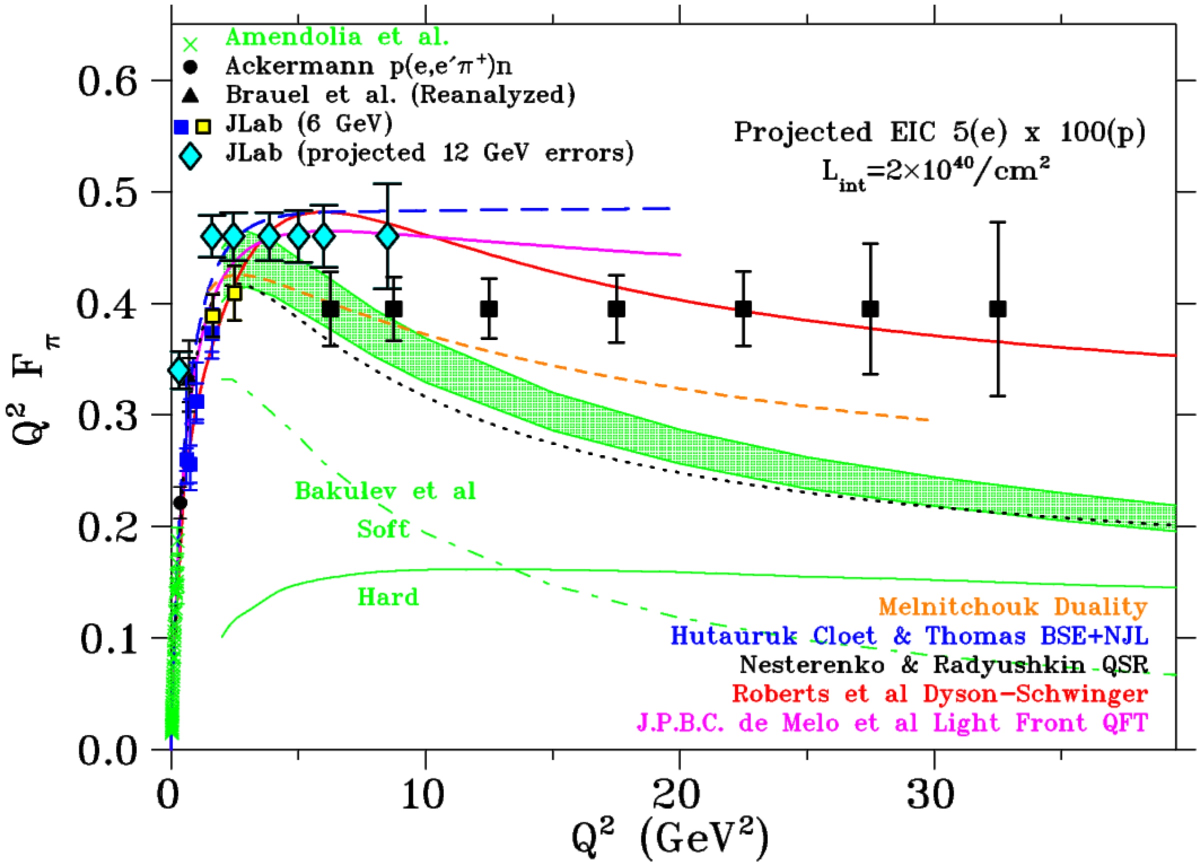
pion_sigl_errors_higherE.ods

F_π Hall C Phase 2 (new VHMS for π^+, SHMS for e')

E_{beam} (GeV)	ε	θ_{SHMS} (e')	P_{SHMS} (e')	$\theta_{q(VHMS)}$ (π^+)	P_{VHMS} (π^+)	Time FOM	#Events /t-bin	$\delta\sigma_{UNS}$ (stat & est uncorrel syst unc)	$\delta\sigma_L$ (stat & est uncorrel+ correlated syst unc)	δF_π (stat & est syst unc / incl est model unc)
Q^2=8.5 W=4.18 $-t_{min}$=0.15 R(VR)=T/L=0.37										
17.0	0.39	21.39	3.63	5.55	13.29	20.5	3.2k	2.5%	11.2%	5.6% / 10.6%
22.0	0.67	12.15	8.63	7.62	13.29	1.8	4.8k	2.2%		
Q^2=10.0 W=4.08 $-t_{min}$=0.21 R(VR)=T/L=0.45										
17.0	0.21	24.49	3.27	5.56	13.62	53.3	3.2k	2.5%	11.2%	5.6% / 10.6%
22.0	0.64	13.46	8.27	9.57	13.62	4.3	4.8k	2.2%		
Q^2=11.5 W=3.95 $-t_{min}$=0.29 R(VR)=T/L=0.50										
17.0	0.32	27.34	3.03	5.55	13.82	124.8	2.8k	2.6%	11.3%	5.6% / 12.4%
22.0	0.63	14.66	8.03	8.12	13.82	9.3	4.2k	2.3%		
Q^2=13.0 W=3.96 $-t_{min}$=0.35 R(VR)=T/L=0.80										
18.0	0.32	27.55	3.18	5.54	14.63	209.5	2.4k	2.7%	14.2%	7.0% / 14.3%
22.0	0.57	16.49	7.18	7.69	14.63	24.4	3.6k	2.4%		
Q^2=15.0 W=3.73 $-t_{min}$=0.52 R(VR)=T/L=1.20										
18.0	0.30	30.24	3.06	5.73	14.66	560	2.0k	2.8%	17.9%	6.0% / 18.6%
22.0	0.56	17.88	7.06	8.07	14.66	65.7	2.0k	2.8%		

pion_sigl_errors_higherE.ods

EIC Kinematic Reach (projection)



Assumptions:

- 5(e^-) x 100(p).
- Integrated L=20 fb^{-1}/yr.
- Clean identification of exclusive $p(e,e'\pi^+n)$ events.
- Syst. Unc: 2.5% pt–pt and 12% scale.
- $R=\sigma_L/\sigma_T$ =0.013–0.14 at lowest $-t$ from VR model, and $\delta R=R$ syst. unc. in model subtraction to isolate σ_L.
- π pole dominance at small $-t$ confirmed in ^{2}H π^-/π^+ ratios.

