An Introduction to the Quark Model

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Particles in Atomic Physics

- View of the particle world as of early 20^{th} Century.
- Particles found in atoms:
  - Electron
  - Nucleons:
    - Proton (nucleus of hydrogen)
    - Neutron (e.g. nucleus of helium – α-particle - has two protons and two neutrons)
- Related particle mediating electromagnetic interactions between electrons and protons:
  - Photon (light!)

Particle	*Electric charge (x 1.6 10^{-19} C)*	*Mass (GeV=x 1.86 10^{-27} kg)*
e	−1	0.0005
p	+1	0.938
n	0	0.940
γ	0	0

Early Evidence for Nucleon Internal Structure

- Apply the Correspondence Principle to the Classical relation for magnetic moment:

$$\mu = \frac{q}{2m} L$$

- Obtain for a point-like spin-½ particle of mass m_p:

$$\mu = \frac{q}{2m_p}\left(\frac{\hbar}{2}\right) = \frac{q}{2e}\left(\frac{e\hbar}{2m_p}\right) = \frac{q}{2e}\mu_N$$

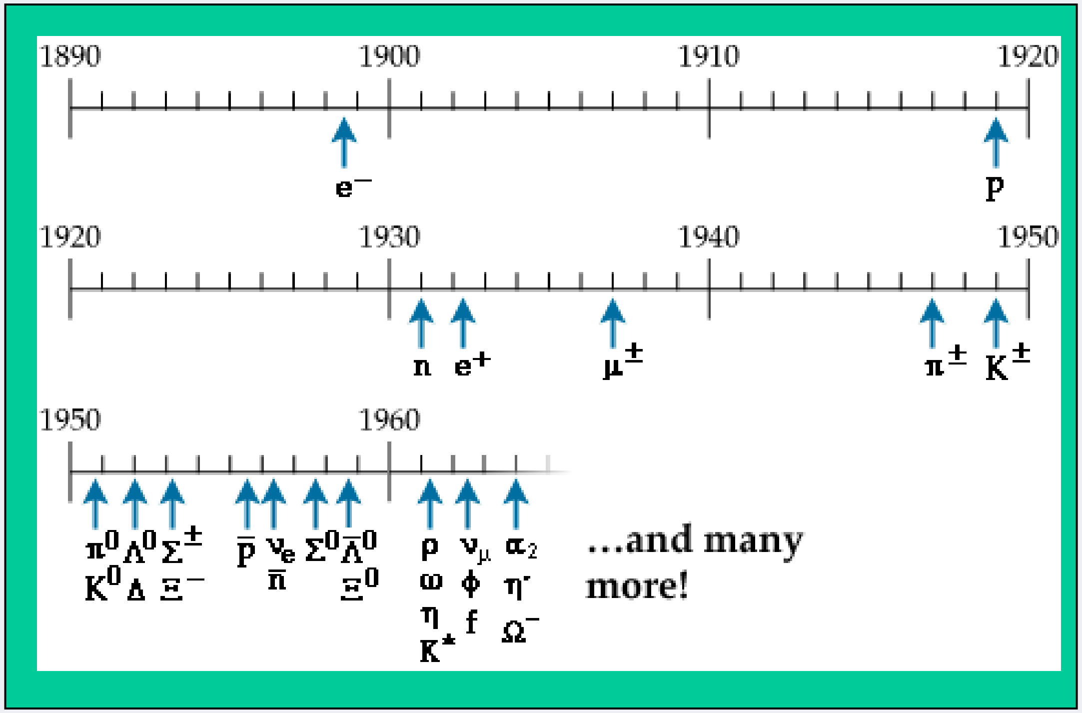
Experimental values:	$\mu_p = 2.79\ \mu_N$ (p)
	$\mu_n = -1.91\ \mu_N$ (n)

- Experimental values inconsistent with point-like assumption.
- In particular, the neutron's magnetic moment does not vanish, as expected for a point-like electrically neutral particle.

This is unequivocal evidence that the neutron (and proton) has an internal structure involving a distribution of charges.

The Particle Zoo

- Circa 1950, the first particle accelerators began to uncover many new particles.
- Most of these particles are unstable and decay very quickly, and hence had not been seen in cosmic ray experiments.
- Could all these particles be fundamental?



Over the years inquiring minds have asked:

"Can we describe the known physics with just a few building blocks ?"

Historically the answer has been yes.

⇒ Elements of Mendeleev's Periodic Table (chemistry).

⇒ Nucleus of atom made of protons, neutrons.

⇒ *p* and *n* really same "particle - NUCLEON" (different isotopic spin).

By 1950's there was evidence for many new particles beyond γ, e, p, n

It was realized that even these new particles fit certain patterns:

pions:	π^+(140 MeV)	π^-(140 MeV)	π^o(135 MeV)
kaons:	K^+(496 MeV)	K^-(496 MeV)	K^o(498 MeV)

Some sort of pattern was emerging, but .... lots of questions.

⇒ **If mass difference between proton neutrons, pions, and kaons is due to electromagnetism then how come:**

$M_n > M_p$ and $M_{k^o} > M_{k^+}$ *but* $M_{\pi^+} > M_{\pi^o}$

Lots of models concocted to try to explain why these particles exist:

⇒ Model of Fermi and Yang (late 1940's-early 50's):

pion is composed of nucleons and anti-nucleons.

$$\pi^+ = p\bar{n}, \quad \pi^- = n\bar{p}, \quad \pi^o = p\bar{p} - n\bar{n}$$

Note: this model was proposed before discovery of anti-proton !

Regularities observed among particles

	Q = -1	***Q = 0***	***Q = +1***
S = +1		K^0	K^+
S = 0	π^+	π^0, η	π^+
S = -1	K^+	K^0	

Q = Electric Charge

Spin 0 Meson Octet

	Q= -1	***Q= 0***	***Q=+1***
S= 0		n	p
S= -1	Σ^-	Σ^0, Λ	Σ^+
S= -2	Ξ^+	Ξ^0	

Spin 1/2 Baryon Octet

	Q = -1	***Q = 0***	***Q = +1***	***Q = +2***
S = 0	Δ^-	Δ^0	Δ^+	Δ^{++}
S = -1	Σ^{*-}	Σ^{*0}	Σ^{*+}	
S = -2	Ξ^{*-}	Ξ^{*0}		
S = -3	Ω^-			

Spin 3/2 Baryon Decuplet

S = Strangeness Quantum Number

Similar masses in each multiplet.