ECCE 2022 projections shown

Projections to be updated soon using latest ePIC detector simulation

Garth Huber, huberg@uregina.ca

F_π EIC 5x100 ε=0.999

Q^2 mean (GeV^2)	W_{mean} (GeV)	$-t_{bin}$ (GeV^2)	R(VR) =T/L	Rate (Hz) $L=10^{34}cm^{-2}s^{-1}$	#Events /t-bin $L_{int}=2x10^{40}cm^{-2}$	$\delta\sigma_L$ (stat & est uncorrel syst / est correlated syst & σ_T subtraction unc)	δF_π (stat and est correlated+ uncorrel syst unc)
5.87	7.71	0.02	0.12	0.29	290k	3.4% / 15.8%	8.4%
		0.30	1.03	0.12	119k	27.2% / 52.2%	
8.48	8.06	0.02	0.073	0.045	45.1k	3.4% / 13.8%	7.4%
		0.30	0.65	0.019	18.9k	17.8% / 41.1%	
11.71	8.53	0.02	0.047	0.012	12.4k	4.4% / 12.8%	7.1%
		0.30	0.42	8.6E-3	8.6k	12.4% / 32.1%	
16.88	8.88	0.06	0.038	3.2E-4	3.5k	7.1% / 12.5%	8.0%
		0.30	0.27	1.2E-3	1.2k	13.8% / 24.4%	
22.02	9.03	0.06	0.027	5.5E-4	550	16.9% / 12.3%	12.6%
		0.30	0.20	3.4E-4	340	21.0% / 20.3%	
27.03	9.09	0.10	0.035	2.0E-4	200	28.3% / 12.5%	20.0%
		0.30	0.16	1.0E-4	100	36.4% / 18.1%	

- Assume 2.5% pt-pt syst error and 12% scale syst error (similar to HERA-H1 pion struct fcn)
- Assume uncertainty in R=T/L (due to lack of L/T sep) is bounded by R, i.e. δR/R=1

fpi_EICerrors_5x100_2204mod.ods

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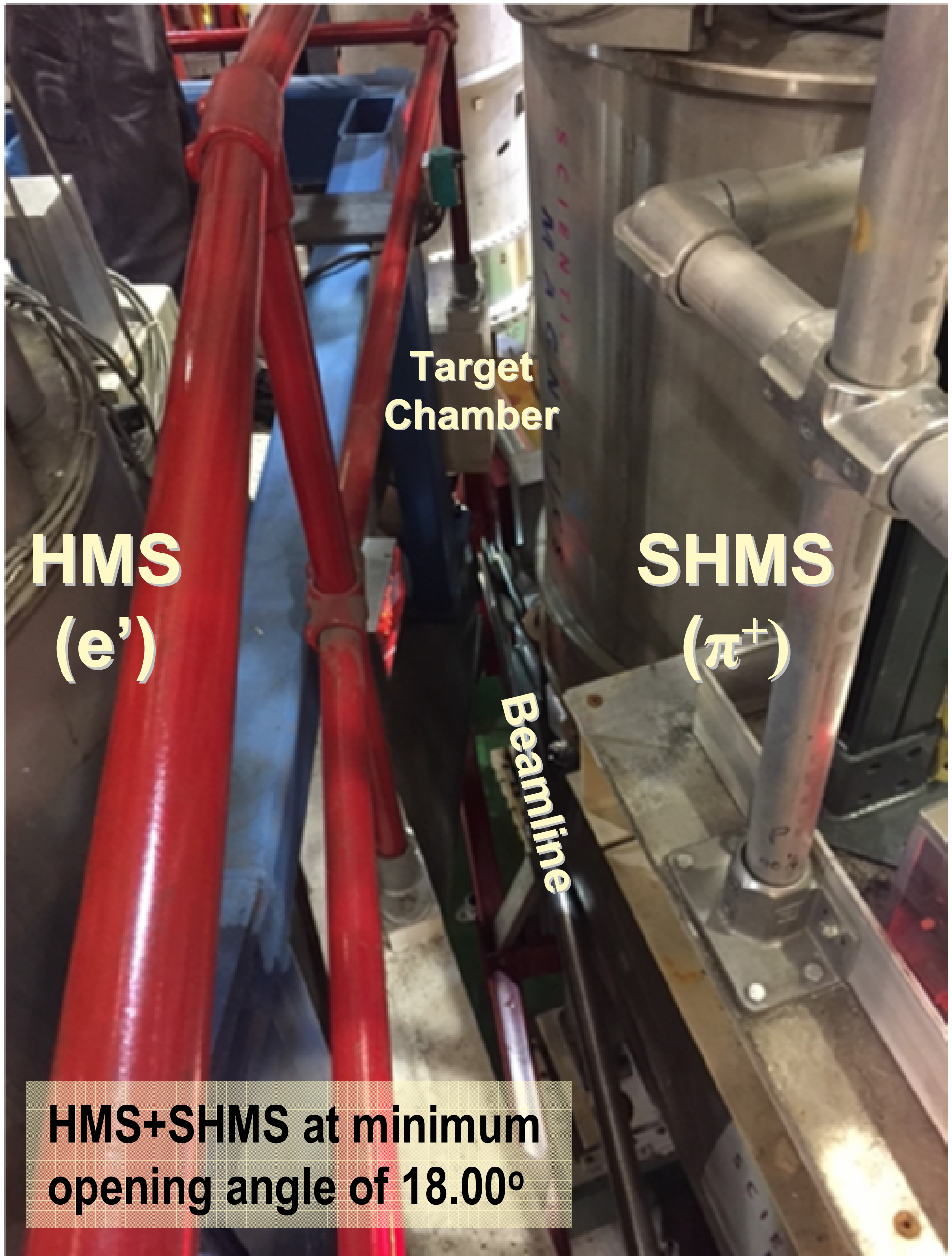
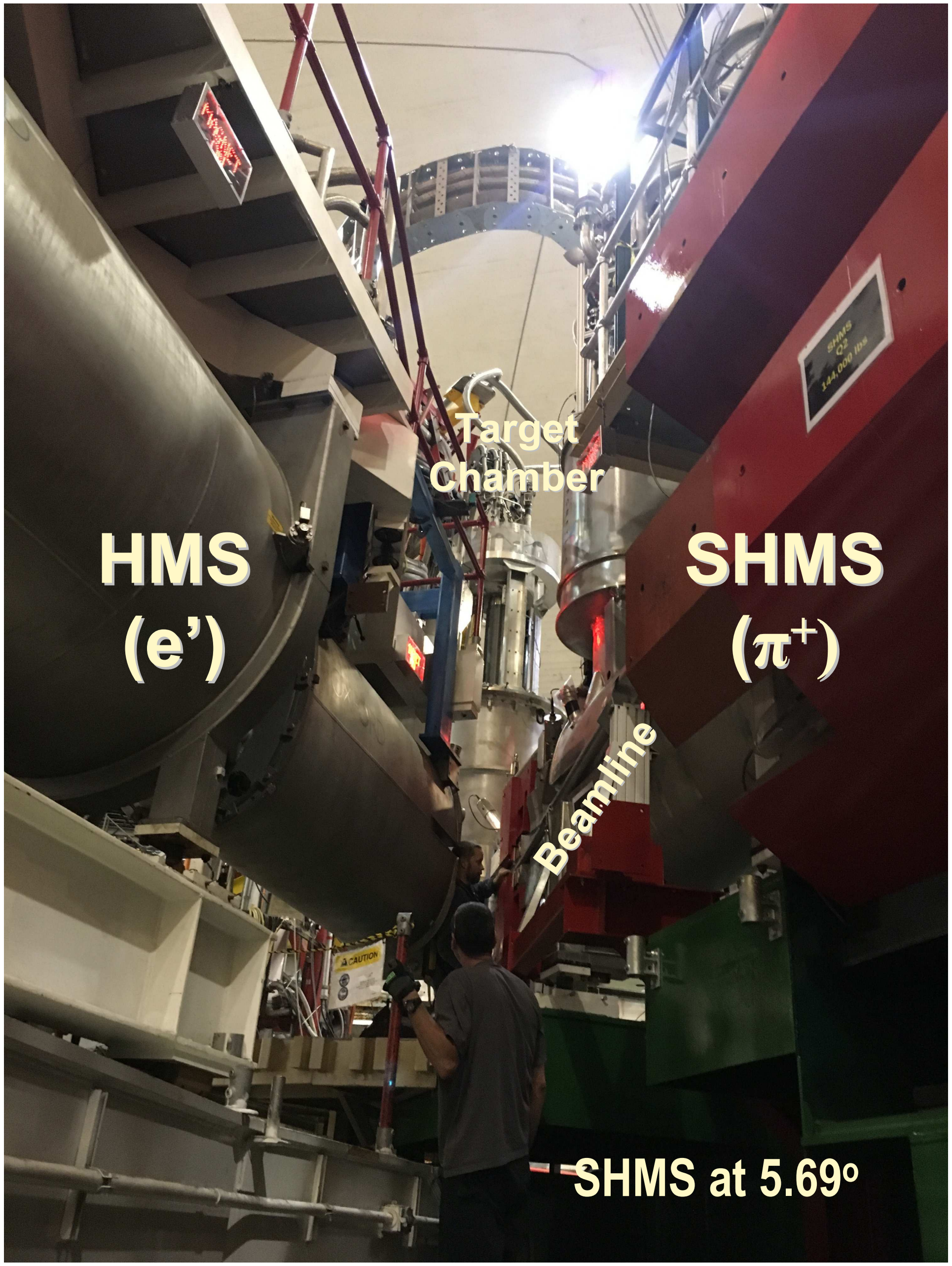
Hall C during Data Taking

π^+/K^+ FF experiments have challenging forward angle requirements



HMS+SHMS at minimum opening angle of 18.00°



SHMS at 5.69°

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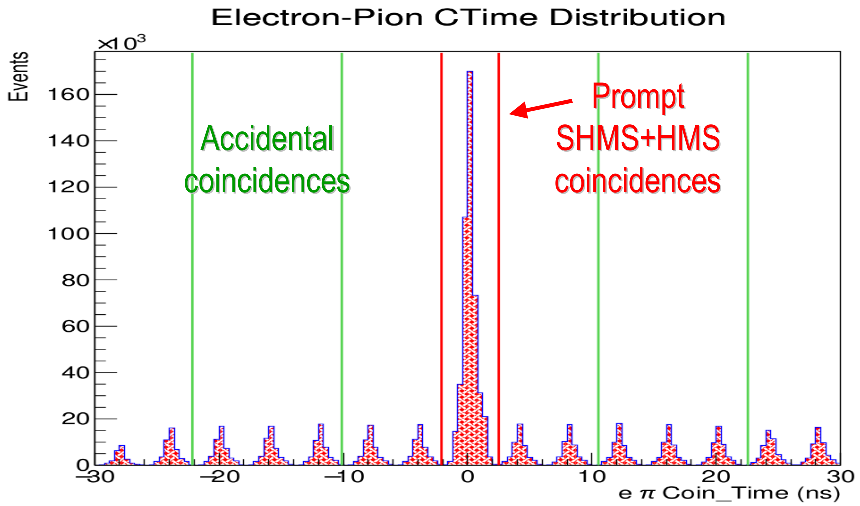
$p(e,e'\pi^+)n$ Event Selection