Quark hypothesis

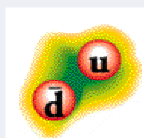
- **Quark model of hadrons** **Gell-Mann, Zweig 1964**

	$Q = -1/3$	$Q = +2/3$
$S = 0$	d	u
$S = -1$	s	

$m_d \cong m_u \cong 0.1$ GeV

$m_s \cong 0.30$ GeV

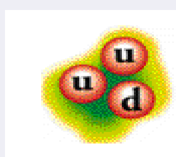
Mesons are bound states of quark-antiquark.



π^+

$\pi^+ = u\bar{d}$, $\pi^- = d\bar{u}$, $\pi^o = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$, $k^+ = d\bar{s}$, $k^o = d\bar{s}$

Baryons are bound states of three quarks.



p

proton = (uud), neutron = (udd), Λ= (uds)

anti-baryons are bound states of 3 anti-quarks:

$\bar{p} = \bar{u}\bar{u}\bar{d}$ $\quad \bar{n} = \bar{u}\bar{d}\bar{d}$ $\quad \bar{\Lambda} = \bar{u}\bar{d}\bar{s}$

These quark objects are:

- point-like.
- spin 1/2 fermions.
- parity = +1 (-1 for anti-quarks).
- two quarks are in isospin doublet (u and d), s is an iso-singlet (=0).
- For every quark there is an anti-quark.
- Quarks feel all interactions (have mass, electric charge, etc).

How do we "construct" baryons and mesons from quarks?

Use SU(3) as the group (1960's model)

This group has 8 generators (n^2-1, n=3).

Each generator is a 3x3 linearly independent traceless hermitian matrix.

Only 2 of the generators are diagonal $\Rightarrow$ 2 quantum numbers.

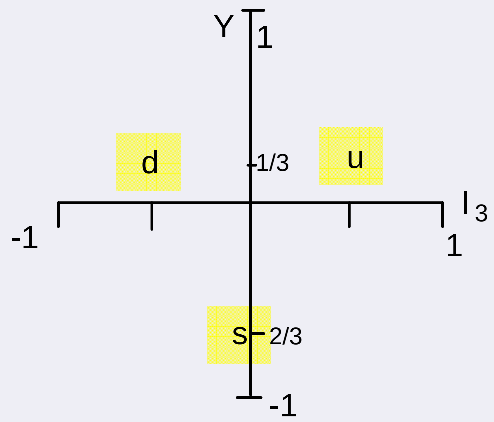
Hypercharge = Strangeness + Baryon number = Y

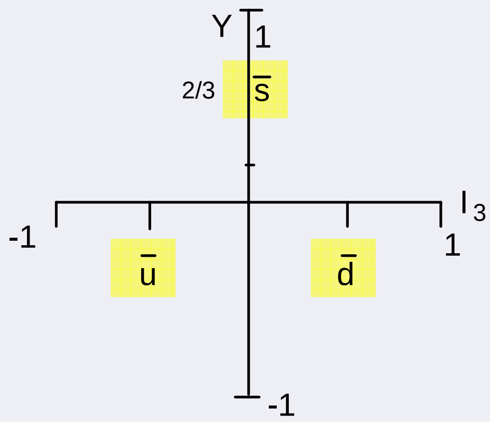
Isospin (I_3)

In this model (1960's) there are 3 quarks, which are the eigenvectors (3 row column vector) of the two diagonal generators (Y and I_3).

Quarks are added together to form mesons and baryons using SU(3) group.

It is interesting to plot Y vs. I_3 for quarks and anti-quarks:





Quarks obey:

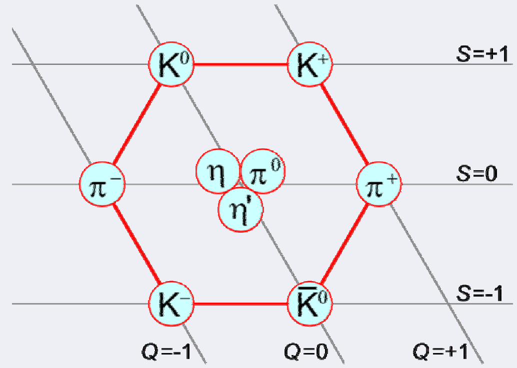
$Q = I_3 + 1/2(S+B) = I_3 + Y/2$

Making Mesons and Baryons with Quarks

Making Mesons (orbital angular momentum L=0)

The properties of SU(3) tell us how many mesons to expect: $3 \otimes \overline{3} = 1 \oplus 8$

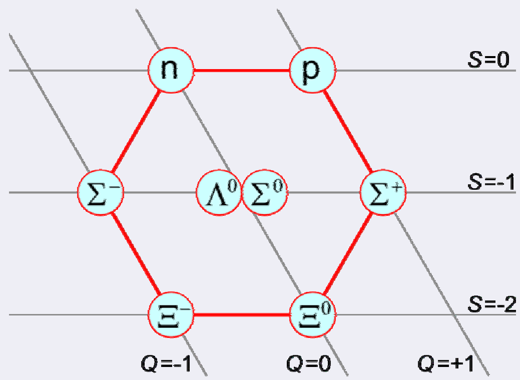
Thus we expect an octet with 8 particles and a singlet with 1 particle.

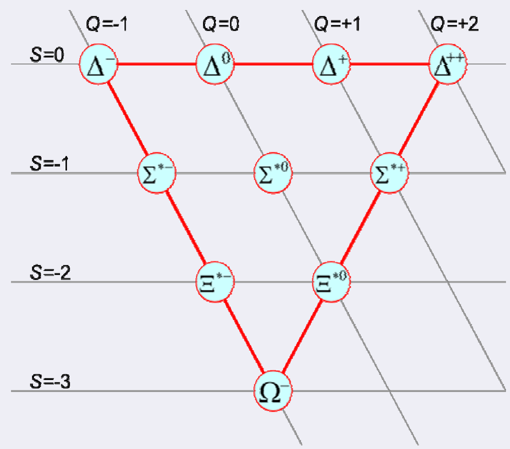


Making Baryons (orbital angular momentum L=0).