Coincidence measurement between charged pions in SHMS and electrons in HMS

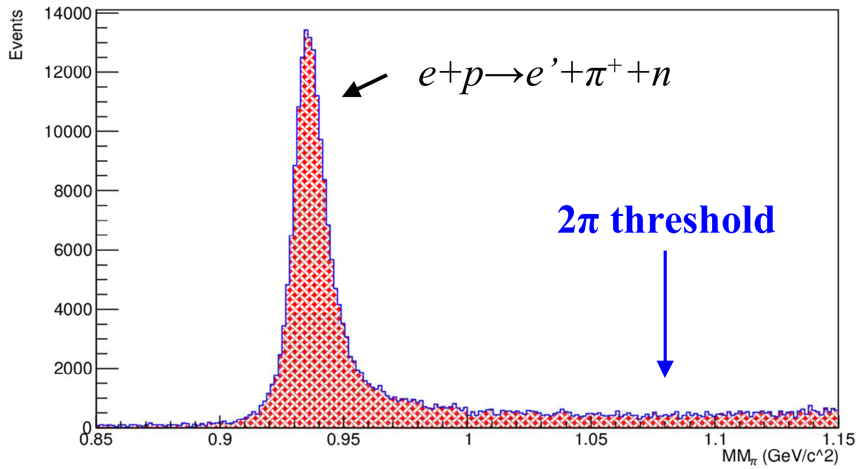
Easy to isolate exclusive channel

- Excellent particle identification
- CW beam minimizes "accidental" coincidences
- Missing mass resolution easily excludes 2–pion contributions

Electron-Pion CTime Distribution



Missing Mass Distribution



PionLT experiment E12–19–006 Data

Q^2=1.60, W=3.08, x= 0.157, ε=0.685

E_{beam}=9.177 GeV, P_{SHMS}=+5.422 GeV/c, θ_{SHMS}= 10.26° (left)

Plots by Muhammad Junaid

Extract $F_\pi(Q^2)$ from JLab σ_L data

Model incorporates π^+ production mechanism and spectator neutron effects:

VGL Regge Model:

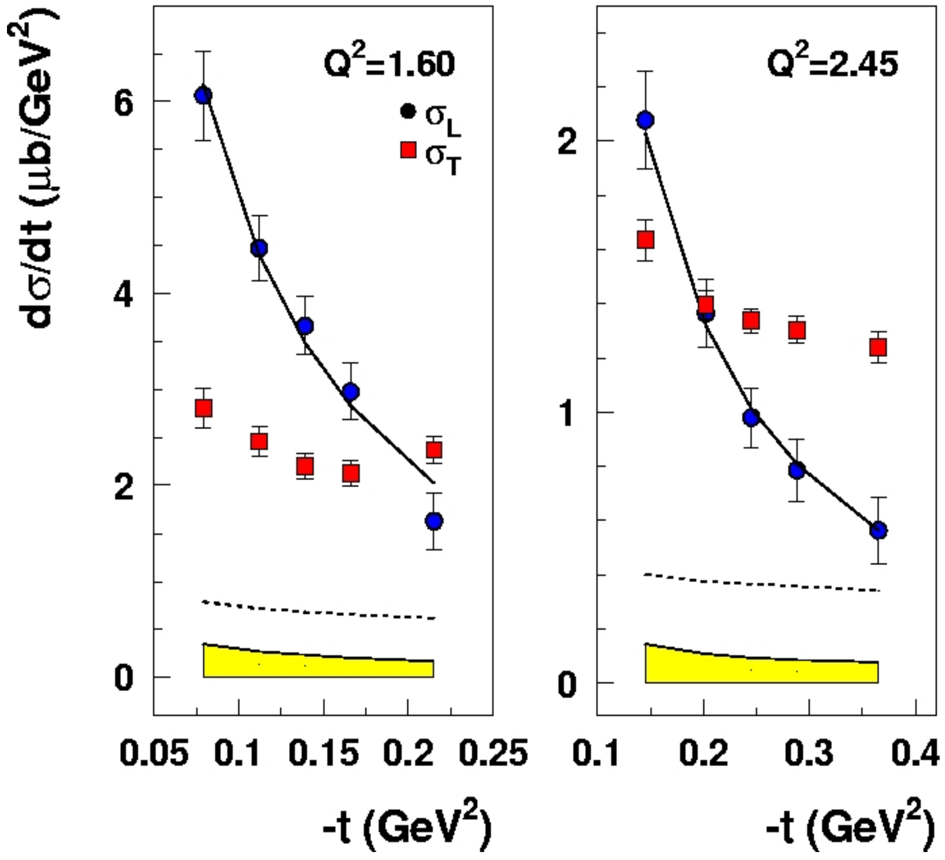
- Feynman propagator $\left(\frac{1}{t-m_\pi^2}\right)$

  replaced by π and ρ Regge propagators.
  - Represents the exchange of a series of particles, compared to a single particle.
- Free parameters: Λ_π, Λ_ρ (trajectory cutoff).

  [Vanderhaeghen, Guidal, Laget, PRC **57**(1998)1454]
- At small $-t$, σ_L only sensitive to F_π

$$\boxed{F_\pi = \frac{1}{1+Q^2/\Lambda_\pi^2}}$$

Fit to σ_L to model gives F_π at each Q^2



Fπ-2 data: T. Horn et al., PRL **97**(2006)192001.

Error bars indicate statistical and random (pt-pt) systematic uncertainties in quadrature.
Yellow band indicates the correlated (scale) and partly correlated (t-corr) systematic uncertainties.

Λ_π^2=0.513, 0.491 GeV², Λ_ρ^2=1.7 GeV².

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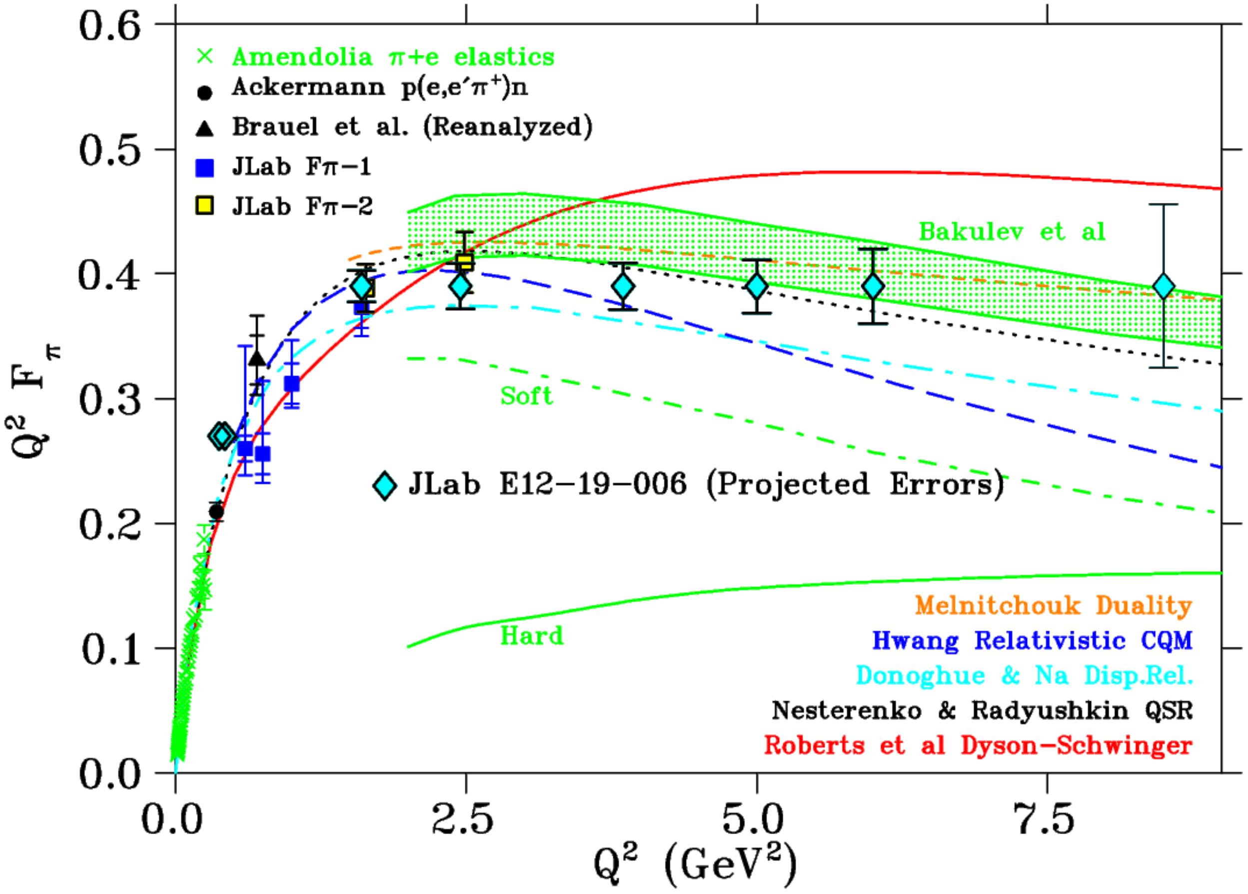
Current and Projected F_π Data

SHMS+HMS will allow measurement of F_π to much higher Q^2.

No other facility worldwide can perform this measurement.

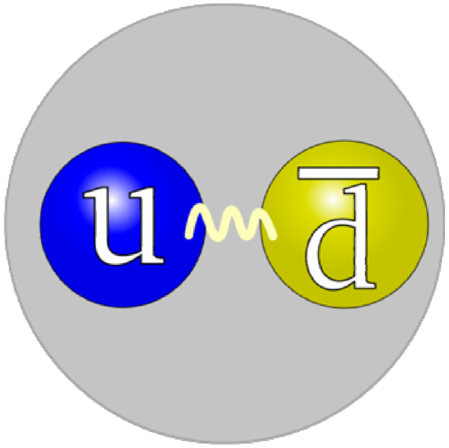
The pion form factor is the clearest test case for studies of QCD's transition from non–perturbative to perturbative regions.



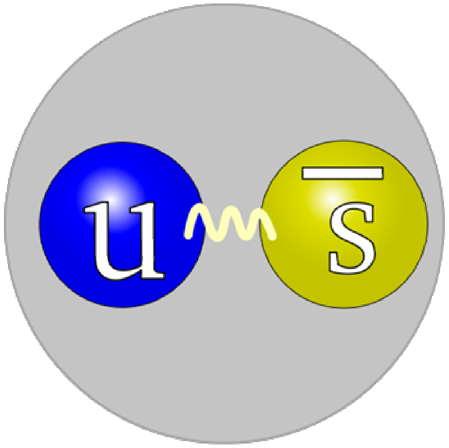
The ~17% measurement of F_π at Q^2=8.5 GeV^2 is at higher $-t_{min}$=0.45 GeV^2

PionLT E12–19–006: D. Gaskell, T. Horn and G. Huber, spokespersons00

The Charged Kaon – a 2nd QCD test case



π^+



K^+

- In the hard scattering limit, pQCD predicts that the π^+ and K^+ form factors will behave similarly