Now must combine 3 quarks together: $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$

Expect a singlet, 2 octets, and a decuplet (10 particles)
$\Rightarrow$ 27 objects total.





If SU(3) were a perfect symmetry then all particles in a multiplet would have the same mass.

Static Quark Model

Quantum #	u	d	s	c	b	t
electric charge	2/3	-1/3	-1/3	2/3	-1/3	2/3
Isospin I_3	1/2	-1/2	0	0	0	0
Strangeness	0	0	-1	0	0	0
Charm	0	0	0	1	0	0
Bottom	0	0	0	0	-1	0
Top	0	0	0	0	0	1
Baryon number	1/3	1/3	1/3	1/3	1/3	1/3
Lepton number	0	0	0	0	0	0

Successes of 1960's Quark Model:

- Classify all known (in the early 1960's) particles in terms of 3 building blocks.
- predict new hadrons (e.g. Ω^-).
- explain why certain particles don't exist (e.g. baryons with S = +1).
- explain mass splitting between meson and baryons.
- explain/predict magnetic moments of mesons and baryons.
- explain/predict scattering cross sections (e.g. $\sigma_{\pi p}/\sigma_{pp} = 2/3$).

Failures of the 1960's model:

- No evidence for free quarks (fixed up by QCD)
- Pauli principle violated (Δ^{++}= uuu wavefunction is totally symmetric) (fixed up by color)
- What holds quarks together in a proton ? (gluons!)
- How many different types of quarks exist ? (6?)

The Need for a "Strong Force"

Why do protons stay together in the nucleus, despite the fact that they have the same electric charge?

➔ **They should repel since they are like charge.**

Why do protons and neutrons in the nucleus bind together?

➔ **Since the neutron is electrically neutral, there should be no EM binding between protons and neutrons.**

- ❑ **Inside the nucleus, the attractive strong force is stronger than the repulsive electromagnetic force.**
- ❑ **Protons and neutrons both "experience" the strong force.**
- ❑ **The actual binding that occurs between *proton-proton* and *proton-neutron* is the residual of the strong interaction.**

Search for a Theory of Strong Interactions

- By the 1960's, Feynman *et al*, had fully developed a "quantum" theory which accounted for all EM phenomenon. This theory is called ***Quantum Electrodynamics*** (or ***QED*** for short).

- Because of this *remarkable success*, scientists developed an analogous theory to describe the strong interaction. It is called ***Quantum Chromodynamics*** (or ***QCD*** for short).

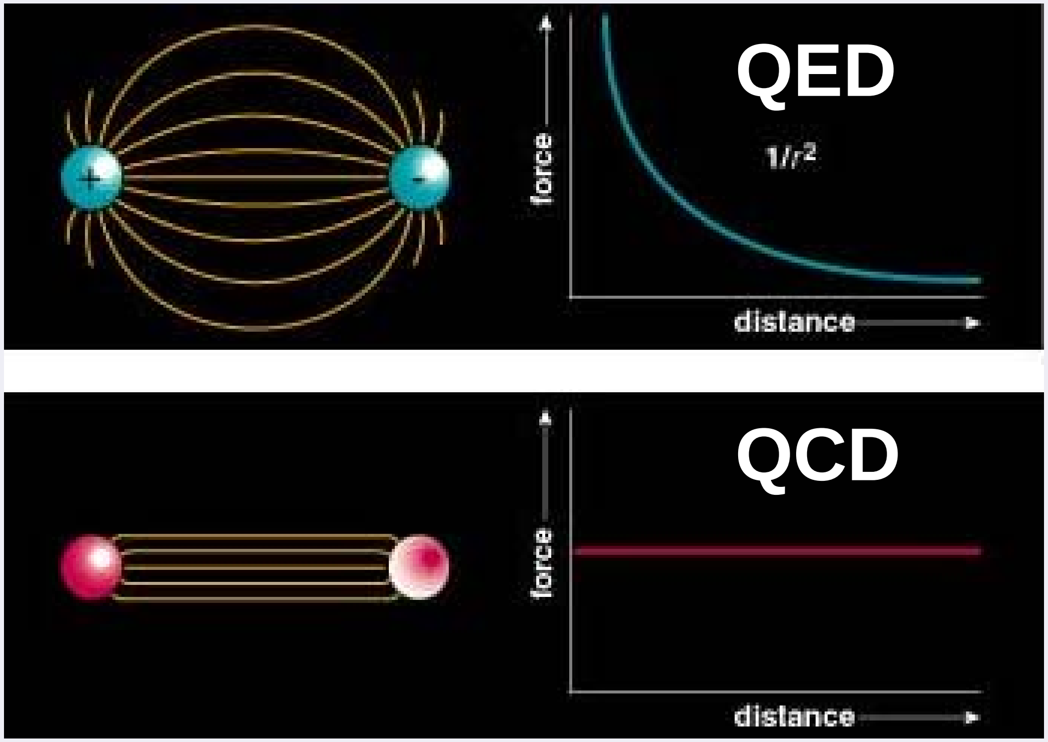
- Scientists conjectured that, like the EM force, there is also a ***quantum of the strong force***, and called it the **gluon**.

Quantum Electrodynamics

vs.

Quantum Chromodynamics

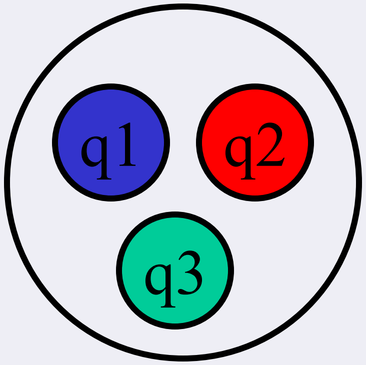
The gluons of QCD carry color charge and interact strongly (in contrast to the photons of QED).