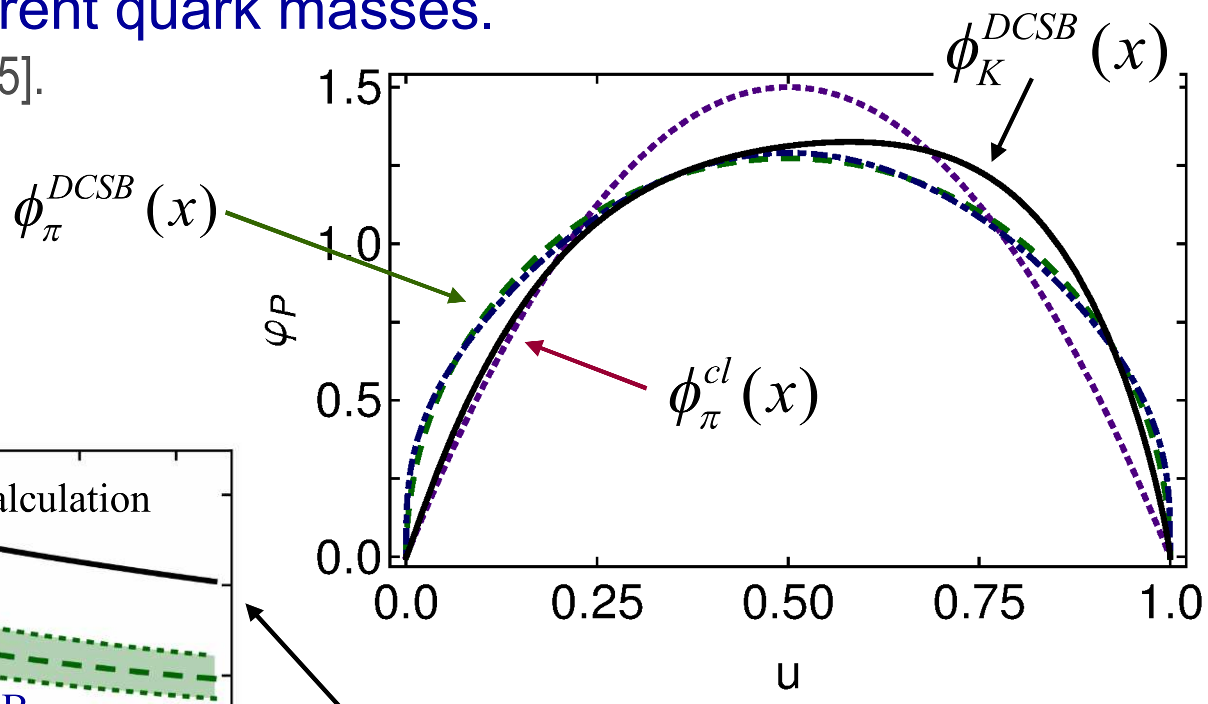
$$\frac{F_K(Q^2)}{F_\pi(Q^2)} \xrightarrow[Q^2\to\infty]{} \frac{f_K^2}{f_\pi^2}$$

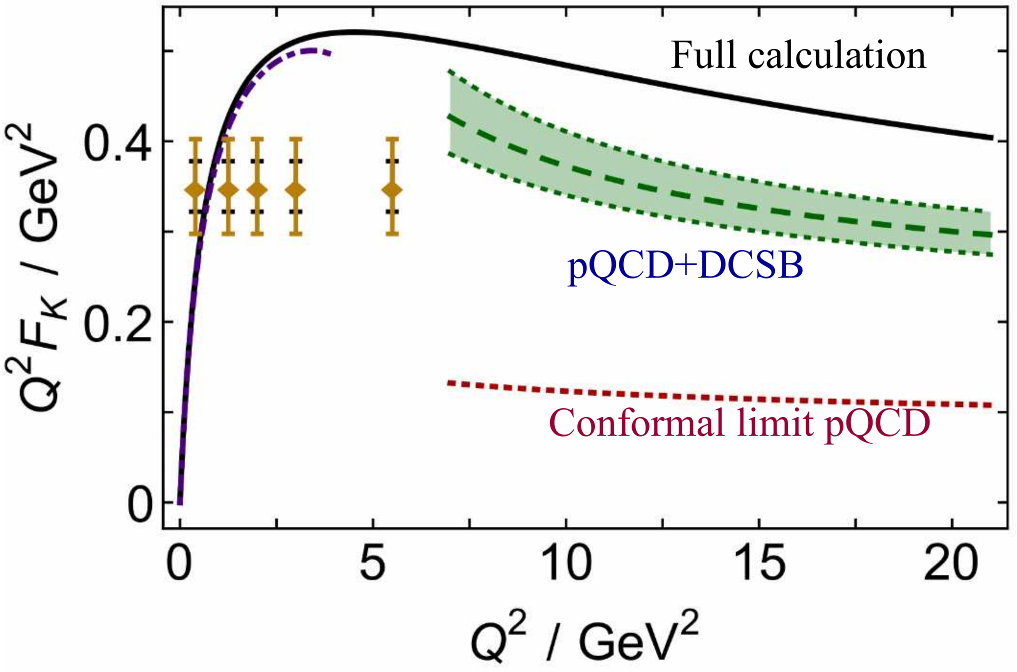
- It is important to compare the magnitudes and Q^2–dependences of both form factors.

K^+ properties also strongly influenced by EHM

- ***K*⁺ PDA also is broad, concave and asymmetric.**
- While the heavier s quark carries more bound state momentum than the u quark, the shift is markedly less than one might naively expect based on the difference of u, s current quark masses.

[C. Shi, et al., PRD **92** (2015) 014035].