Strong and EM Force Comparison

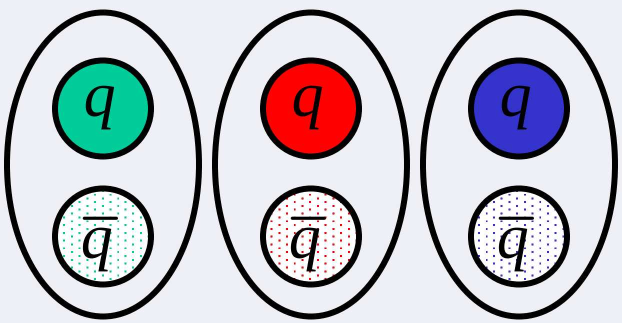
Property	EM	Strong
Force Carrier	Photon (γ)	Gluon (g)
Mass	0	0
Charge ?	None	Yes, color charge.
Charge types	+, -	red, green, blue
Couples to:	All objects with electrical charge.	All objects with color charge.
Range	Infinite ($1/d^2$)	$\lesssim 10^{-15}$ [m] (inside hadrons)

Color of Hadrons



BARYONS

RED + BLUE + GREEN = "WHITE"
or "COLORLESS"



MESONS

GREEN + ANTIGREEN = "COLORLESS"
RED + ANTIRED = "COLORLESS"
BLUE + ANTIBLUE = "COLORLESS"

Color of Gluons

$r\bar{b}$

$r\bar{g}$

$b\bar{g}$

$b\bar{r}$

$g\bar{b}$

$g\bar{r}$

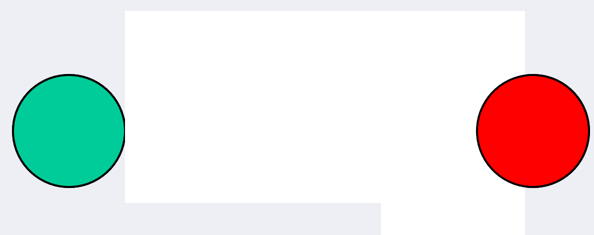
$r\bar{r} + g\bar{g} - 2b\bar{b}$

$r\bar{r} - g\bar{g}$

Don't worry about what this means.

Each of the 8 color combinations have a "color" and an "anti-color".

When quarks interact, they **"exchange" color charge.**

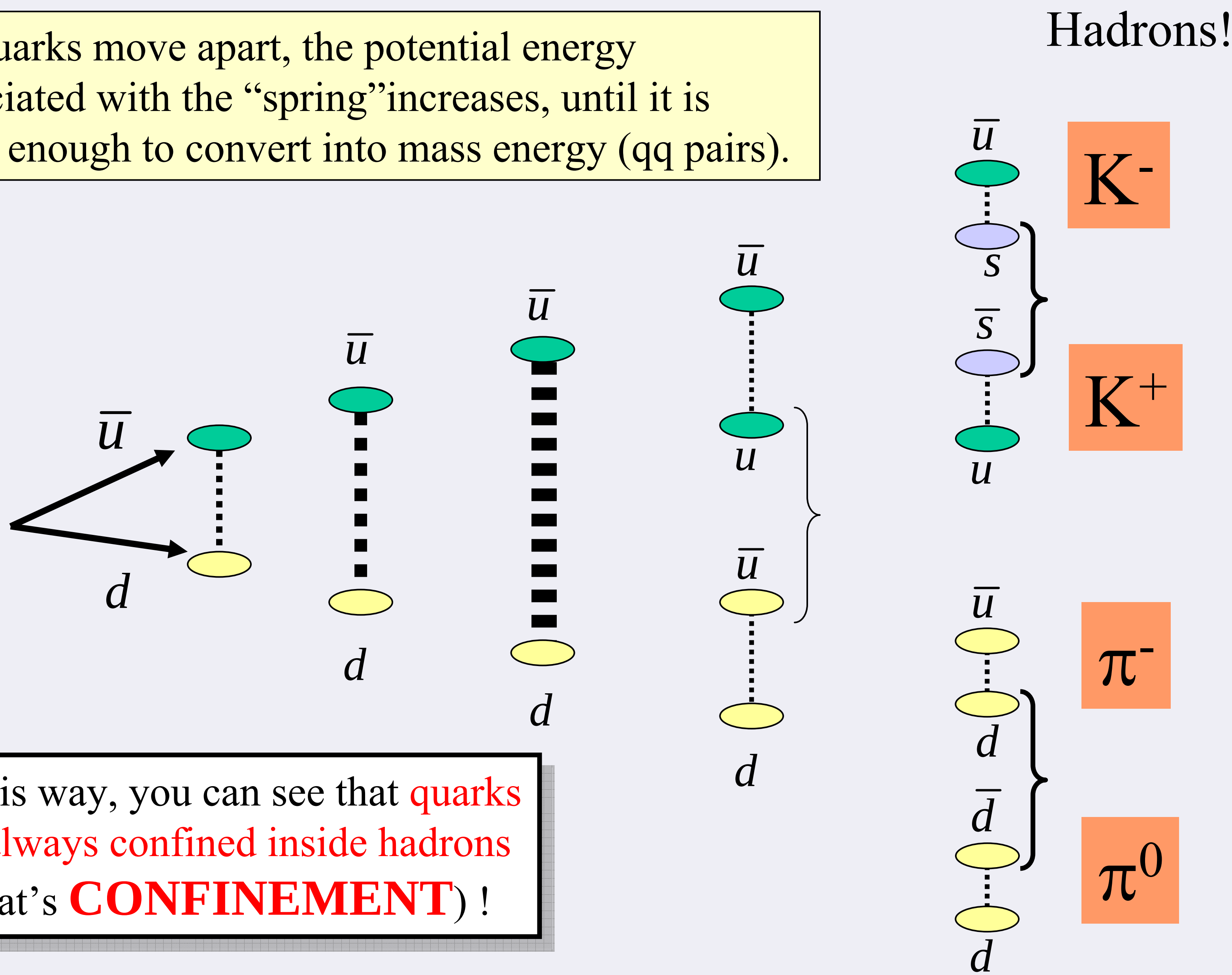


Quark 1

Quark 2

Confinement

As quarks move apart, the potential energy associated with the "spring"increases, until it is large enough to convert into mass energy (qq pairs).



In this way, you can see that quarks are always confined inside hadrons (that's **CONFINEMENT**) !

Dynamic Quark Model

Dynamic Quark Model (mid 70's to now!)

- Theory of quark-quark interaction ⇒ QCD.
- includes gluons.

Successes of QCD:

- "Real" Field Theory i.e.
  - Gluons instead of Photons.
  - Color Charge instead of Electric Charge.
- explains why no free quarks ⇒ confinement of quarks.
- calculate cross sections, e.g. $e^+e^- \rightarrow q\bar{q}$
- calculate lifetimes of baryons, mesons.

Failures/problems of the model:

- Hard to do calculations in QCD (non-perturbative).
- Polarization of hadrons (e.g. Λ's) in high energy collisions.
- How many quark-antiquark pairs are there?

Food for thought

Recall: Mass of Proton **~ 938 [MeV/c^2]**

Proton constituents:

2 up quarks: 2 * (3 [MeV/c^2]) = 6 [MeV/c^2]

1 down quark: 1 * 6 [MeV/c^2] = 6 [MeV/c^2]

Total quark mass in proton: ~ 12 [MeV/c^2]

Where does the proton's mass come from ?????

It's incorporated in the binding energy associated with the gluons !

➔~99% of our mass comes from quark-gluon interactions in the nucleon which are still poorly understood!

Physics Problems for the Next Millennium

Selected by:

Michael Duff, David Gross, Edward Witten

Strings 2000

1. Size of dimensionless parameters.
2. Origin of the Universe.
3. Lifetime of the Proton.
4. Is Nature Supersymmetric?
5. Why is there 3+1 Space-time dimensions?
6. Cosmological Constant problem.
7. Is M-theory fundamental?
8. Black Hole Information Paradox.
9. The weakness of gravity.
10. **Quark confinement and the strong force.**

Experimental Evidence

"Particles, particles, particles."

- Energetic particles provide a way to probe, or "see" matter at small distance scales. (e.g.Electron microscope).

- Electron accelerators produce energetic beams which allow us to probe matter at its most fundamental level.

- As we go to higher energy particle collisions:
  1) Wavelength probe is smaller ➔ **see finer detail.**
  2) Can produce more **massive objects**, via **$E=mc^2$.**

Are Quarks really inside the Proton?

Try to look inside a proton (or neutron) by shooting high energy electrons at it and see how they scatter.

Review of scatterings and differential cross section.

The cross section (σ) gives the probability for a scattering to occur.

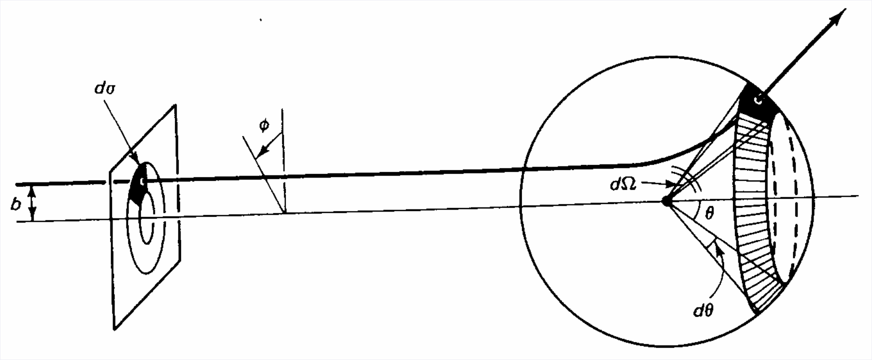
unit of cross section is area (barn=10^{-24} cm^2)

differential cross section= $d\sigma/d\Omega$

number of scatters into a given amount of solid angle: $d\Omega = d\phi\, d\cos\theta$

Total amount of solid angle (Ω):

$$\int_{-1}^{+1}\int_{0}^{2\pi} d\Omega = \int_{-1}^{+1} d\cos\theta \int_{0}^{2\pi} d\phi = 4\pi$$



Cross section (σ) and Impact parameter (b) and relationship between dσ and db:

$d\sigma = |b\,db\,d\phi|$

Solid angle: $d\Omega = |\sin\theta\, d\theta\, d\phi|$

Examples of scattering cross sections

Rutherford Scattering:

A spin-less, point particle with initial kinetic energy E and electric charge e scatters off a stationary point-like target with electric charge also=e:

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4E\sin^2(\theta/2)} \right)^2$$

note: $\sigma = \infty$ which is not too surprising since the Coulomb force is long range.

This formula can be derived using either classical mechanics or non-relativistic QM. The quantum mechanics treatment usually uses the Born Approximation:

$$\frac{d\sigma}{d\Omega} \propto \left| f(q^2) \right|^2$$

with $f(q^2)$ given by the Fourier transform of the scattering potential V:

$$f(q^2) = \int e^{i\vec{q}\bullet\vec{r}} V(\vec{r}) d\vec{r}$$

Now the Projectile has Spin 1/2

Mott Scattering: A relativistic spin 1/2 point particle with mass m, initial momentum **p** and electric charge e scatters off a stationary point-like target with electric charge e:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar}{2p^2\sin^2(\theta/2)}\right)^2 \left[(mc)^2 + p^2\cos^2\frac{\theta}{2}\right]$$

In the high energy limit $p \gg mc^2$ and $E \approx p$ we have:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar\cos(\theta/2)}{2E\sin^2(\theta/2)}\right)^2$$

"Dirac" proton: The scattering of a relativistic electron with initial energy E and final energy E' by a heavy point-like spin 1/2 particle with finite mass M and electric charge e is:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar\cos(\theta/2)}{2E\sin^2(\theta/2)}\right)^2 \frac{E'}{E}\left[1 - \frac{q^2}{2M^2}\tan^2(\theta/2)\right]$$

scattering with recoil, neglect mass of electron, $E \gg m_e$.

q^2 is the electron four momentum transfer: $(p'-p)^2 = -4EE'\sin^2(\theta/2)$

The final electron energy E' depends on the scattering angle θ:

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2(\theta/2)}$$

What if the target is not point-like?