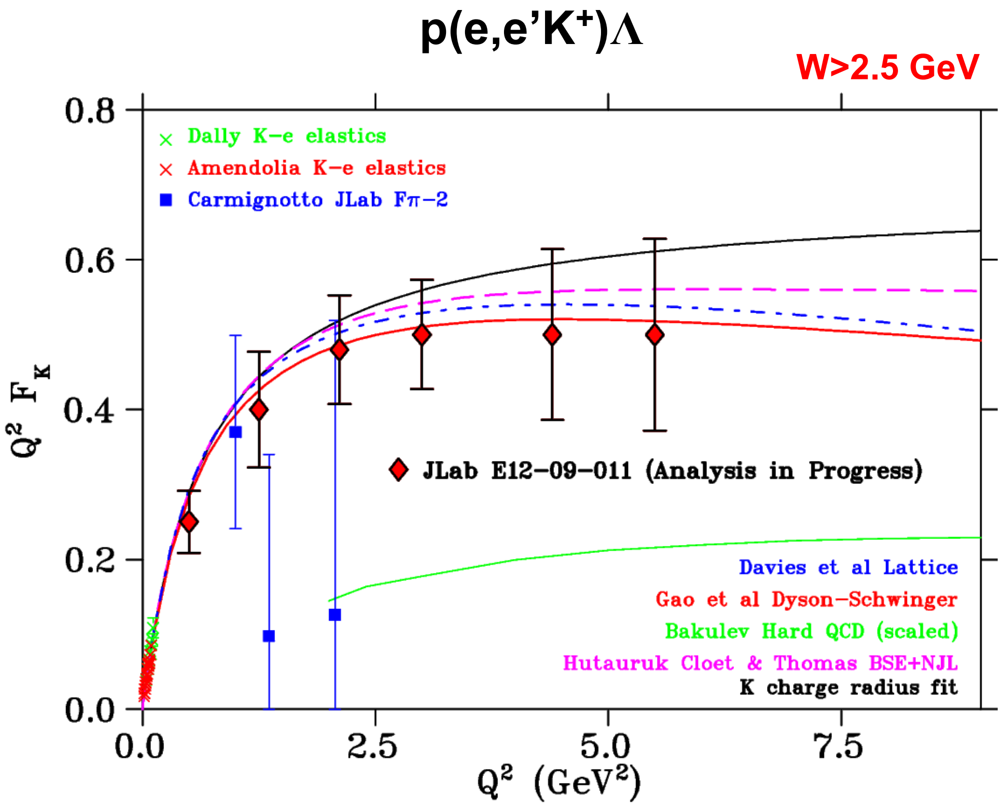


- **F_K DCSB model prediction for JLab kinematics**

[F. Guo, et al., arXiv: 1703.04875].

Projected Uncertainties for K^+ Form Factor

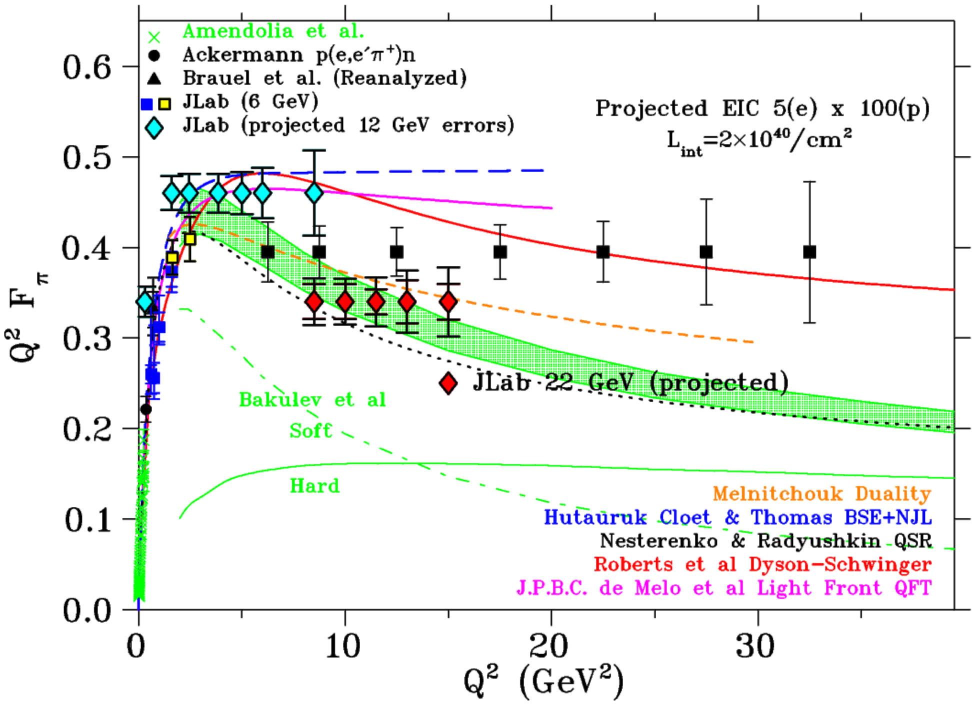
- First measurement of F_K well above the resonance region.
- Measure form factor to Q^2=3 GeV^2 with good overlap with elastic scattering data.
  - Limited by –t<0.2 GeV^2 requirement to minimize non–pole contributions.
- Data will provide an important second $q\overline{q}$ system for theoretical models, this time involving a strange quark.



KaonLT E12–09–011: T. Horn, G. Huber and P. Markowitz, spokespersons

Phase 2 Scenario: π^+ Form Factor

- **Replace HMS with VHMS for π^+, use SHMS for e'**
  - Assume θ_{min}=5.5°, θ_{open}=15.0°
  - VHMS: ΔΩ, ΔP/P similar SHMS
- P_{VHMS}=15.0 GeV/c is sufficient, constrained by max beam energy
- θ_{VHMS}~5.5° allows improved Δε, but does not affect maximum Q^2 reach
- Dramatic increase in upper Q^2 11.5 → 15.0 GeV^2
- Error bars for Q^2=8.5–11.5 GeV^2 substantially decrease due to smaller $-t_{min}$ (better $R=\sigma_T/\sigma_L$) and shorter running times
- Highest Q^2 running time is "expensive" but would have very high scientific priority.