What happens if we don't have a point-like target,

→ i.e. there is some structure inside the target?

The most common example is when the electric charge is spread out over space and is not just a "point" charge.

Example: Scattering off of a charge distribution. The Rutherford cross section is modified to be:

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4E\sin^2(\theta/2)} \right)^2 \left| F(q^2) \right|^2 \qquad \text{with: E=E}' \text{ and } q^2 = -4E\sin^2(\theta/2)$$

The new term |F(q²)| is often called the form factor.

The form factor is related to Fourier transform of the charge distribution ρ(r) by:

$$F(q^2) = \int \rho(r) e^{i\vec{q}\bullet\vec{r}} d^3r \qquad \text{usually} \int \rho(r) d^3r = 1$$

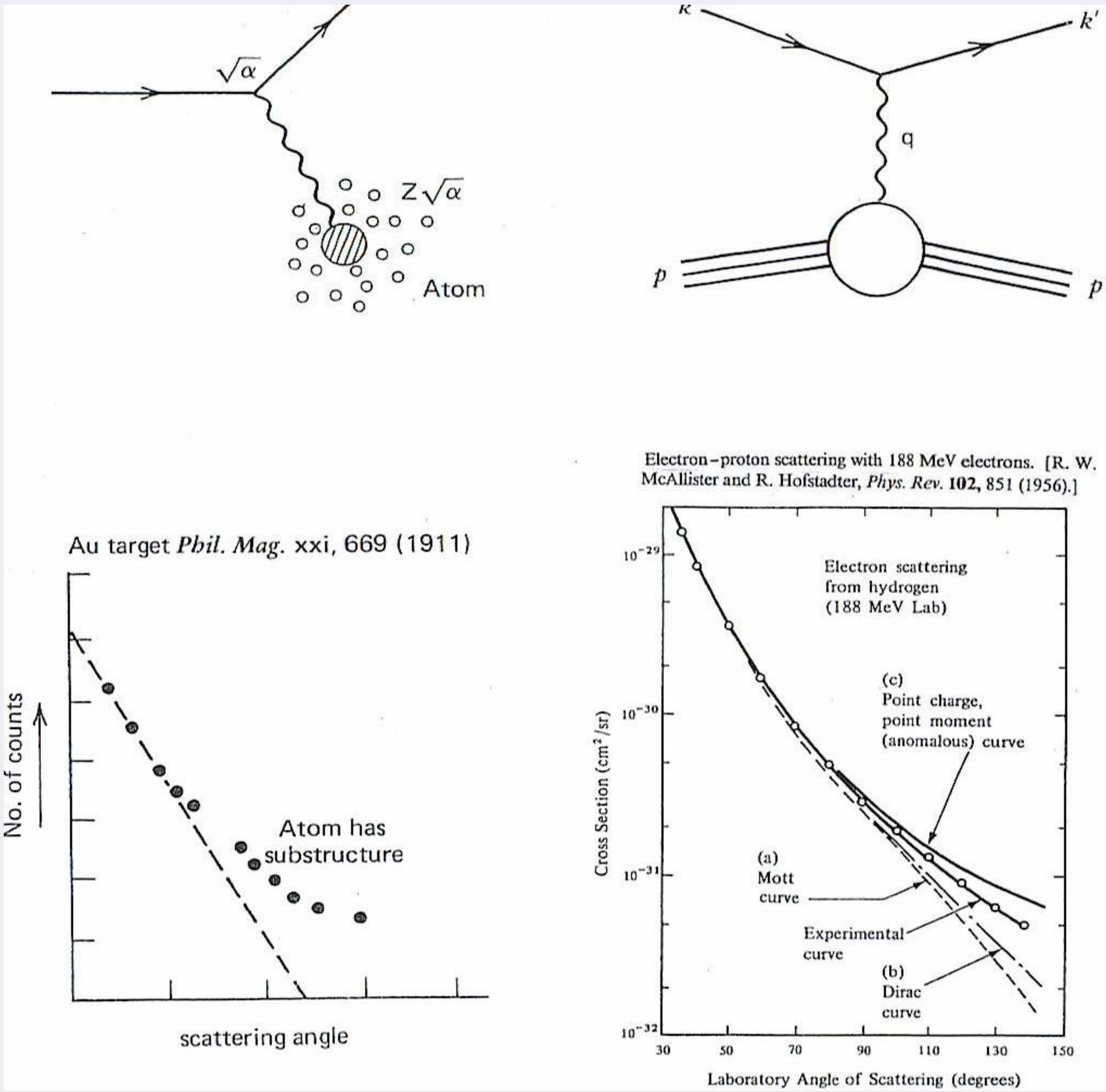
In this simple model we could learn about an unknown charge distribution (structure) by measuring how many scatters occur in an angular region and comparing this measurement with what is expected for a "point charge" $|F(q^2)|^2=1$ (what's the charge distribution here?) and our favorite theoretical mode of the charge distribution.

Rutherford Scattering

Proton Form Factor

Scattering of α particle by atom

Scattering of electron by proton



Pioneering Proton Form Factor Measurements

The charge radius of the proton was first measured via an extensive experimental program of electron nucleon (e.g. proton, neutron) scattering carried out by Hofstadter (Nobel Prize 1961) and collaborators at Stanford.