- Extends region of high quality F_π values to Q^2=13 GeV^2
- Somewhat larger errors to Q^2=15 GeV^2
- Provides MUCH improved overlap of F_π data set between JLab and EIC

JLab L–T Separations in the EIC Era

- **Hall C is world's only facility that can do L–T separations over a wide kinematic range**
- The error magnification in L–T separations depends crucially on the achievable difference in the virtual photon polarization parameter, ε.
  - Errors magnify as 1/Δε, where $\Delta\varepsilon=\varepsilon_{High}-\varepsilon_{Low}$
  - To keep the magnification <500%, one desires Δε>0.2
  - This is not feasible at the EIC, as the high ion ring energy constrains ε>0.98
- **As the interpretation of some EIC data (e.g. GPD extraction) will depend on extrapolation of Hall C L–T separated data, maximizing overlap between Hall C and EIC data sets should be a high priority**
  - An important motivation for extending reach of Hall C data using 22 GeV beam

14 GeV/c HMS Scenario: π^+ Form Factor

- **Replace HMS with a higher momentum spectrometer**
  - For high *z* reactions, such as DEMP, usable beam energy constrained by sum of HMS+SHMS maximum momenta
  - i.e. 22 GeV beam energy is a larger constraint than the maximum HMS momentum
- **New HMS would not extend the Q^2 reach beyond Scenario 1.** However, it would result in smaller errors due to larger Δε and faster high ε data rates

$p(e,e'\pi^+)n$ Kinematics

E_{beam}	θ_{HMS} (e')	P_{HMS} (e')	$\theta_{q(SHMS}$ (π^+)	P_{SHMS} (π^+)	Time FOM
Q^2=8.5 W=3.64 $-t_{min}$=0.24 Δε=0.53					
13.0	34.30	1.88	5.29	10.99	64.7
22.0	10.81	10.88	10.23	10.99	0.6
Q^2=10.0 W=3.44 $-t_{min}$=0.37 Δε=0.54					
13.0	37.78	1.83	5.56	10.97	122.7
22.0	11.76	10.83	10.97	10.97	1.3
Q^2=11.5 W=3.24 $-t_{min}$=0.54 Δε=0.29					
14.0	31.73	2.75	7.06	10.96	82.4
22.0	12.66	10.75	11.56	10.96	2.5

- **This scenario is judged to not be worth it, at least for this reaction channel**

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Upgrade HMS Momentum and Angle: F_π

Garth Huber, huberg@uregina.ca

- **Upgrade both HMS momentum and forward angle capabilities**
  - 7 GeV/c → 11 GeV/c
  - θ_{min}=10.50° → 7.5°
  - θ_{open}=18.00° → 15.00°
- **This upgrade also does not extend the Q^2 reach beyond Scenario 1.**
- However, it would result in smaller errors due to larger $\Delta\varepsilon$ and faster high ε data rates

$p(e,e'\pi^+)n$ Kinematics

E_{beam}	θ_{HMS} (e')	P_{HMS} (e')	$\theta_{q(SHMS)}$ (π^+)	P_{SHMS} (π^+)	Time FOM
Q^2=8.5 W=3.64 $-t_{min}$=0.24 $\Delta\varepsilon$=0.53					
13.0	34.30	1.88	5.29	10.99	64.7
22.0	10.81	10.88	10.23	10.99	0.6
Q^2=10.0 W=3.44 $-t_{min}$=0.37 $\Delta\varepsilon$=0.54					
13.0	37.78	1.83	5.56	10.97	122.7
22.0	11.76	10.83	10.97	10.97	1.3
Q^2=11.5 W=3.24 $-t_{min}$=0.54 $\Delta\varepsilon$=0.29					
14.0	31.73	2.75	7.06	10.96	82.4
22.0	12.66	10.75	11.56	10.96	2.5

- **Basically the same as Scenario 2. Not worth it, at least for this channel**

15 GeV/c SHMS Scenario: π^+ Form Factor

University of Regina

- **Replace SHMS with higher momentum spectrometer, but keep HMS as is**
- Dramatic increase in upper Q^2 11.5 → 15.0 GeV^2
- Error bars for Q^2=8.5–11.5 GeV^2 would substantially decrease due to smaller $-t_{min}$ (better $R=\sigma_T/\sigma_L$) and shorter running times
- The Q^2=15.0 GeV^2 point would be "expensive" in terms of running time, but its high scientific priority would make it worthwhile
- **This seems a compelling scenario for a Phase 2 Upgrade**

$p(e,e'\pi^+)n$ Kinematics

E_{beam}	θ_{HMS} (e')	P_{HMS} (e')	$\theta_{q(SHMS)}$ (π^+)	P_{SHMS} (π^+)	Time FOM
Q^2=8.5 W=4.06 $-t_{min}$=0.17 $\Delta\varepsilon$=0.26					
16.0	23.68	3.15	5.52	12.75	17.7
20.0	14.00	7.15	7.55	12.75	1.9
Q^2=10.0 W=3.96 $-t_{min}$=0.23 $\Delta\varepsilon$=0.28					
16.0	27.41	2.78	5.41	13.09	47.7
20.0	15.60	6.78	7.72	13.09	4.5
Q^2=11.5 W=3.96 $-t_{min}$=0.29 $\Delta\varepsilon$=0.27					
17.0	27.54	2.98	5.49	13.86	76.3
21.0	16.10	6.98	7.72	13.86	8.1
Q^2=13.0 W=3.96 $-t_{min}$=0.35 $\Delta\varepsilon$=0.25					
18.0	27.55	3.18	5.54	14.63	123.6
22.0	16.49	7.18	7.69	14.63	14.4
Q^2=15.0 W=3.78 $-t_{min}$=0.50 $\Delta\varepsilon$=0.27					
18.0	31.30	2.86	5.46	14.87	391
22.0	18.14	6.86	7.86	14.87	41.4

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