Note:
$$F(q^2) = \int \rho(r) e^{i\vec{q}\bullet\vec{r}} d^3r \approx \int \rho(r) \left\{ 1 + i\vec{q}\bullet\vec{r} - \frac{(\vec{q}\bullet\vec{r})^2}{2!} + + + \right\} d^3r$$

For a <u>spherically</u> symmetric charge distribution we have:
$$F(q^2) \approx 1 - \frac{q^2}{6}\int r^2 \rho(r) dr = 1 - \frac{q^2}{6}\left\langle r^2 \right\rangle$$

Hofstadter et al. measured the root mean square radii of the proton charge to be:
$$\left\langle r^2 \right\rangle^{1/2}_{\text{charge}} = (0.74 \pm 0.24) \times 10^{-15} m$$

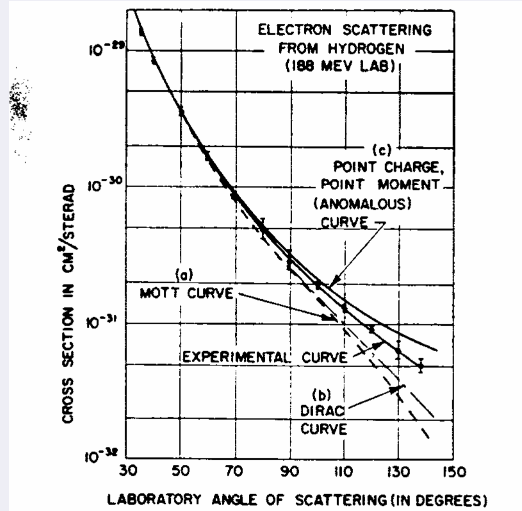
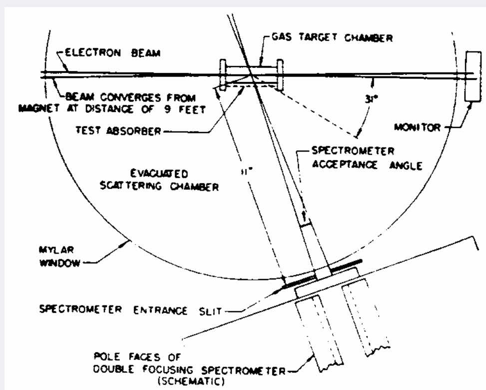


FIG. 5. Curve (a) shows the theoretical Mott curve for a spinless point proton. Curve (b) shows the theoretical curve for a point proton with the Dirac magnetic moment, curve (c) the theoretical curve for a point proton having the anomalous contribution in addition to the Dirac value of magnetic moment. The theoretical curves (b) and (c) are due to Rosenbluth.[8] The experimental curve falls between curves (b) and (c). This deviation from the theoretical curves represents the effect of a form factor for the proton and indicates structure within the proton, or alternatively, a breakdown of the Coulomb law. The best fit indicates a size of 0.70×10^{-13} cm.

McAllister and Hofstadter,
PR, V102, May 1, 1956.
Scattering of 188 MeV electrons from protons and helium.



Inelastic Electron-Proton Scattering

Hofstadter's experiment is an example of an <u>elastic scattering</u>. In an elastic scattering we have the same kind and number of particles in the initial and final states.

$$e+p \rightarrow e+p$$

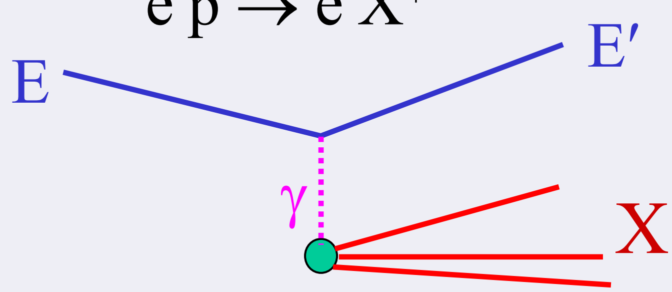
In an <u>inelastic collision</u> there are "new" particles in the final state.

Examples of inelastic e^-p scatterings include:

$e^-p \rightarrow e^-p\pi^o$ $e^-p \rightarrow e^-\Delta^+$ $e^-p \rightarrow e^-pK^+K^-$

Since there are many inelastic final states it is convenient to define a quantity called the <u>inclusive cross section</u>. Here we are interested in the reaction:

$$e^-p \rightarrow e^-X^+$$



- Called an <u>inclusive</u> reaction because don't measure any of the properties of "X", hence <u>include</u> all available final states.
- Nucleon substructure information via Structure Functions (instead of Form Factors).

What if spin ½ point-like objects are inside the proton?

- As Q^2 increases, wavelength of the virtual photon decreases
  → at some point should be able to see "inside" the proton.
- Analyses made by many theorists (Bjorken, Feynman, Callan and Gross) for spin-1/2 point-like objects inside nucleon.
- Predictions quickly verified by new generation of electron scattering experiments.

Write the scattering cross section in terms of F_1 and F_2:

$$\frac{d\sigma}{dE' d\Omega} = \left(\frac{\alpha \hbar}{2E \sin^2(\theta/2)} \right)^2 \left[\frac{2F_1(x,Q^2)}{M} \sin^2(\theta/2) + \frac{2MxF_2(x,Q^2)}{Q^2} \cos^2(\theta/2) \right]$$

Where the *scaling* variable x:

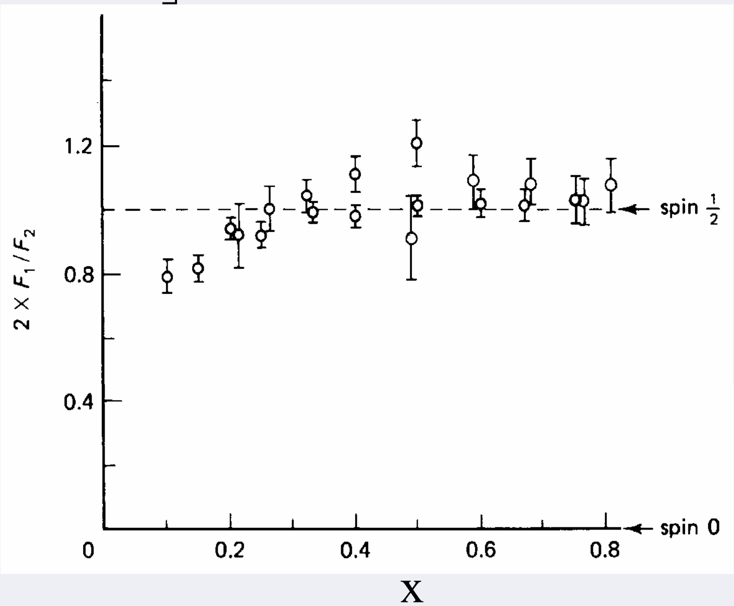
$$x = Q^2/2pq$$

If there are point-like spin ½ objects inside the proton, Callan and Gross predicted that the two structure functions would be related:

$$2xF_1(x)=F_2(x)$$

If however the objects had spin 0, then:

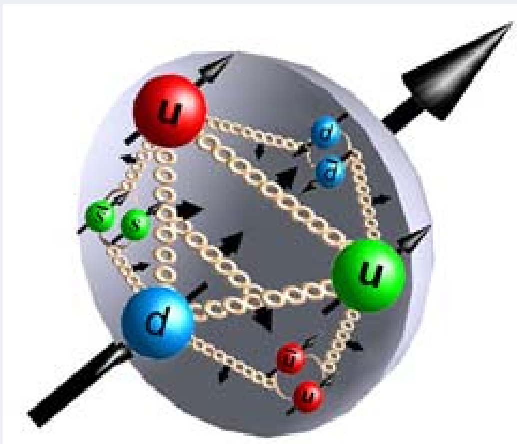
$$2xF_1(x)/F_2(x)=0$$

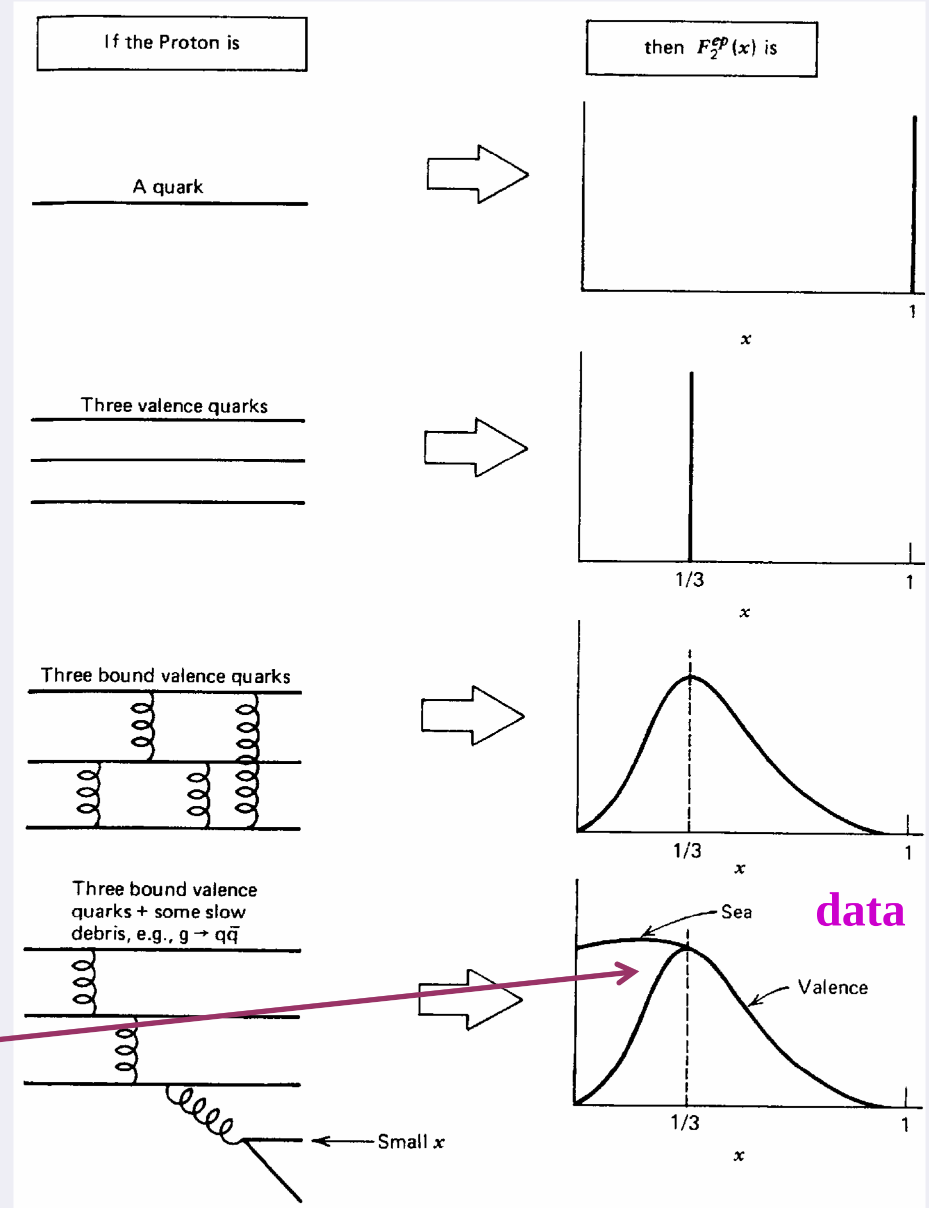


Good agreement with spin ½ point-like objects inside proton!

Quark Momentum Distributions within Proton

- *x* represents fraction of proton momentum carried by struck parton (quark).
- Quarks inside proton have probability (*P*) distribution (*f(x)=dP/dx*) to have momentum fraction *x*.





VALENCE QUARKS: qqq required for correct proton quantum numbers.
SEA QUARKS: virtual q:anti-q pairs allowed by uncertainty principle.

Summary (I)

- The property which gives rise to the strong force is "color charge".
- There are 3 types of color charges, RED, GREEN and BLUE.
- Quarks have color charge, and interact via the mediator of the strong force, the **gluon**.
- The gluon is **massless** like the photon, but differs dramatically in that:
  - **It has color charge.**
  - It's force acts over a very **short range** (inside the nucleus).

Summary (II)

- ❑ **Quarks and gluons are confined inside hadrons** because of the nature of the strong force.

- ❑ Only ~1% of the proton's rest mass is due to valence quark masses. ~99% is due to quark-gluon interactions.

- ❑ We learn about the internal structure of the proton by performing electron scattering experiments.

- ❑ The structure of the proton is a very rich field and there is much we still do not understand.

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